ECE 6720: Lecture #7

Last Class:
- IT-match example
- Smith chart
  - IT-match design on Smith chart
- S-parameters
  - Useful for high frequency 2-port characterization
- Modulation
  - Performance criteria
  - Analog modulation (AM, FM, PM)

Reading: Lee 19.2.3 - 19.3
Razavi 5.1 - 5.2
Analog Modulation

1. Amplitude Modulation: For a baseband signal \( x_b(t) \), the amplitude modulated waveform can be expressed as:

\[
x_{am}(t) = A_c [1 + m \cdot x_b(t)] \cos \omega t
\]

- Can be generated with a mixer:
  \[
  \text{mixer} \quad \xrightarrow{\chi} \quad \text{amplitude modulator}
  \]

- Spectra are convolved, so passband signal has twice the bandwidth of the baseband signal.

There are other more efficient forms of AM (discussed in Lee) but we won't worry about them.

Demodulation: mix with carrier and LPF, or use an envelope detector.

AM is not used much in modern wireless systems.

2. Phase/Frequency Modulation: Since phase and frequency are related by \( \omega = \frac{d\phi}{dt} \), we will consider these together.

- We can express a phase modulated signal as:
  \[
x_{pm}(t) = A_c \cos(\omega t + m \cdot x_b(t))
  \]

- And a frequency modulated signal as:
  \[
x_{fm}(t) = A_c \cos(\omega t + m \int_0^t x_b(t) \, dt)
  \]

Since the passband signals are not linearly dependent on the baseband signals, calculating the bandwidth of the modulated signal is not as easy as for AM. Approximations can be made at certain cases.

Modulation can be done by varying the frequency of an oscillator using the baseband signal:

\[
x_b(t) \rightarrow \text{VCO} \rightarrow X_{fm}(t)
\]

Demodulation is commonly performed by a PLL (phase-locked loop), something we will discuss later in the course.

Kagavi & Lee have discussion of FM bandwidth, we will not cover this in class.
Digital Modulation

- Most modern wireless systems use digital modulation, where the carrier is modulated with discrete bits.

- As mentioned previously, in a digital system we evaluate the quality of the link with the resulting bit error rate (BER), and in analysis we compare modulation methods according to their probability of error ($P_e$).

Concepts:

1. **Basis Functions** - Any modulation scheme can be represented by a set of orthogonal basis functions.

   Let $\mathbf{\Phi}(t) = \{\phi_0(t), \phi_1(t), \ldots\}$ be a set of basis functions.

   Orthogonality condition:
   
   $$\int \phi_m(t) \phi_n(t) \, dt = 0 \quad \text{for} \quad m \neq n$$

   - Integrate over one symbol period.

2. **Frequency Shift Keying (FSK):**

   - Each bit can be represented by $x(t) = A_c \cdot \cos(\omega_c t)$ if $b_n = 0$.
   - Each bit can be represented by $x(t) = A_c \cdot \cos(\omega_c t + \phi(t))$ if $b_n = 1$.

3. **Amplitude Shift Keying (ASK):**

   - Each bit can be represented by $x(t) = A_1 \cdot \cos(\omega_c t)$ if $b_n = 0$.
   - Each bit can be represented by $x(t) = A_2 \cdot \cos(\omega_c t)$ if $b_n = 1$.

4. **Signal Constellations** - Modulated waveforms can be viewed in terms of their coefficients and basis functions.

   - Each basis function contributes an axis to the plot.
   - The distance between points in the signal constellation tells us how susceptible the modulation is to noise effects.

   **FSK:**
   
   - $A_1$, $A_2$, $\omega_c$ are the parameters of the FSK signal.
   - A decision threshold is used for detection.

   **ASK:**
   
   - $A_1$, $A_2$, $\omega_c$ are the parameters of the ASK signal.
   - A decision threshold is used for detection.
Noise introduces variation in received signal constellation.

ASK:

\[ \text{pdf} \quad \text{pdf} \]

\[ A_1 \quad A_2 \]

- Errors are introduced when noise spreads the signal to appear on the other side of the decision threshold.

- To find the probability of error, we can integrate the Gaussian function for the tail on the other side of the decision threshold and add them.

- Intuitive Notes:
  - Higher signal power is equivalent to spreading the points in the signal constellation further apart, so clearly, the probability of error will be reduced.
  - Higher noise power can be related to the variance of the Gaussian pdf, so if this increases, the noise spread will increase, and the probability of error will increase.

