EE 538B CMOS RF IC DESIGN

Midterm Examination No. 2: May 15, 2002

Time Allowed: 110 Minutes

Student Name: Solutions

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You are allowed two sheets of notes. Write legibly. Show all work. State assumptions.

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\[100\]
1. (20 points) Find the noise factor of the circuit shown with respect to the source resistance $R_s$. Neglect channel length modulation effects and all parasitic capacitors. Consider only the drain current noise component of $M_1$.

\[ V_s^2 = 4kT R_s \Delta f \]

\[ i_d^2 = 4kT \delta g_{do} \Delta f \]

\[ i_d^2 = \frac{4kT}{R_d} \]

The three noise sources are uncorrelated:

(i) Consider $i_d^2$:

\[ i_d^2 = \frac{4kT}{R_d} \Delta f \]

(ii) Consider $i_{ind}^2$:

\[ i_{ind}^2 = 4kT \delta g_{do} \Delta f \]

(iii) Consider $V_s^2$:

\[ V_s = \frac{S_L}{S_L + R_s} \]

\[ V_s = \frac{S_L / R_s}{S_L / R_s + 1} \]

\[ V_{gs}^2 = \frac{(WL/R_s)^2}{(WL/R_s)^2 + 1} \]

\[ \frac{1 \Delta f}{L_3} = g_m V_{gs} = \frac{g_m (WL/R_s)^2}{(WL/R_s)^2 + 1} \]

\[ F = \frac{i_d^2 + i_{ind}^2 + i_s^2}{i_d^2} = 1 + \frac{4kT}{R_0} \Delta f + \frac{4kT \delta g_{do} \Delta f}{g_m (WL/R_s)^2} \]

\[ = 1 + \frac{[1 + (WL/R_s)^2]}{g_m R_s R_0 (WL/R_s)^2} + \alpha \frac{[1 + (WL/R_s)^2]}{R_m (WL/R_s)^2} \]

Where $\alpha = \frac{g_m}{g_m (WL/R_s)^2}$.
2. (30 points) Assume that M₁ and M₂ switch completely and instantaneously at the zero crossings of the RF signal that has a 50% duty cycle. The LO signal is \( V_{\text{LO}} \cos \omega_{\text{LO}} t \) and the tail current is \( I_{\text{DC}} + I_{\text{RF}} \cos \omega_{\text{RF}} t \). For simplicity, let \( \omega_{\text{LO}} = 10 \) and \( \omega_{\text{RF}} = 9 \).

Determine and plot on the graph below the peak voltage frequency components in \( V_{\text{out}} \). Carefully label both the amplitude and frequency of each component. (Neglect switching frequency components higher than the third harmonic)

![Graph showing frequency components]

Let's assume that \( V_{\text{LO}} \) is large enough to fully switch M₁-M₂, \( V_{\text{OUT}} \).

Since the circuit is unbalanced, either all or none of the tail current is passed through \( R \) to create \( V_{\text{RF}} \). Hence, the appropriate switch waveform is at \( V_{\text{LO}} \) is:

\( 0 \quad M_1 \text{ on} \quad M_1 \text{ off} \)

\( \quad T_{\text{LO}} \quad \rightarrow \)

The Fourier series of this switching waveform is:

\[
V(t) = \frac{1}{2} + \sum_{n=1}^{\infty} a_n \cos n \omega_{\text{LO}} t
\]

\[a_n = \begin{cases} \frac{2}{\pi n} & n = 1 \\ 0 & n > 1 \end{cases} \]

DC Component of switching waveform

where \( a_n = \frac{\sin n \pi / 2}{n \pi / 2} \)
2. (cont—blank work page)

\[ V_{IF} = \left[ \frac{1}{2} + \frac{2}{\pi} \cos \omega_{LO} - \frac{2}{3\pi} \cos 3\omega_{LO} \right] R \left[ I_{DC} + I_{RF} \cos \omega_{RF} t \right] \]

\[ = \left[ \frac{I_{DC} R}{2} \right] + \left[ \frac{I_{RF} R}{2} \cos \omega_{RF} t \right] + \left[ \frac{2I_{DC} R}{\pi} \cos \omega_{LO} t \right] + \left[ \frac{2I_{RF} R}{\pi} \cos \omega_{LO} t \cos \omega_{RF} t \right] - \left[ \frac{2I_{RF} R}{3\pi} \cos 3\omega_{LO} t \cos \omega_{RF} t \right] \]

**Interpretation of terms:** (Plotted on graph on previous page.)

1. \( \frac{I_{DC} R}{2} \rightarrow \text{DC term at output} \)
2. \( \frac{I_{RF} R}{2} \cos \omega_{RF} t \rightarrow \text{RF feedthrough term at output—} \)
   \( \text{Not present in balanced version of circuit} \)
3. \( \frac{2I_{DC} R}{\pi} \cos \omega_{LO} t \rightarrow \text{LO feedthrough term at output} \)
4. \( \frac{I_{RF} R}{\pi} \left[ \cos (\omega_{LO} + \omega_{RF}) t + \cos (\omega_{LO} - \omega_{RF}) t \right] \)
   \( \text{Upconverted term} \)cerpted from a document. It seems to be an excerpt from a problem or question related to electrical engineering, specifically dealing with signal processing or modulation. The expression involves terms that include direct current (DC) components, radio frequency (RF) components, and local oscillator (LO) components, with various trigonometric functions. The notation and terms suggest a focus on analyzing the output voltage of a system involving these components. The interpretation of terms includes a graphical representation on a previous page. The document is part of a midterm exam from the course EE 538B, Midterm No. 2 – Spring 2002.
3. (20 points) An amplifier with NF₁ = 6dB, power gain $A_{P₁} = 4$dB, and $IIP_{3,1} = 0$dBm is cascaded with a second amplifier with NF₂ = 10dB, power gain $A_{P₂} = 10$dB, and $IIP_{3,1} = -6$dBm. Assuming a matched condition at all nodes, what are the overall NF, $A_{P}$ and $IIP_{3}$ values?

