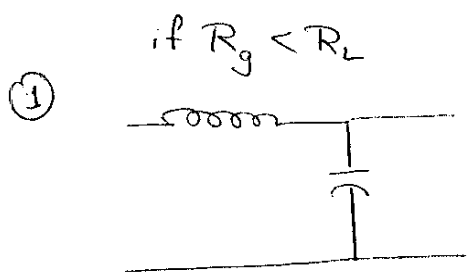
L-Networks

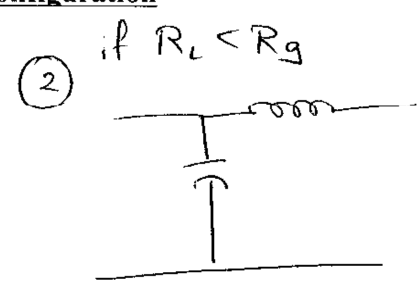
1. Simplest and most widely used network
2. The circuit receives it's name due to the component orientation which has the shape L

There are three different types of L-Net problems
1. L-Net with Real Load
2. L-NET with complex Load
3. L-Net with Complex Load and Source

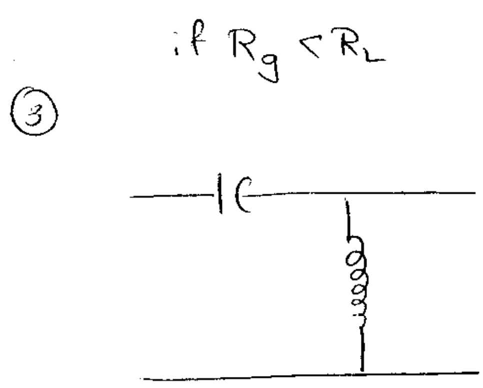
There can be 4 different circuit configurations for L-Nets

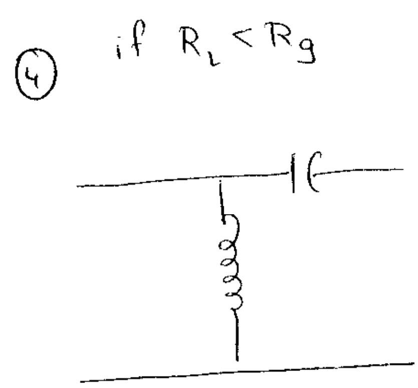
Low Pass Configuration





High Pass Configuration





Cook Book for solving L-Net problem with Real Load

1 Choose R_p and R_s

Note the impedance with greater value should be R_p

eg Design L-matching N/w to match an antenna with 73Ω to 50Ω line (R_g)

$R_L = 73\Omega = R_p$

$R_g = 50\Omega = R_s$

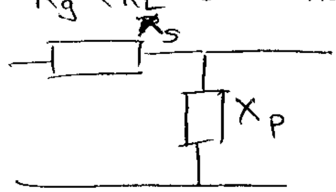
2 Choose Appropriate circuit

Choose one of the circuits from the above configurations

Eg [Look whether you want to design Low Pass or High Pass]

Let us design Low Pass

Since $R_g < R_L$ we choose ckt (1)



