Problem 2) Sample-and-hold phase detector

- $x(t)$: Sinusoidal signal from VCO
- $y(t)$: Impulse train supplied by reference
- $y_{out}(t)$: Output

(a) Draw the waveforms $x(t)$, $y(t)$, and $y_{out}(t)$ for equal input frequencies.

(b) What is the frequency of the phase detector output for case (a)?

The output signal is DC. The DC level depends on the phase relationship.

How is this behavior better or worse than other phase detectors?

There is no periodic disturbance at the output when the PLL is locked (no reference spurs will be present).

Continued
Problem 2) cont'd

c) Draw the waveforms \( x(t) \), \( y(t) \), and \( v_{out}(t) \) for unequal frequency.

\[ 
\begin{align*}
\text{\( x(t) \)} & \\
\text{\( y(t) \)} & \\
\text{\( v_{out}(t) \)} & 
\end{align*}
\]

d) Sketch the phase error/output voltage characteristic of the phase detector.

From part (a), it is clear that the characteristic will be sinusoidal.

If we define the positive zero crossing as the zero phase reference, the characteristic will look like:

\[ 
\begin{align*}
\text{Phase Error} & \\
\text{\( \Theta \)} & \\
\text{\( v_{out} \)} & 
\end{align*}
\]
Problem 2: Charge pump PLL with PFD and N divider and simple resistive loop

a) What is the type and order of the PLL?

Type = I (one pure integration from VCO)

Order = 1

\[ F(s) = R \quad H(s) = \frac{K_{pR \cdot K_{VCO}}}{s} \frac{N}{sN + \frac{K_{pR \cdot K_{VCO}}}{s+1}} = \frac{N}{SN + \frac{K_{pR \cdot K_{VCO}}}{s+1}} \]

First order \( H(s) \) (single pole)

b) Provide an expression for the bandwidth of the PLL.

From \( H(s) \) in (a) above with \( K_{pR} = \frac{I_{p}}{s} \) for charge pump

\[ BW = \frac{\pi}{\omega_0} \quad \text{(rad/s)} \]

\[ \omega_0 = \frac{K_{pR \cdot K_{VCO}}}{SN} \]

\[ C_{S} = \frac{1}{K_{pR \cdot K_{VCO}} + 1} \]

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Error

\[ \frac{\Delta f_{in}}{\Delta \phi_{in}} = 1 - \frac{\phi_{err}}{\phi_{in}} = 1 - \frac{1}{N} \left( \frac{N}{SN + \frac{K_{pR \cdot K_{VCO}}}{s+1}} \right) \]

OR

\[ \frac{\Delta f_{in}}{\Delta \phi_{in}} = \frac{C_{S} \cdot K_{pR \cdot K_{VCO}}}{SN + \frac{K_{pR \cdot K_{VCO}}}{s+1}} \]

\[ \frac{\Delta f_{in}}{\Delta \phi_{in}} = \frac{C_{S} \cdot K_{pR \cdot K_{VCO}}}{SN + \frac{K_{pR \cdot K_{VCO}}}{s+1}} \]

\[ \text{Freq step} \rightarrow \text{Phase ramp} \rightarrow \phi_{err} = \Delta \phi \quad \text{(Phase integral of frequency)} \]

\[ \text{Use Final Value Theorem:} \quad \lim_{s \to 0} f(t) = \lim_{s \to 0} s \cdot F(s) \]

\[ \frac{\text{SSerr}}{\phi_{in}} = \frac{\text{SSerr}}{\phi_{in}} \cdot \frac{C_{S} \cdot K_{pR \cdot K_{VCO}}}{SN + \frac{K_{pR \cdot K_{VCO}}}{s+1}} \]

\[ \text{SSerr} = \lim_{s \to 0} \frac{\text{SSerr}}{\phi_{in}} \cdot \frac{C_{S} \cdot K_{pR \cdot K_{VCO}}}{SN + \frac{K_{pR \cdot K_{VCO}}}{s+1}} = \frac{\text{SSerr}}{\phi_{in}} \cdot \frac{\text{SSerr}}{\phi_{in}} = \frac{\text{SSerr}}{\phi_{in}} \cdot \frac{\text{SSerr}}{\phi_{in}} = \frac{\text{SSerr}}{\phi_{in}} \cdot \frac{\text{SSerr}}{\phi_{in}} \]

\[ \frac{\text{SSerr}}{\phi_{in}} = \frac{N \Delta f_{in}}{I_{pR \cdot K_{VCO}}} \quad \text{(rad/s)} \]
Problem 3: Active charge pump loop filter

\[ \text{KCL@Vin: } -I_m + \frac{V_{in} - V_A}{R_x} + \frac{V_{in} - V_{out}^+}{R_y} = 0 \]

\[ \text{KCL@VA: } V_A + \frac{V_{in}}{R_x} + V_{ASC} = 0 \]

**Note:** \( V_A = V_{out}^+ \)

\[ I_m = \frac{V_{in} - V_{out}^+}{R_x} + \frac{V_{in} + V_{out}^+}{R_y} \quad \frac{V_{out}^+ - V_{in}}{R_x} + V_{out}^+ S_C = 0 \]

**Solve for Vin:**

\[ \text{Vin} = \text{Vin} \left( \frac{S_R C + 1}{S_R C} \right) \]

\[ I_m = \text{Vin} \left( 1 - \frac{S_R C}{R_x} \right) + \frac{S_R C}{R_x (S_R C + 1)} \]

\[ \frac{V_{in}}{I_m} = \frac{R_x R_y}{R_y + R_x R_y} \left( \frac{S_R C + 1}{S_R C} \right) \]

\[ \frac{V_{in}}{I_m} = \frac{R_y}{R_x R_y} \left( \frac{S_R C}{S_R C + 1} \right) \]

b) Assuming that this is the only filtering that takes place on the charge pump output, which of \( V_{in} \) or \( V_{out}^+ \) should be taken as the output?

