Midterm Exam

March 3, 9:10-10:30am

Name: Oprah Winfrey

(54 points total)

Problem 1: We would like to use the matching network shown in Fig. 1 to transform $R_L = 50\,\Omega$ up to $R_{in} = 100\,\Omega$. The matching network should be designed for operation at a frequency of 1 GHz, and should have an overall quality factor of $Q = 100$. In all series-parallel transformation, you may use the high $Q$ approximation ($Q^2 >> 1$). [16 points]

(a) Choose appropriate values for $C_1$, $L_1$, and $L_2$. [14]

(b) Now assume that the matching network you have just designed is flipped from right to left, so that $L_2$ is in parallel with the input and $C$ is in parallel with $R_L$. What does $R_{in}$ become? [2]

Figure 1: Matching network for Problem 1.
PROBLEM 1 (cont'd)

Now, \( R_{in} = \frac{R_L Q_{out}^2}{Q_L^2} \) \( \Rightarrow \) \( Q_L^2 = \frac{R_L}{R_{in}} \) \( \Rightarrow \) \( Q_L = \left( \frac{50}{10^9} \right)^{1/2} \approx \sqrt{50} \approx 7.1 \)

Also, \( Q_{in} = \frac{P_{in}}{\omega L} \) \( \Rightarrow \) \( L = \frac{P_{in}}{\omega Q_{in} \cdot Q_{in}} = \frac{10^9}{10^{12} \cdot 2\pi \cdot 1E9} = 1.59E-10 = L = L_1 + L_2 \)

To find \( L_1 \) \& \( L_2 \), use \( Q_R \): \( Q_R = \frac{P_R}{\omega L} \Rightarrow L_2 = \frac{P_R}{Q_R \cdot Q_R \cdot \omega} = \frac{50}{7.1 \cdot 2\pi \cdot 1E9} = 1.13E-10 = L_2 \)

\( L_1 = L - L_2 = 1.59E-10 - 1.13E-10 = 4.6E-11 = L_1 \)

We use to find \( C_1 \): \( \omega_0 = \frac{1}{\sqrt{L \cdot C_1}} \Rightarrow C_1 = \frac{1}{\omega_0^2 \cdot L} = \frac{1}{\left(7.1E9\right)^2 \cdot 1.59E10} = 1.59E-10 = C_1 \)

(b) Looking at part (a), we see twice the resistance at port 2, so if we reverse 1 \& 2 then we would expect \( R_i = 50 \Omega \) to be transformed down by the same amount to \( \underline{R_{in} = 25 \Omega} \).
**Problem 2:** Consider the common source amplifier shown in Fig. 2, where $C_\infty$ and $R_B$ have been added to allow the dc gate voltage to be set with $V_{Bias}$. **You may assume that $C_\infty$ is a short at the signal frequency.** In your small signal transistor model, you may neglect $r_o$ as well as all capacitances except $C_{gs}$. [16 points]

(a) Determine an expression for the noise factor of this circuit. You should sub in the appropriate expressions for the noise sources, but all component values may be left as variables. [12]

(b) How should $R_B$ be sized for optimal noise performance, and why? [2]

(c) What should the operating frequency of the amplifier be limited to for the best noise performance, and why? [2]

![Diagram of the common source amplifier](image)

**Figure 2:** Common source amplifier for Problem 2.

**Small signal model:**

![Small signal model diagram](image)

1. Define $R_{eq} = R_S \parallel R_g$.

First noise at output due to each of the four noise sources:

1. $i_{nS} \cdot V_{nS} = i_{nS} \cdot R_{eq \parallel SC_g} = \frac{i_{nS} \cdot R_{eq}}{1 + s R_{eq} C_g} \Rightarrow i_{out} = i_{nS} \frac{g_m R_{eq}}{1 + s R_{eq} C_g}$

2. $i_{nG}$: Same as in 1, $i_{out} = i_{nG} \frac{g_m R_{eq}}{1 + s R_{eq} C_g}$
PROBLEM 2 (cont'd)

(3) \( \text{ind} \): goes directly to output, \( \text{out} = \text{ind} \)

(4) \( \text{in}_L \): Same as (3), \( \text{out} = \text{in}_L \)

Now, find power, sub into expression for \( F \), along with
\[
\text{in}_S = \frac{4\eta T A_f}{R_s}, \quad \text{in}_0 = \frac{4\eta T A_f}{R_L},
\]
\[
\text{in}_L = 4\eta T g_{ds} A_f, \quad \text{in}_G = \frac{4\eta T A_f}{R_L}
\]

\[
F = 1 + \frac{R_s}{R_b} + \frac{8 \cdot g_{ds} \cdot R_s \cdot (1 + s \cdot R_b \cdot C_g)^2}{2 \cdot R_b^2 \cdot R_b \cdot C_g^2} + \frac{R_s}{R_L} \cdot \frac{1 + s \cdot R_b \cdot C_g}{R_L^2 \cdot g_{ds} \cdot C_g^2}
\]

(5) Size \( R_s \) as large as possible, to minimize term 2 in the \( F \) expression.

