Last class: Low Noise Amplifiers
- Derived 2-port noise parameters for a MOSFET.
- 

LNA Architectures

- Last class, we derived the optimal source impedance to obtain the best noise performance from a MOSFET.
- Since an LNA usually interfaces with external components, we must provide an input impedance in the neighborhood of 50 Ω to yield acceptable power gain.
- Today: Present architectures for achieving a 50 Ω input impedance, evaluate their noise performance.

1) Shunt Input Resistor

- One simple way to provide a broadband 50 Ω real impedance is to put a 50 Ω resistor across the gate of a common source amplifier.

- We would expect this to be a suboptimal solution, since $R_s$ will contribute noise to the input signal.

- Evaluate noise factor:

$$ F = 1 + 1 + \frac{R_s}{2} + \frac{g_{ds}}{g_m R} $$

- $\alpha = \frac{g_m}{g_{ds}}$

So,

$$ F = 1 + 1 + R_s \cdot \frac{g_{ds} \cdot 4}{g_m^2 \cdot R} = 2 + R_s \cdot \frac{4}{\alpha g_m R} $$

where $\alpha = \frac{g_m}{g_{ds}}$.
We have neglected induced gate noise as well as 1/f noise, so this will provide a conservative estimate.

- Assume \( \delta = 1 \), \( \alpha = 1 \) (\( g_{ds} \approx g_m \)), and \( g_m R = 1 \) (all conservative)
  - Then, \( F = 6 \), leading to a NF of \( \approx 8 \) dB.

For reasonable values in the \( F \) \& \( \alpha \) expression for a standalone MosFET, we got a noise figure of \( 0.5 - 1 \) dB, so this is very bad.
- Let's explore other options.

2) Shunt-Series Amplifier

- Another way to present a broadband real input impedance is through the use of feedback:

![Diagram]

**Analysis:**
- Source degeneration resistor alters transconductance to:
  \[ g_m = \frac{1}{g_m + R_1} \]
  - Check: \( R = 0 \), \( g_m = g_m \), as expected.

- Assume \( R_1 \gg g_m \), then \( g_m \approx \frac{1}{R_1} \)
- Also assume \( R_f \gg R_1 \), then the small-signal voltage gain is
  \[ A_v = \frac{-R_f}{R_1} \]

- Now find input impedance:
  - Neglect \( g_{ds} \), then input impedance is due solely to current flowing in \( R_f \).
  - Due to gain across \( R_f \), we can use the Miller effect to calculate the effective impedance looking into \( R_f \) (and hence the 1/P impedance) as:

\[ R_{in} = \frac{R_f}{1 - A_v} = \frac{R_f}{1 + R_f/R_1} \]

- Can show that the output impedance is of the same form.
- All of the above can also be derived with small signal models or KCL, using the proper assumptions.
For the noise figure, draw out the small signal model, including noise sources:

- We have again neglected MOSFET capacitances as well as induced gate noise and VHF noise.
- We still assume that RF ≫ RL and R1 ≫ \frac{1}{\beta_2}

\[ F = 1 + \frac{R_F + R_i + R_S + V_{gdo} \cdot R_S}{R_S} \]

So, by proper selection of the resistor values, we should be able to get the NF claim to an acceptable level.

- This circuit performs better than the first one, but the resistors still contribute additional noise at the output.
- We would like an architecture where we don’t need explicit resistors to achieve the input match.

3) Common Gate Amplifier

\[ F = 1 + \frac{V_{gdo}}{\beta_2 \cdot R_L} \]

It is easy to show that the input impedance is 1/\beta_2.

- By proper choice of \beta_2, we can achieve a 50 Ohm input impedance.

- Investigate noise figure (neglect capacitances, VHF, gate induced noise).

- We have broken the drain noise into two correlated sources.

- Using the above circuit, we can solve for the noise factor as

\[ F = 1 + \frac{V_{gdo}}{\beta_2} + \frac{1 + \frac{1}{\beta_2 \cdot R_L}}{\beta_2 \cdot R_S} \text{ (gain aside)} \]
From inspection, we should be able to reduce the noise factor by increasing $g_m$, however $g_m$ is constrained to a certain value by the input metal requirement.

- Assume $\gamma = 1$, $g_{m0} = 1$, then our noise figure will be at a minimum of 3 dB, plus contributions from the third term (due to the load resistor) and contributions from induced gate and 1/f noise (which we have neglected).

So, while we do not have explicit resistors in the circuit for achieving the input match, our performance is still limited because the signal encounters the noisy channel resistance.

4) Common Source Amplifier with Inductive Source Degeneration.