Optimum Detector:

- It can be shown that the optimum detector is a correlator, which can be built by a multiplier (mixer) followed by an integrator.

\[ x(t) \rightarrow \times \rightarrow \int_{\frac{T_b}{2}}^{T_b} \rightarrow -d - \rightarrow y(t) \]

\[ p(t) \quad \text{(pulse shape)} \quad T_b \quad \text{(sample at time } T_b) \]

- The integrator provides an averaging that reduces the effects of the noise.

- In most modern receivers, after downconversion the signal is digitized by an A/D and all of the demodulation is done in the digital domain, so we won't worry about it much.

Types of Digital Modulation:

1. Binary Phase Shift Keying (BPSK)

\[ X(t) = A_c \cos(\omega t + \phi) \text{ where } \phi = 0^\circ \text{ or } 180^\circ \text{ depending on whether a 0 or 1 is being transmitted.} \]

Signal Constellation:

- Reflected to as "Antipodal" signalling.

\[ \begin{array}{c}
A_c \\
- A_c
\end{array} \]

- Basis function = \[ \cos(\omega t) \]

- Energy per bit:

\[ \int_{-\infty}^{\infty} \left( \frac{2E_s}{N_0} \right) \text{ energy per bit} \]

- Can show that \[ P_e = Q \left( \sqrt{\frac{2E_s}{N_0}} \right) \]

- AWGN
2. **Binary Frequency Shift Keying (BFSK)**:

\[ x(t) = a_1 \cos(\omega t) + a_2 \cos(\omega t) \] where \( a_1, a_2 = 0, A_c \) or \( A_c, 0 \)

**Signal Constellation**:

- Can show that \( P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right) \)

- Increased likelihood of error can be seen as for a given signal power \((A_c^2)\), the points in the constellation are closer together.

- For a given \( P_e \), BFSK must have double the bit energy as BPSK, so BPSK has an inherent 3-dB advantage in the SNR.

- BPSK is still popular (used in pages) since it lends itself to simple receiver design.

3. **Quadrature Modulation**

- Employ the basis functions \( \cos(\omega t) \) and \( \sin(\omega t) \).

a. **Quadrature Phase Shift Keying (QPSK)**

- Send 2 bits with one symbol using weights of \([a_1, a_2] = [A_c, A_c], [-A_c, A_c], [A_c, -A_c], [-A_c, -A_c]\)

- **Signal constellation**:

- A problem with QPSK is the sharp transitions that occur:

- For equal bit energy, BPSK and QPSK have nearly identical error probabilities:

\[ P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right) \]

- These can cause problems for amplifiers, so sometimes modifications are made to standard QPSK to improve this: QOPSK & E-QPSK

b. **Quadrature Amplitude Modulation (QAM)**

- Can be extended to more bits per symbol by varying the amplitudes of the two basis functions:

- 16-QAM - need higher SNR for these to be effective

- Some systems employ adaptive modulation where they adjust the modulation (and data rate) as the SNR changes.
Receiver Architectures

Why not perform channel selection and demodulation at the received RF frequency?

Channel select filter

1. Tuneable filters are much harder to realize than fixed ones
   - multiple LC elements to be tuned (for higher order filters)
   - VCOs have only a single LC tank to tune (as we will see)

2. Channel select filter at \( f_c \) would require a prohibitively high \( Q \).
   - recall \( Q \) can be defined as a ratio of 3-dB bandwidth to center frequency \( f_c \).
   - example: IS-54 has 0.3 kHz channels at 900 MHz. If 60 dB of attenuation is needed at 45 kHz offset, a 2nd order L-C filter would require a \( Q \) of \( 10^7 \) (impossible)
   - higher \( Q \) filters also generally have higher loss.

By downconverting the signal to a lower intermediate frequency (IF) before channel selection using a variable oscillator, we can use a fixed, lower-\( Q \) filter.

By 10 dB reg. in the same, \( f_c \) has been reduced.

1. Heterodyne Receivers

   Also called "super-heterodyne" for historical reasons.

