Multiplier - Based Mixers

- Extra non-linearities in previous mixer result in undesired spurious components.
- Also suffer from poor isolation.
- Want a 3-port mixer, with separate ports for LO, RF & IF.
- Explicit multiplication exhibits only the desired IF product at the O/p.
- CMOS provides good switches, so a multiplier is a good option.

2) Single-Balanced Mixers

\[ \text{I}_{\text{out}} = \text{I}_{\text{out-}} \]

\[ V_{\text{o+}} \rightarrow \begin{bmatrix} M1 \quad M2 \end{bmatrix} \rightarrow V_{\text{o-}} \]

\[ \text{I}_{\text{LO}} + \text{I}_{\text{RF}} \cos(\omega_{\text{RF}} t) \]

- Differential output (can be converted to a voltage with a load).
- Need differential LO to drive it.
- Here we have assumed an ideal trans-conductor for the RF signal.

Operation:
- M1 and M2 act as switches that commutate the current back and forth between the + and - output branches.
- If we assume that \( I_{\text{out}} = I_{\text{out+}} - I_{\text{out-}} \) then this is equivalent to multiplying the RF signal by a square wave that assumes values of \( \pm I_0 \) at the LO freq.

Fourier Series expansion of a square wave:

\[ \text{ff(t)} = \sum_{n=1,3,5}^{\infty} \frac{4}{n\pi} \sin(n\omega_{\text{LO}} t) \]

\[ = \frac{4}{\pi} \sin(\omega_{\text{LO}} t) + \frac{4}{3\pi} \sin(3\omega_{\text{LO}} t) + \frac{4}{5\pi} \sin(5\omega_{\text{LO}} t) + \ldots \]

So, \( I_{\text{out}} = \left[I_{\text{LO}} + \text{I}_{\text{RF}} \cos(\omega_{\text{RF}} t)\right] \left[\frac{4}{\pi} \sin(\omega_{\text{LO}} t) + \frac{4}{3\pi} \sin(3\omega_{\text{LO}} t) + \frac{4}{5\pi} \sin(5\omega_{\text{LO}} t) + \ldots \right] \]

(neglected higher order terms)
We can see immediately that the term due to the dc bias current will result in the LO (and harmonics) being present at the O/P.
- This in LO feedthrough and undesirable.
- Thus we cry it is called a "single-balanced mixer."

Plot the output spectrum:

- Only the lowest term is desired, all others must be filtered out.
- Note that in the absence of any unintended non-linearities, we don't need to worry about spurious components falling in band.

This assumes a perfectly linear transconductor, which will not be the case in the real world:
- If RF transconductor has significant non-linearities, then the RF harmonics will also be present at the output, and we can run into spurious problems.
- So, enhancing the linearity of the RF transconductor is an important design challenge.

Transconductor implementations:
- Most common: $Vin \xrightarrow{\text{gm}} I_{out}$
- Linear with small signal approximation.

$$I_{out} = \frac{gm}{2} \cdot (Vin - Ve)^2$$

- Non-linearity introduced through quadratic behavior.

Most common way to improve the linearity is with source degeneration.
Now, \( I_{out} = C \cdot \left( \frac{V_{in} - V_{t}}{Z_{gs} + Z_{s}} \right)^2 \)

- New \( I_{out} \) curve:

\[
I_{out} \sim V_{in}
\]

- Reduces effective \( g_m \) and makes behavior more linear.

- Inductive degeneration is usually preferred since it doesn't add thermal noise or dc voltage drop.

- Another option is a source-degenerated common gate amplifier:

\[
\text{if } Z_s \gg Z_{in} \text{ (impedance looks into the source),}
\]

Can show using small signal analysis that

\[
g_m = \frac{1}{Z_s}
\]

- So, now the transconducance is not impacted at all by device nonlinearities (changing \( g_m \)), and it should be totally linear.

- Conversion gain for single balanced mixer?

\[
\text{transconducance}
\]

\[
\text{desired signal at output: } G_m V_{RF} \frac{Z}{Z_s} \cos\left(\frac{V_{LO} - V_{RF}}{Z_s}\right)
\]

\[
\text{RF signal at input: } V_{RF} \cos(\omega_{RF} t)
\]

\[
G_c = \frac{g_m Z_s}{Z}
\]

(b) Double-balanced Mixer

- We saw that the last design had undesirable LO feedthrough.

- To minimize this, we'd like a design where the LO components cancel at the output.

When \( V_{io} \) is high:

\[
\begin{align*}
I_{+} &= I_{Loc} + I_{RF} \cos(V_{RF}) \\
I_{-} &= -I_{Loc} - I_{RF} \cos(V_{RF})
\end{align*}
\]

\[
\begin{align*}
I_{+} + I_{-} &= [I_{Loc} + I_{RF} \cos(V_{RF})] - [I_{Loc} - I_{RF} \cos(V_{RF})] \\
&= 2I_{RF} \cos(V_{RF})
\end{align*}
\]
When \( V_{no.\,1} \) is low: \( I_{at} = [I_{pe} - I_{RF}\cos(\omega_{RF}t)] - [I_{oc} + I_{RF}\cos(\omega_{RF}t)] \)

\[ = -2I_{RF}\cos(\omega_{RF}t) \]

Now the bias term that caused the \( V_{no.\,1} \) feed through is cancelled out, left with only RF component multiplied by square wave.