\[ P_{out} = V_{in}^2 \left( \frac{R_{i}}{R_{s}+R_{i}} \right)^2 \cdot A_{V₁}^2 \cdot \left( \frac{R_{z}}{R_{0₁}+R_{z}} \right)^2 \cdot A_{V₂}^2 \cdot \left( \frac{R_{L}}{R_{L}+R_{0₂}} \right)^2 / R_{L} \]

\[ P_{in} = V_{in}^2 \left( \frac{R_{i}}{R_{s}+R_{i}} \right)^2 / R_{i} \]

\[ A_{P} = \frac{P_{out}}{P_{in}} = A_{V₁}^2 \left( \frac{R_{z}}{R_{0₁}+R_{z}} \right)^2 \cdot A_{V₂}^2 \left( \frac{R_{L}}{R_{L}+R_{0₂}} \right)^2 \cdot \frac{R_{i}}{R_{L}} \]

Also, \[ A_{P₁} = \frac{P_{out₁}}{P_{in₁}} = A_{V₁}^2 \left( \frac{R_{z}}{R_{0₁}+R_{z}} \right)^2 \cdot \frac{R_{i}}{R_{z}} = \frac{A_{V₁}^2}{4} \] with matched condition.

\[ A_{P} = A_{P₁} \cdot A_{P₂} = 14 \text{dB} \]

\[ NF₂ = 10 \text{dB} \quad \Rightarrow \quad F₂ = 10^{10} = 10 \quad A_{P₁} = 4 \text{dB} \Rightarrow 10 = 2.51 \]

\[ NF₁ = 6 \text{dB} \quad \Rightarrow \quad F₁ = 10^{6} = 3.98 \]

\[ F\text{riis Equation} \quad \Rightarrow \quad NF = 1 + (F₁-1) + \frac{(F₂-1)}{A_{P₁}} \]

\[ = 1 + (3.98-1) + \frac{(10-1)}{2.51} = 7.566 \quad NF \]

\[ \frac{1}{IIP_{3}} = \frac{1}{IIP_{3,1} A_{P₁}} + \frac{A_{V₁}^2}{IIP_{3,2} A_{P₂}} \]

\[ A_{V₁} = 4 \text{dB} = 2.51 \]

\[ IIP_{3,1} A_{P₁} = 0 \text{dBm} = 10 \log \frac{V_{rms}^2}{10^{-3}} \quad \Rightarrow \quad V_{rms} = 50 \times 10^{-3} V \]

\[ IIP_{3,2} A_{P₂} = -6 \text{dBm} = 10 \log \frac{V_{rms}^2}{12.5 \times 10^{-3} V^2} \]

\[ \Rightarrow \quad IIP_{3} = 34.9 \times 10^{-3} V \Rightarrow 10 \log \frac{V_{rms}^2}{12.5 \times 10^{-3}} = -16.1 \text{dBm} \]
4. (20 points) A passive mixer circuit is shown below. Assume \( V_{RF, DC}=3.85\,V \), \( V_{LO, DC}=1.15\,V \), \( V_{DD}=5.0\,V \), and \( V_T=1.0\,V \). Also assume that \( V_{RF}=V_{cos \omega_{RF}} \) and \( V_{LO}=V_{cos \omega_{LO}} \). Determine the maximum value of \( V \) for which the circuit behaves properly as a passive mixer.

\[
\begin{align*}
\text{Note } V_{RF} &= V_{RF}^+ - V_{RF}^- \quad \text{and } V_{LO} = V_{LO}^+ - V_{LO}^- \\
V_{RF}^+ &= V_{RF, DC} + \frac{V}{2} \cos \omega_{RF} \quad \text{and } V_{RF}^- = V_{RF, DC} - \frac{V}{2} \cos \omega_{RF} \quad \text{and} \quad V_{LO}^+ = V_{LO, DC} + \frac{V}{2} \cos \omega_{LO} \quad \text{and } V_{LO}^- = V_{LO, DC} - \frac{V}{2} \cos \omega_{LO} \\
\text{Transistor } m_1 - m_4 \text{ must remain on and in non-saturation:} \\
\text{(i) } (V_{GS} - V_T) \geq V_T \quad \Rightarrow V_{RF, DC} - \frac{V}{2} - V_{LO, DC} - V_T \geq 0 \\
\text{Cosine } \omega_{RF} = 1 \quad \Rightarrow V_{RF, DC} - \frac{V}{2} - V_{LO, DC} - V_T \geq 0 \quad 3.85V - \frac{V}{2} - 1.15V - 1.0V \geq 0 \\
\Rightarrow V \leq 3.4V \\
\text{(ii) } V_{DS} < (V_{GS} - V_T) \text{ for } m_4 \text{ in non-saturation:} \\
\Rightarrow (V_{LO}^+ - V_{LO, DC}) < (V_{RF} - V_{LO, DC} - V_T) \\
\Rightarrow V_{LO}^+ < (V_{RF} - V_T) \\
(V_{LO, DC} + \frac{V}{2} \cos \omega_{LO} T) < (V_{RF, DC} - \frac{V}{2} \cos \omega_{RF} - V_T) \\
\text{Worst-case condition } \left\{ \begin{array}{l} (1.15V + \frac{V}{2}) < (3.85V - \frac{V}{2} - 1.0) \\
V < 1.7V \\
\end{array} \right. \\
\text{Condition (i) dominates } \Rightarrow V < 1.7V
\end{align*}
\]