3 Calculate Q

$$Q = \sqrt{\frac{R_p}{R_s} - 1}$$

eg :-

$$Q = \sqrt{\frac{73}{50} - 1} = 0.678 = Q_s = Q_p$$

4 Calculate X_p and X_s

$$X_s = Q_s R_s$$

$$X_p = R_p / Q_p$$

eg :-

$$X_s = 0.678 \times 50 = 33.9\Omega$$

$$X_p = \frac{73}{0.678} = 107.67\Omega$$

5 Calculate L and C

$X_s = \omega L$

$X_p = \frac{1}{\omega C}$

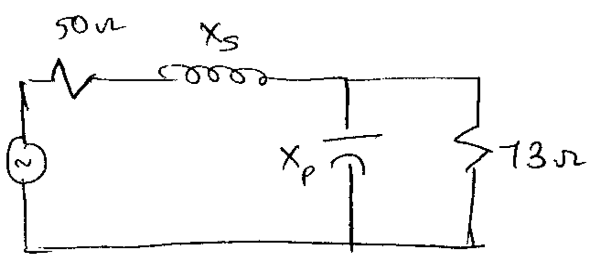
} for our case

eg

$X_s = \omega L \rightarrow L = \frac{X_s}{\omega} =$

$X_p = \frac{1}{\omega C} \qquad C = \frac{1}{\omega X_p} =$

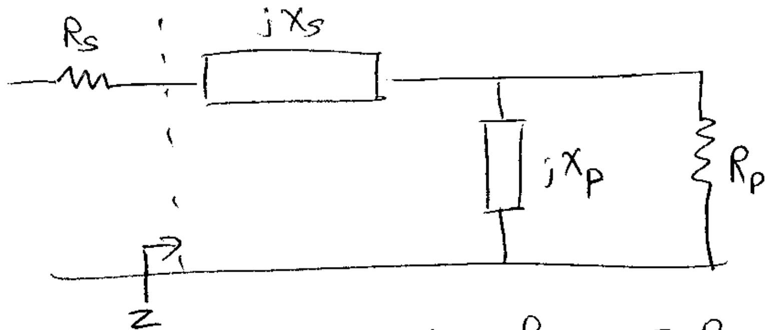
6 Draw the circuit with the calculated values



How do you get the expressions for X_p and X_s ?

Let us assume $R_g < R_L$

$\therefore R_s = R_g$ & $R_p = R_L$



For Maximum power transfer $Z = R_s$

from the ckt we get

$$Z = jX_s + (jX_p \| R_p)$$

$$\Rightarrow R_s = jX_s + \frac{jX_pR_p}{jX_p + R_p} \quad \text{— (1)}$$

Multiply eqn(1) by $R_p + jX_p$

$$R_s(R_p + jX_p) = jX_s(R_p + jX_p) + jX_pR_p$$

$$R_s(R_p) + jX_pR_s = jX_sR_p - X_sX_p + jX_pR_p \quad \text{— (2)}$$

Comparing Real parts of eqn (2)

$$R_sR_p = -X_sX_p \Rightarrow X_s = \frac{-R_sR_p}{X_p} \quad \text{— (3)}$$

Comparing Imaginary parts of eqn (2)

$$X_pR_s = X_sR_p + X_pR_p$$

$$X_p R_s = \frac{-R_p R_s}{X_p} R_p + X_p R_p$$

$$X_p^2 R_s = -R_p^2 R_s + X_p^2 R_p$$

$$X_p^2 (R_p - R_s) = R_p^2 R_s$$

$$X_p = \pm R_p \sqrt{\frac{R_s}{(R_p - R_s)}} \quad -(4)$$

Substituting eqn (4) in eqn (3)

$$X_s = \frac{-R_s R_p}{\pm R_p \sqrt{\frac{R_s}{R_p - R_s}}} = \mp \sqrt{R_s (R_p - R_s)} \quad -(5)$$

In the problems you see $Q_s = X_s R_s$? & $Q_p = \frac{R_p}{X_p}$

What do you know about Q

$$Q = \sqrt{\frac{R_p}{R_s} - 1} = \sqrt{\frac{R_p - R_s}{R_s}}$$

$$Q_s = X_s / R_s$$

$$X_s = Q_s R_s = \sqrt{R_s (R_p - R_s)} \quad -(6)$$

[look eqn (5) & (6) are same]

$$X_p = \frac{R_p}{Q_p} = \frac{R_p}{\sqrt{\frac{R_p - R_s}{R_s}}} = R_p \sqrt{\frac{R_s}{R_p - R_s}} \quad -(7)$$

[look eqn (7) & (4) are same]

2. (30 points) Design an L-matching network to match a load with $Z_L = 200\ \Omega$ to a $50\ \Omega$ line at 1 MHz. The circuit must pass 200 Hz waves.