- Consider a common source amplifier with an inductor in the source:

- Draw small signal model to find 1/P impedance:

$$\begin{align*}
\text{Using KVL, we can show that:} \\
Z_{in} &= \frac{1}{\frac{1}{2} + \frac{g_m}{g_{ds}}} \\
&\approx \frac{1}{\frac{1}{2} + \frac{g_m}{g_{ds}}} \\
&\text{where } W_I = \frac{g_m}{g_{ds}}.
\end{align*}$$

- At resonance between $g_{ds}$ and $L$, we see that the first two terms will cancel out, and we will be left with a purely real input resistance, determined by $W_I$ and $L$.

Aside: What is $W_I$?

- Defined as unity current gain frequency
- Drive input with current source $I_{in}$. Small signal short circuit the output (drain), find freq where $\frac{V_{in}}{V_{out}} = 1.0$

Considering only $g_{ds}$,

$$\begin{align*}
|Z_{in}| &= \frac{1}{\omega L g_{ds}} \\
V_{in} &= \frac{I_{in}}{\omega L g_{ds}} \\
V_{out} &= g_m V_{in} = \frac{I_{in} g_m}{\omega L g_{ds}}
\end{align*}$$
\[ |\text{Input}| = \frac{g_m}{\omega C_{es}} = 1 \Rightarrow \omega T = \frac{g_m}{\frac{1}{C_{es}}} \]

→ a lot of approximations involved here, but this is a standard result.

→ Back to the text at hand:

- We are losing a degree of freedom in design because if we choose \( L \) for a given input impedance (in conjunction with \( g_m \) and \( C_{es} \)), we are stuck with the resonant frequency of \( C_{es} \) and \( L \).

- We need to specify both the input impedance and the resonant frequency.

→ Solution: Add another inductor at the input

\[ Z_{in} = \frac{5(L_s + L_g) + 1}{\frac{g_m L_s}{C_{gs}}} + \frac{g_m L_s}{C_{gs}} \]

→ Choose \( L_g \) for desired input impedance
→ Choose \( L_s \) for desired resonant frequency.

Note: Clearly this technique will provide only a narrowband match, but in some cases this is desirable.

Techniques exist for extending the BW of this amplifier, but we won't cover them (see Ch. 9 if interested).

→ A side effect of this technique is that due to the resonant behavior of the input circuit, the effective transconductance of the device is increased.

→ Recall that for a series resonant RLC circuit, at resonance the voltage across \( L \) or \( C \) is \( Q \) times the voltage across the circuit as a whole.

\[ G_m = Q \cdot g_m \]

Sub in expression for series \( Q \) : \( Q_s = \frac{1}{\omega R C} \)

\[ G_m = \frac{g_m}{\frac{1}{\omega L_g} \cdot (L_s + \omega T L_g)} = \frac{\omega T}{2 \omega L s R_s} \quad \text{(subbed in } \omega T = \frac{g_m}{\frac{1}{C_{es}}}) \]

→ Note that the second expression does not depend on \( g_m \).
Let’s consider the effects of a finite transistor output resistance, $r_o$.

- We can rederive the input impedance expression for this case, and we find:
\[ Z_{in} = \frac{1}{j \omega L} + \frac{1}{C_{gs}} \left( \frac{r_o}{r_o + j \omega L + Z_L} \right) \]

- as $r_o \to \infty$ we converge to our prior result, as expected.

- For smaller $r_o$, we will have a mismatch at the input and in the resonant frequency.
- This will particularly be a problem for circuits using a resonant load, in which case $Z_L$ is very large at resonance.
- At resonance, real input impedance is reduced.

**Noise performance:** since we have used only reactive components, the noise figure will be no worse than for the best MOSFET (which we derived previously), if we neglect the load.

- If a resonant load is used then we can approach this limit, as far as the quality of our passive components will allow.
- Of course real components and parasitics will prevent us from reaching the limit of the standalone MOSFET.

**Resonant Load:**

- $L$ maximizes headroom by providing a bias current with no voltage drop.
- $C_L$ is used to shunt parasitic capacitances of the MOSFET.
- Narrowband design (depends on resonance at $1/p$ or $Q/p$).
- No explicit resistor: minimize thermal noise.
- Biasing details not shown.

**How to size $M_1$?**

- Possibility: use our previously derived expression for $G_{opt}$ of a MOSFET:
\[ G_{opt} = \frac{2 \omega C_{gs} \sqrt{g_s}}{V_{SV}} \left( 1 - \frac{1}{C_{gs}} \right) \]
- Set $g_s$ so that $G_{opt} = G_{s}$ (the actual driving source impedance).
- Choose device width to yield desired $g_s$.

- Unfortunately, this results in unrealistic device sizes (see Lee Ch. 12.3).