\( V_{in} \) should be taken as the output because it provides a stabilizing zero. Using \( V_{out}^+ \) would result in a double integrator system with a phase margin of \( 0^\circ \) (unstable)

(Cont'd)
Problem 3 cont'd c) Now assume that $V_{out}$ is taken and a stabilizing zero is added elsewhere. What advantage would this circuit have over a simple capacitor?

For $R_x \gg R_y$, $R_x/R_y \rightarrow R_y$

$$\frac{V_{out}}{I_{in}} = R_y \frac{1}{SC R_x}$$

$$\frac{V_{out}}{I_{in}} = \frac{1}{SC R_y}$$

By choosing the proper ratio of $R_x/R_y$, the value of $C$ may be reduced, saving chip area.
Problem 4: Noisy VCO Bufferer

Close loop using two options:

Option A

Option B

1. Assume \( F(s) = k_{ref} \) and \( \Phi_{out} \) is white.

a) Sketch the phase noise spectrum \( S_{\Phi}(f) \) at the output for option A assuming all devices are noise free (except the buffer).

\[
S_{\Phi}(f) \equiv \| H(f) \|^2 \Phi_{in}(f) \]  \quad \text{Here} \quad H(f) = \frac{k_{ref}}{s + k_{ref}} - 1

\[
S_{\Phi}(f)
\]

b) Repeat for option B.

\[
H(f) = \frac{1}{1 + k_{ref}k_{out} \frac{\gamma_{out}}{s}} = \frac{s}{s + k_{ref}k_{out} \frac{\gamma_{out}}{s}} = \frac{1}{k_{ref}k_{out} \frac{\gamma_{out}}{s} \left( \frac{s}{k_{ref}k_{out} \frac{\gamma_{out}}{s} + 1} \right)}
\]

\[
H(f) = \frac{1}{k} - \frac{j2\pi f}{(2\pi f)^2 + 1} \quad \text{K = k_{ref}k_{out}}
\]

\[
\| H(f) \|^2 = \frac{1}{K^2} \frac{(\frac{\frac{\gamma_{out}}{s}}{k} + j2\pi f)^2}{\left( \frac{\gamma_{out}}{k} \right)^2 + 1^2}
\]

\[
S_{\Phi}(f)
\]

c) Which choice leads to a cleaner output spectrum?

Option B
Problem 5) Class A Power Amplifier

\[ V_{DD} = 3.3 \]
\[ R_L = 50 \Omega \]

Biased for MAX EFFICIENCY in Class A
\[ f = 1 \text{GHz} \]

\[ P_L = \frac{(V_{peak})^2}{2R_L} \]

\[ R_L' = \frac{R_L}{2} = \frac{3.3V^2}{4(2W)} \]

\[ R_L' = 2.192 \Omega \]

a) What must the load be transformed to to deliver 2W to the load?

\[ V_{peak} = V_{DD} \]

\[ R_L = \frac{(V_{peak})^2}{2P_L} \]

\[ R_L' = \frac{R_L}{Q+1} \]

\[ Q = \frac{R_L}{R_L'} \]

\[ Q = \frac{R_L}{R_L'} = \frac{R_L}{\frac{R_L}{Q+1}} \]

\[ Q = \frac{Q+1}{R_L} \]

\[ Q = \frac{R_L'}{R_L} \]

\[ R_L' = \frac{1}{Q} \]

\[ Q = \frac{1}{R_L'} \]

\[ L_1 = 1.19 \text{mH} \]

\[ C_1 = \frac{1}{2\pi f L_1} \]

\[ C_1 = \frac{1}{2\pi (1 \text{GHz})(1.91 \text{mH})} \]

\[ C_1 = 14.0 \text{pF} \]

b) \[ R_L' = \frac{R_L}{Q+1} \]

\[ Q = \frac{R_L'}{R_L} \]

\[ Q = \frac{R_L'}{R_L} \]

\[ L_1 = 1.19 \text{mH} \]

\[ C_1 = \frac{1}{2\pi f L_1} \]

\[ C_1 = \frac{1}{2\pi (1 \text{GHz})(1.91 \text{mH})} \]

\[ C_1 = 14.0 \text{pF} \]

\[ Q = \frac{2\pi f L_1}{R_L} \]

\[ Q = \frac{2\pi (1 \text{GHz})(1.91 \text{mH})}{R_L} \]

\[ Q = 50 \text{pF} \]

\[ I_{DC} = \frac{V_{DD}}{R_L} \]

\[ I_{DC} = \frac{3.3V}{50 \Omega} \]

\[ I_{DC} = 0.066 \text{mA} \]

\[ I_{DC} = 0.066 \text{mA} \]

\[ I_{DC} = 12.12 \text{mA} \]
for \( Q_L = 5 \) we have \( R_{in} \) in parallel with \( R_L \):

\[
\frac{1}{R_{in}} = 3 \frac{1}{R_{L}} + \frac{1}{R_L}
\]

\[
R_{in} = \frac{Q^2}{Q^2 + 1}
\]

\[
R_{in} = \frac{25}{25 + 1} = \frac{25}{26}
\]

\[
R'_{in} = \frac{25}{26} \cdot 2 \Omega
\]

\[
R'_L = 4.12 \Omega
\]

So \( P_L = \frac{V_{in}^2}{2R'_L} = \frac{3.3V}{2(4.12\Omega)} \)

\[
P_L = 0.373 \text{ W}
\]

Assume IPC is unchanged

\[
P_{out} = 4 \text{ W}
\]

Now \( \eta = \frac{P_L}{4 \text{ W}} \)

\[
\eta = 0.373
\]