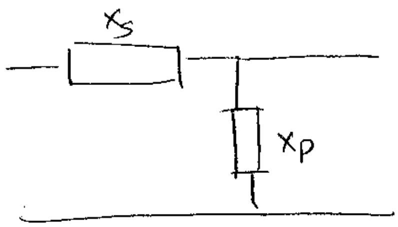
1] Choose R_p & R_s

[The impedance with greater value is R_p]

$R_p = 200\Omega$

$R_s = 50\Omega$

2] Choose appropriate ckt



(Refer notes for the reason)

3] Calculate Q

$$Q = \sqrt{\frac{R_p}{R_s} - 1} = \sqrt{\frac{200}{50} - 1} = 1.73$$

$Q_s = Q_p = Q = 1.73$

4) Calculate X_s & X_p

$X_s = Q_s R_s = (1.73)\,50 = 86.6\Omega$

$$X_p = \frac{R_p}{Q_p} = \frac{200}{1.73} = 115.6\Omega$$

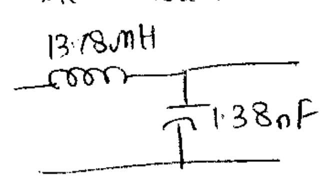
5) Calculate the components L & C

$X_s = \omega L = (2\pi \times 10^6)\,L = 86.6\Omega$

$L = 13.78\,\mu H$

$C = \frac{1}{\omega X_c} = 115.6\Omega \qquad C = 1.38\,nF$

6) Draw the ckt with the calculated values



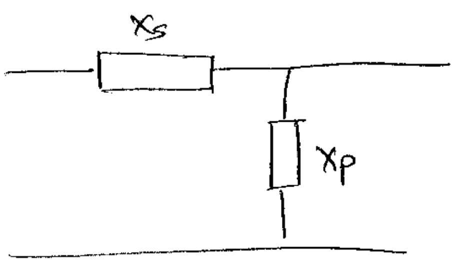
Design a L-Network to match a 100 ohm source to a 1000 ohm load at 100 MHz.Assume DC voltage must be transmitted.

1) Choose R_p & R_s [The impedance with greater value is R_p]

$R_p = 1000\Omega$

$R_s = 100\Omega$

2) Choose appropriate ckt



(Refer notes for Reason)

3) Calculate Q

$$Q = \sqrt{\frac{R_p}{R_s} - 1} = \sqrt{\frac{1000}{100} - 1} = 3$$

$Q = Q_s = Q_p = 3$

4) Calculate X_s & X_p

$X_s = Q_s R_s = 3 \times 100 = 300\Omega$

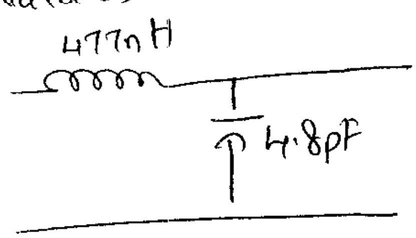
$$X_p = \frac{R_p}{Q_p} = \frac{1000}{3} = 333.33\Omega$$

5) Calculate the components L & C

$$L = \frac{X_s}{\omega} = 477nH$$

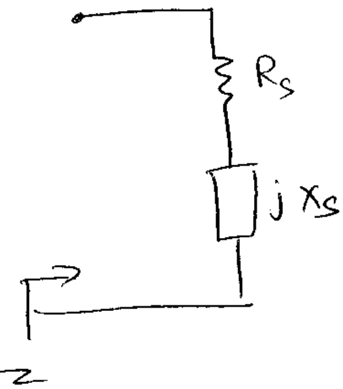
$$C = \frac{1}{\omega X_p} = 4.8pF$$

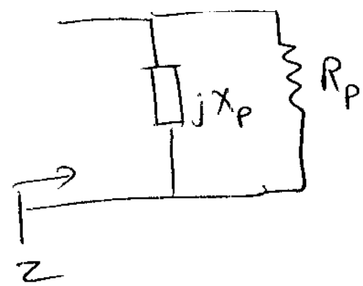
6) Draw the ckt with calculated values



Series to parallel Conversion

Convert





$$Z = R_s + jX_s = \frac{R_p jX_p}{R_p + jX_p}$$

Multiply by $(R_p + jX_p)$

$$(R_s + jX_s)(R_p + jX_p) = jR_pX_p$$

$$R_sR_p + jX_sR_p + jX_pR_s - X_sX_p = jR_pX_p \quad \text{— (1)}$$

Real part from eqn(1)

