PROBLEM 1: Shown in Fig. 1 is a BJT operating as a mixer. Assuming that $V_{LO} = V_1 \cos \omega_{LO} t$ and $V_{RF} = V_2 \cos \omega_{RF} t$, calculate the conversion gain of this mixer (Hint: the power series expansion of an exponential will come in handy for this problem, neglect terms of order 4 or higher).

![Figure 1: BJT mixer for Problem 1.](image)

PROBLEM 2: In class we calculated the conversion gain for an ideal single-balanced mixer, under the assumption that the LO transistors act as ideal switches driven by a perfect square wave. Using the same assumptions except the square wave driving the switches now has a duty cycle of 40% (instead of the ideal case of 50%), calculate the reduction in conversion gain.

PROBLEM 3: Consider the VCO shown in Fig. 2. Here $L_1 = 5 \text{ nH}$, $L_2 = 20 \text{ nH}$, $C = 1 \text{ pF}$, and for $M_1$ the transconductance is $g_m = 7.85 \text{ mS}$. In your calculations it will be helpful to make the high Q assumption for impedance transformations.

(a) What is the frequency of oscillation?

(b) What is the loaded Q of the tank (assume that all components are ideal and lossless)?

PROBLEM 4: Consider the VCO shown in Fig. 3, where $L = 2 \text{ nH}$ and $C = 6.3 \text{ pF}$. Assume that the inductor dominates the tank losses, with $Q_L = 10$. For the NMOS transistors, $2\mu_n C_{ox}(W/L) = 200 \mu\text{S/V}$, and for the PMOS transistors $2\mu_p C_{ox}(W/L) = 100 \mu\text{S/V}$. 


Figure 2: Hartley VCO for Problem 3.

(a) What is the frequency of oscillation?
(b) What is the bias current required to maintain oscillation?

Figure 3: Negative-$g_m$ VCO for Problem 4.