The current sources in the schematic are typically implemented by a differential pair transconductor, with each of the linearization techniques discussed previously applied.

Noise Performance of Multiplier Mixers

- Noise due to the input transconductor can be analyzed in a similar manner to the LNA.

- Input matching network can be used as it the LNA to optimize the noise performance of the transistor for a given power budget.

- Also need to keep in mind conversion gain and linearity when designing the input transconductor.

- Noise due to differential pair should also be considered.

- Consider the case during switching when \( V_{no.\,1} = V_{no.\,2} = 0 \) and all transistors are on simultaneously:

\[ I_{at} = \frac{1}{2}[I_{pe} + I_{RF}\cos(\omega_{RF}t)] + \frac{1}{2}(I_{pe} - I_{RF}\cos(\omega_{RF}t)) \]

\[ = I_{pe} \]

(1) Same holds true for \( I_{at} \), so during this time no signal current appears at the output, degrading the noise figure.

(2) Also, since all 4 switch transistors are on, they are all contributing drain current noise to the output.
(3) Finally, in this configuration, the transistors are acting as amplifiers, so noise at the LO signal ($V_{10}$) is amplified.

- For these three reasons, the best noise performance will be achieved if the switch transistors switch infinitely fast.
  - Need to provide sufficient LO drive for complete switching.
  - Size switcher small enough so they can switch quickly.

- For the case where the switch transistors are fully switched, the circuit looks like a cascaded common source:

$$
V_{gs} = \frac{1}{C_p} \int \frac{1}{C_p} \Delta \phi
$$

- We saw previously that parasitic capacitance $C_p$ at the source of the switch transistor will increase its noise contribution, so this should also be minimized.

**Linearity of Multiplier Mixers**

- Assuming that the LO driven transistors act as good switches, the linearity is more or less determined by the transconductor.

- While for noise we mentioned that we want high LO drive for complete switching, excessive LO drive will degrade the linearity.

- If $V_{gs}$ is too large, $V_{10}$ will be large.
- Recall that at saturation, $V_{ds} > (V_{gs} - V_{th})$

- If $V_{gs} = V_{10}$ is fixed, if $V_{gs}$ gets too large, the switch will enter the triode region, leading to non-linear behavior.

- Need to be careful not to over-drive the switches.

**Passive Mixers**

- Much of the difficulty in designing the multiplier-based mixers stems from the transconductor.

- We can avoid this if we work directly in the voltage domain without converting to current.

- Passive mixers work in this manner, sacrificing conversion gain for good linearity and low-power operation:
- When $V_{ds}$ is high, $V_{IF} = V_{RF} - V_{IF} = V_{RF}$
- When $V_{ds}$ is low, $V_{IF} = -V_{RF}$.

So, again, the RF signal is multiplied by a unit amplitude square wave.

And we can then show that $G_{c} = \frac{2}{1}$

-similar to that of the active multiplier mixer, but now there is no associated $g_{m}$ and associated load impedance to provide gain.

- Need high $R_{in}$ to drive to optimize noise and linearity.

Lee text covers a number of other mixer designs and many more can be found in papers, you are free to use any topology you want to for your project.

A note on short channel effects

- Last class there was some controversy concerning short channel (high field) behaviour of MOSFETS.

- As $V_{ds}$ is increased, electric field in channel increases → carrier velocity increases → $I_{DS}$ increases.

- At $V_{ds} = V_{GS} - U_{T}$, channel pinched off and device enters saturation:

$$I_{D} = \frac{2}{L} \frac{W}{2} (V_{GS} - U_{T}) V_{DS} = \frac{W}{2} L \frac{V_{GS} - U_{T}}{2}$$

- Carrier velocity saturates at field strength of $\sim 10^6 \text{ V/m}$ in silicon due to scattering.

- If a device is short enough, this occurs for $V_{DS} < V_{GS} - U_{T}$.

Now, $I_{D} = \frac{W}{2} L \frac{(V_{GS} - U_{T}) V_{DS}}{2}$ where $V_{DS} < V_{GS} - U_{T}$, so it will have a sub-square dependence on $V_{DS}$.

- $V_{DS}$ can be expressed as $V_{DS} \approx (V_{GS} - U_{T}) L \frac{(V_{DS})}{L \cdot E_{sat}} \approx (V_{GS} - U_{T}) - E_{sat}$

- In a long channel device, $V_{GS} - U_{T} < L \cdot E_{sat}$, so $V_{DS} \approx V_{GS} - U_{T}$ and we have square-law behaviour.

- So, the definition of a long channel device is one where $V_{GS} - U_{T} \ll L \cdot E_{sat}$.

- For a very short channel device, we have $V_{DS} \approx L \cdot E_{sat}$, so

$$I_{D} = \frac{W}{2} L \frac{(V_{GS} - U_{T}) \cdot E_{sat}}{2} \quad \text{(linear dependence on $V_{GS}$!)}$$