$$R_sR_p - X_sX_p = 0 \qquad R_p = \frac{X_sX_p}{R_s} \quad \text{— (2)}$$

Imaginary part from eqn(1)

$$X_sR_p + X_pR_s - R_pX_p = 0$$

$$R_p(X_s - X_p) + X_pR_s = 0$$

$$R_sX_p = R_p(X_p - X_s)$$

$$R_sX_p = \frac{X_sX_p}{R_s}(X_p - X_s)$$

$$R_s^2 X_p = X_s X_p^2 - X_s^2 X_p$$

$$X_p (R_s^2 + X_s^2) = X_s X_p^2$$

$$\boxed{X_p = \frac{R_s^2 + X_s^2}{X_s}}$$

$$X_p = X_s + \frac{R_s^2}{X_s}$$

$$R_p = R_s + \frac{X_s^2}{R_s}$$

Matching L-Networks with Complex Loads

Mostly the loads which are available are complex loads (Transmission lines, mixers, antennas etc)

There are two basic approaches for handling complex loads

1 Absorption - To actually absorb any stray reactances into the impedance matching network itself. This can be done by prudent placement of each matching network such that element capacitors are placed parallel to stray capacitances. the element inductors are placed in series with any stray inductance. The stray components values are substracted from the calculated element values leaving new element values

2 Resonance :- To resonate any ~~circuit~~ stray reactance with an equal & opposite reactance at freq of interest

Note:

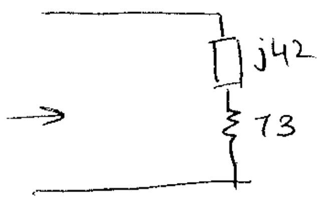
[If stray capacitance value is greater than the calculated elements values absorption cannot take place]

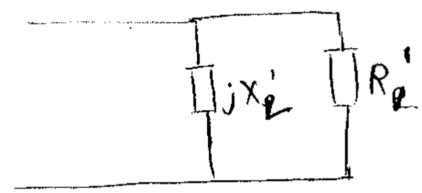
[Definitions from
RF Circuit Design
Chris Bowick
Newnes Press © 1982]

Cook Book for solving L-Net problem with ~~Real~~ Complex Load

1 Convert series impedance to shunt impedance

eg Design an L-net to match 73+j42Ω load to 50Ω source





$R_\ell' = \frac{R_L^2 + X_L^2}{R_L} = 97.16\Omega \quad X_\ell' = \frac{R_\ell' R_L}{X_L} = 168.87\Omega$

2 Choose R_p^* and R_s

Note the impedance with greater value should be R_p

eg

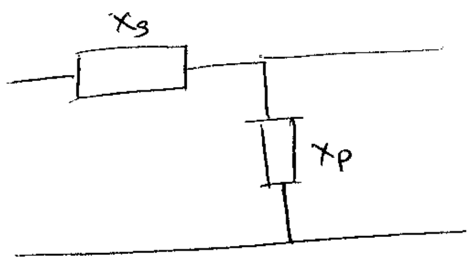
$R_p^* = R_L' = 97.16\Omega$

$R_s = 50\Omega$

3 Choose Appropriate circuit

Choose one of the circuits from the above configurations (HP circuit)

Eg



(Refer notes for reason)

