Last class:
  - Analog Modulation (AM, FM, PM)
  - Digital Modulation
    - basis functions
    - signal constellations
    - BPSK, BFSK
    - QPSK, QAM
  - Receiver Architectures
    - Heterodyne receivers (to relax BPF, Q)

Reading Survey...

→ Taping out chips through MEP?
  - 6 x 1.5mm x 1.5mm in Axial 0.5 μm (April 23, June 4)
  - (3 retal, 2 poly)

→ Tapped capacitor...

→ How too long?
E.g. 5 MHz channels, 20 channels, centered around 500 MHz. IF is 300 MHz.

1. LO tuning range for both types of injection
   1. Low side: LO must tune from 150 MHz to 250 MHz (TR = 1.67)
   2. High side: LO must tune from 750 MHz to 850 MHz (TR = 1.13)

   - Drawback of higher LO is that you burn more power.

   - How do we choose the IF?
     - Proceeding discussion indicates we should choose it as low as possible to relax BPF (channel select) requirements, however there is another conflicting requirement...

   The Image Problem:

   - LO converts frequencies both above and below by an offset of WIF.
     - Assume high-side injection:

   \[
   \begin{align*}
   W_{\text{lo}} & \rightarrow W_{\text{in}} \\
   \text{desired} & \rightarrow \text{interferer} \\
   W_{\text{wif}} & \leftarrow W_{\text{wif}}
   \end{align*}
   \]

   - Desired signal sees the LO as high-side injection
   - Image frequency sees the LO as low-side injection
   - Both are down-converted to WIF, where image signal appears as interference.

   - Most common way to alleviate this problem is to place an image reject filter before the mixer to filter out interference at the image frequency.

   \[
   \begin{align*}
   \text{LNA} & \rightarrow \text{Image Filter} \rightarrow \text{cross} \rightarrow \Rightarrow \\
   \text{Image reject filter.}
   \end{align*}
   \]

   - Now for the image reject filter design, we want WIF to be as high as possible, in direct conflict with the requirements for the channel select filter.

   - Choosing the IF is a tradeoff between these factors, and also which image frequencies might have worse interference.
To lessen the tradeoff between image rejection and channel selection, sometimes a dual-IF topology is used:

- 1st mixer converts to a relatively high IF to allow image rejection
- Some channel select filtering is performed here
- 2nd mixer converts to a low IF to allow channel selection

- Taken to the extreme, the first IF can actually be chosen higher than $\nu_e$ for image rejection purposes.

1. Image Reject Receivers

- Some receiver architectures have been introduced specifically to combat the image problem.

(a) Hartley Architecture

Effect of a 90° phase shift on a narrowband signal is to multiply its spectrum by $G(w) = -j \cdot \text{sign}(w)$

- Assume the RF input is $x(t) = A_{RF} \cos \omega_{RF} t + A_{IM} \cos \omega_{IM} t$
- $\omega_{LO} = \omega_{RF} + \omega_{IF} = \omega_{IM} - \omega_{IF}$ (High side injection)

- At point $A$, we have

$$X_A(t) = \frac{A_{RF}}{2} \sin (\omega_{LO} - \omega_{RF}) t = \frac{A_{IM}}{2} \sin (\omega_{IM} - \omega_{LO}) t$$

- Sum frequency components have been removed by LFFs.
At point B: \[ x_B(t) = \frac{A_{pe}}{2} \cos(w_0 - \omega_f)t + \frac{A_m}{2} \cos(w_m - w_0)t \]

At point C: (with a 90° phase shift, \( \sin \rightarrow -\cos, \cos \rightarrow \sin \))
\[ x_C(t) = \frac{A_{pe}}{2} \sin(w_0 - \omega_f)t + \frac{A_m}{2} \sin(w_m - w_0)t \]

Now sum \( x_A(t) \) and \( x_C(t) \), and
\[ x_{IF}(t) = A_{pe} \sin(w_0 - \omega_f)t \]

- Image components cancel out.
- 90° phase shift allows us to differentiate between sum and difference frequency components, its placement differs depending on if high or low-side injection is used.

How do we generate a 90° phase shift?

- Phase shift must only be relative to each other.

Consider 2-C sections: \( V_{in} \) \( \rightarrow \) \( m \) \( \rightarrow \) \( V_{out} \) \( \rightarrow \) \( 20 \log \omega \)

Low pass:

\[ V_{l} \rightarrow \frac{1}{2} \rightarrow V_{o} \rightarrow \frac{1}{2} \rightarrow 0 \]

\( \Delta \angle H(j\omega) = 90° \)

High pass:

\[ V_{h} \rightarrow 0 \rightarrow V_{o} \rightarrow 90° \rightarrow \frac{1}{2} \rightarrow W_{c} \]

Difference between phases will be 90° at all frequencies.
Amplitudes will only be equal at 3dB points, \( W_p = W_c \)
- If relative amplitudes are important (we will see that they are), then \( W_p \) and \( W_c \) must be well-matched, and signal must be narrowband.

Put high-pass in one branch, low-pass in the other.

Note: 3-dB loss can be significant, need enough gain before

Effect of gain 1: Phase mismatch

In a physical implementation, there will be finite gain of phase error introduced between the two branches. This will lead to incomplete image cancellation.
For analysis, assume $A_{RF} = A_{LO} = 1$ and assume lower path has a gain of $1 + \varepsilon$ while upper path has a gain of $1$.