   LNA

   \[ W_{IF} \]

\[ W_{IF} = W_c - W_{Lo} \]

- \( W_{Lo} \) can be chosen to be either above or below \( W_c \) by \( W_{IF} \)
  - called "high-side" or "low-side" injection
  - high-side injection has the advantage of requiring a reduced tuning range for the VCO.
E.g. 5 MHz channels, 20 channels, centered around 500 MHz. IF is 300 MHz:

- Low side: 10 must be from 250 MHz to 350 MHz (\( f_{IF} = 1.4 \))
- High side: 10 must be from 750 MHz to 850 MHz (\( f_{IF} = 1.13 \))

Drawback of higher IF is that you have more power.

How do we choose the IF?

- Preceding discussion indicates we should choose it as low as possible to relax the BPF (Channel select) requirements, however, there is another conflicting requirement...

The Image Problem

- Who converts frequencies both above and below by an offset of \( W_{IF} \).
- Assume high-side injection:

\[
\begin{align*}
& W_{IF} \\
\Rightarrow & W_{IF} \\
& W_{IF} = W_o - W_e = W_{IM} - W_o
\end{align*}
\]

- Desired signal sees the LO as high-side injection
- Image frequency sees the LO as low-side injection
- Both are down-converted to \( W_{IF} \), where image signal appears as interference.

- Most common way to alleviate this problem is to place an image reject filter before the mixer to filter out interference at the image frequency.

Now, for the image reject filter design, we want \( W_{IF} \) to be as high as possible, in direct conflict with the requirements for the channel select filter.

Choosing the IF is a tradeoff between these factors, and also which image frequencies might have worse interference.
To lessen the tradeoff between image rejection and channel selection, sometimes a dual-IF topology is used:

- 1st mixer converts to a relatively high IF to allow image rejection.
- Some channel select filtering is performed here.
- 2nd mixer converts to a low IF to allow channel selection.

Taken to the extreme, the first IF can actually be chosen higher than $W_L$, for image rejection purposes.

2 Image Rejection Receivers

Some receiver architectures have been introduced specifically to combat the image problem.

(a) Hartley Architecture

Effect of a 90° phase shift on a narrowband signal is to multiply the spectrum by $G(w) = -j \cdot \text{sgn}(w)$.

Assume the RF input is $x(t) = A_{RF} \cos \omega_{RF} t + A_{IM} \cos \omega_{IM} t$.

At point A, we have

$X_A(t) = \frac{A_{RF}}{2} \sin (\omega_{RF} - \omega_{IM}) t - \frac{A_{IM}}{2} \sin (\omega_{IM} - \omega_{RF}) t$

Used $\sin(-x) = -\sin(x)$

Sum frequency components have been removed by LPFs.
At point B:

\[ x_b(t) = \frac{A_F \cdot \cos(\omega_0 - \omega_f) t + A_m \cdot \cos(\omega_m - \omega_0) t}{2} \]

\[ \Rightarrow \text{At point C: (with 90° phase shift, } \sin \rightarrow \cos, \cos \rightarrow \sin) \]

\[ x_c(t) = \frac{A_F \cdot \sin(\omega_0 - \omega_f) t + A_m \cdot \sin(\omega_m - \omega_0) t}{2} \]

\[ \Rightarrow \text{By summing } x_a(t) \text{ and } x_c(t), \text{ we get:} \]

\[ x_{ac}(t) = A_F \cdot \sin(\omega_0 - \omega_f) t \]

- Image components cancel out.
- 90° phase shift allows us to differentiate between sum and difference frequency components, it's placement differs depending on if high or low-side injection is used.

How do we generate a 90° phase shift?

- Phase shift must only be relative to each other.

Consider RC Sections:

- Low pass:

\[ V_{in} \quad M \quad V_{out} \quad 0° \quad W_p \quad 20 \log_{10} A \]

\[ \Delta \angle \theta_{jw} = 90° \]

- High pass:

\[ V_{in} \quad M \quad V_{out} \quad 90° \quad W_p \]

\[ \Delta \angle \theta_{jw} = 90° \]

- Difference between phases will be 90° at all frequencies.
- Amplitudes will only be equal at 3 dB points, \( W_p = W_z \).

- If relative amplitudes are important (we will see that they are), then \( W_p = W_z \) must be well-matched, and signal must be narrowband.

- Put high-pass in one branch, low-pass in the other.

Notes: - 3-dB loss can be significant, need enough gain before.

**Effect of Gain + Phase Mismatch**

- In a physical implementation, there will be finite gain + phase errors introduced between the two branches.
  - This will lead to incomplete image cancellation.