(6) Terms 3 \& 4 each have a zero at \( \omega_z = \frac{1}{R_b \cdot C_g} \), so noise performance

will be degraded at higher frequencies (limit \( \omega \) to less than \( \omega_z \)).
**Problem 3:** You are analyzing an RF front end that has an IIP3 of 10 dBm, and a total gain of 30 dB. [8 points]

(a) Label the IP3 point on the plot provided in Fig. 3. [2]

(b) If the input-referred noise floor is -110 dB, use Fig. 3 to graphically determine the spurious-free dynamic range. [4]

(c) Why is extrapolation used to measure the IP3, instead of just increasing the input power until the power in the third order IM component is equal to the power in the fundamental? [2]

![Figure 3: SFDR plot for Problem 3.](image)

(b) $SFDR = 80$ dB

(c) Both compression would cause the power of the first order term to drop before intersecting the third order term.
**PROBLEM 4:** You are analyzing a heterodyne receiver with an incoming RF signal centered around 1 GHz with a bandwidth of 1 MHz. The receiver uses **low-side injection** with an intermediate frequency of 250 MHz, a gain of 40 dB, and a noise figure of 10 dB. [12 points]

(a) What is the total noise power at the output of the receiver, assuming that the specified bandwidth of 1 MHz is the equivalent noise bandwidth? (Hint: as a starting point for this problem, use the noise factor definition of \( F = \frac{P_{\text{noise, total}}}{P_{\text{noise, in}}} \). [6]

(b) Assuming that the image reject filter starts abruptly at 1 GHz and has a roll-off of -40 dB/dec, what is the maximum allowable power at the image frequency to keep the interference power less than or equal to the noise floor? [4]

(c) How would the maximum allowable power at the image frequency change (increase or decrease) if high-side injection were used instead of low-side, and why? [2]

\[
(a) \quad F = \frac{P_{\text{noise, tot, input}}} {P_{\text{noise, source}}} \Rightarrow P_{\text{noise, tot, output}} = F \cdot P_{\text{noise, source}} \cdot \text{Gain} = F \cdot KT \cdot \text{BW} \cdot \text{Gain} \\
\text{Total available noise power :} \quad P_{\text{noise, wo}} = NF + 10 \cdot \log(KT) + 10 \cdot \log(\text{BW}) + 10 \cdot \log(Gain) \\
\text{in dB :} \quad P_{\text{noise, wo}} = 10 \text{ dB} + (-174 \text{ dBm}) + 60 \text{ dB} + 140 \text{ dB} \\
\text{Total noise power :} \quad P_{\text{noise}} = -64 \text{ dBm} \\
\]

(b) 

\[1000 \cdot 10^x = 500 \quad 10^x = 0.5 \quad x = \log(0.5) = -0.3 \text{ decades,} \]

\[\text{attenuation is } 40 \text{ dB} \cdot 0.3 = 12 \text{ dB.} \]

\[\text{noise power at the input is } -104 \text{ dBm, interference can be } 12 \text{ dB above, } -92 \text{ dBm} \]

(c) 

\[1000 \cdot 10^x = 1500 \Rightarrow x = 0.18 \text{ decades,} \]

\[\text{allowable power would decrease because the image freq. is only } 0.18 \text{ decades away from the desired signal and will be attenuated less.} \]
PROBLEM 5: Which of the following was a question that Ali G (Fig. 4) posed in his interview with Buzz Aldrin (the second person to walk on the moon)? [2 points]

(a) When you arrived on the moon, was the people who lived there very friendly, or was they scared of you?
(b) Do you think man will ever walk on the sun?
(c) What do you say to all those conspiracy theorists who come up to you and say, “Does the moon really exist?”?
(d) Was you upset that Michael Jackson got all the credit for inventing the moonwalk, but you was the first geezer to actually do it?
(e) All of the above.

Figure 4: Booyakasha!