Now, $\chi_A(t) = \frac{1}{2}[\sin(W_{1A} - W_{RF})t - \sin(W_{1A} - W_{2A})t]$

$\chi_C(t) = \frac{(1 + \varepsilon)[\sin(W_{2A} - W_{RF})t + \sin(W_{2A} - W_{LO})t]}{2}$

$\Rightarrow$ Desired signal amplitude $= \frac{2 + \varepsilon}{2}$, Image signal amplitude $= \frac{\varepsilon}{2}$

Relative Image Rejection Ratio as power ratio of desired to image at output:

$IRR = \frac{(2 + \varepsilon)^2}{\varepsilon^2} = \frac{4}{\varepsilon^2}$ (assuming $\varepsilon \ll 2$)

Following a similar procedure, we can show that the impact of a phase error $\Delta \phi$ between branches is:

$IRR = \frac{4}{\Delta \phi^2}$ in radians

So, the total $IRR$ due to gain and phase mismatches is:

$IRR_{total} = \frac{4}{\Delta \phi^2 + \varepsilon^2}$

Note that $\Delta \phi$ is in radians!

$\Rightarrow \Delta \phi = 1^\circ$ and $\varepsilon = 0.001$ correspond to $IRR = 41$ dB.

This typically not enough image rejection, so this architecture must still be used in conjunction with image reject filters (albeit with relaxed specifications).

(b) Weaver Architecture

- Generating the 90° phase shift in the signal can be problematic, so we need an alternate image reject architecture called the Weaver Architecture can be used.

Note: The adder is actually subtracting the two signals.
Analyze operation pictorially:

\[ \begin{align*}
    \text{RF input} & \quad \rightarrow \quad \text{output} \\
    -w_1 & \quad \rightarrow \quad w_1 \\
    -w_2 & \quad \rightarrow \quad w_2 \\
\end{align*} \]

**Useful facts for analysis:**

\[ \begin{align*}
    F(\sin wt) &= \frac{1}{2} (\delta(w + w_o) - \delta(w - w_o)) \\
    F(\cos wt) &= \frac{1}{2} (\delta(w + w_o) + \delta(w - w_o)) \\
    f(w) \ast \delta(w - w_o) &= f(w - w_o) \\
\end{align*} \]

Can run into the problem of secondary image, where the image frequency of the second set of mixers is not suppressed.

- Can replace LPFs by BPFs to get around this problem.

**Homodyne Receivers**

- Also referred to as direct conversion receivers.

- Mix incoming RF signal with \( w_o = w_c \) to downconvert directly to dc, only LPF is needed for channel selection.

The above topology works only for double sideband AM signals since they contain the same information on both sides of the spectrum.

- For PM, PM signals the different sides of the signal contain different information and after downconversion they overlap, resulting in a loss of information.

- For PM/FM use quadrature downconversion.

This is explored in a problem on the next HW assignment.
**Advantages:**
- No need to drive off-chip filters
- Higher level of integration
- Fewer components

**Drawbacks:**

1. **DC Offset:** DC offset voltages resulting from down-conversion appear as part of the signal and can saturate subsequent gain stages. Typically arises from finite isolation between mixer ports.

   ![Diagram of DC Offset](image)

   - LO mixes with itself, producing a DC component in the mixer output
   - Can also arise from interference leaking to LO mixer port.

   Even more problematic is a time-varying offset arising from LO power that leaks to the antenna and is reflected back.

2. **2 Solutions for this:**
   i. High pass filter the downconverted signal to remove DC offset.
      - May be used to ensure signal contains no energy (information) at dc
      - Need low cutoff frequency (large components)
      - Wastes spectrum
   ii. Perform periodic offset cancellation
      - Periodically sample the offset and cancel it out
      - Works best for TDM systems
      - Most common solution.

2. **I-Q Mismatch:** Mismatches in gain and phase between I/Q LO signals introduce shifts in the received signal constellation.
   - Leads to higher Pe

   For received QPSK signal: \( x(t) = \frac{a \cos(\omega t) + b \sin(\omega t)}{1} \)

   a, b = ±1, and effect of amplitude mismatch 6.

   Mix with: \( 2(1 + \frac{\xi}{2}) \cdot \cos(\omega t) \) (I-branch)
   Mix with: \( 2(1 - \frac{\xi}{2}) \cdot \sin(\omega t) \) (Q-branch)

   I output: \( a (1 + \frac{\xi}{2}) \cdot \cos(0) \)
   Q output: \( b (1 - \frac{\xi}{2}) \cdot \cos(0) \)

   ![Spectrum Diagram](image)
→ We show that a phase mismatch shifts the spectrum:

- The shift is not just a constant scale factor, but is influenced by the data in the other channel, and so appears as noise.

- Typical mismatches might be 1dB at 5°.

2. Even Order Distortion

- Up until now we have mainly discussed detrimental effects of 3rd order IM components, but for direct-conversion receivers the 2nd order IM components are also problematic.

- Due to IM2 in LNA:

- Due to mixer feed-through and IM2 of mixer:

→ Differential circuit topologies will help suppress 2nd order components.

3. Flicker Noise

- Since signal levels are still low when the spectrum has been translated to DC (only sees gain of LNA & mixer), IM noise of subsequent devices becomes important:

- May be necessary to use an active mixer to get more gain from the LNA/mixer combination.

- Despite all of these difficulties, direct conversion receivers are a very active research area due to the huge advantages of increased levels of integration:

- Standard in some applications that are amenable to their implementations (up to 1000x)
- Becoming more common in other applications.