4 Calculate Q

$Q_s = Q_p = Q = \sqrt{\frac{R_p}{R_s} - 1} = \sqrt{\frac{97.16}{50} - 1} = 0.9711$

eg

$\sqrt{\frac{97.16}{50} - 1} = 0.9711$

5 Calculate X_p' and X_s

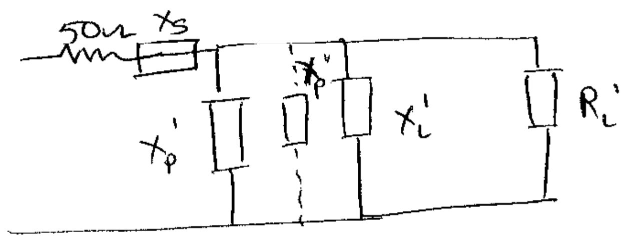
$$X_p' = \frac{R_p}{Q_p} \qquad X_s = Q_s R_s$$

eg

$$X_p' = \frac{97.16}{0.9711} = 100.05\,\Omega$$

$$X_s = 0.9711 \times 50 = 48.555\,\Omega$$

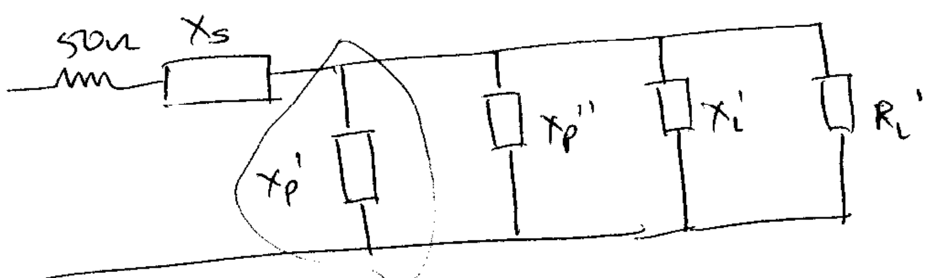
6 Draw the circuit with the calculated values



7 Absorption Method

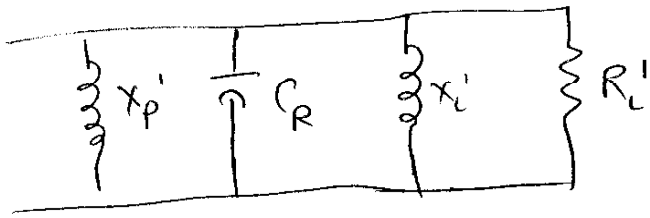
$$X_p'' = \frac{X_p' X_L'}{(X_L' - X_p')} = \frac{48.555 \times 168.87}{168.87 + 48.555}$$

$$= 37.71$$



not needed [is replaced using Xp''

8 Resonance



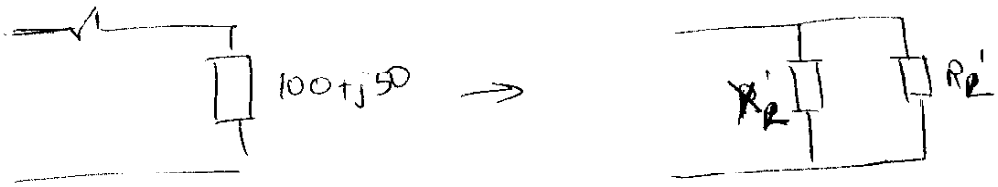
$$\omega^2 = \frac{1}{LC}$$

$$X_L' = X_p = \frac{1}{\omega C_R}$$

Calculate C_R

Design a L-Network to match a 100+j50 ohm load to a 50 ohm source. Try using Absorption and resonance (Design HPF)

1) Convert Series impedance to shunt (parallel) impedance



$$R_R' = \frac{R_L^2 + X_L^2}{R_L} = \frac{(100)^2 + (50)^2}{100} = 125\Omega$$

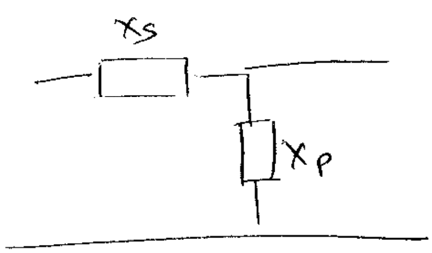
$$X_R' = \frac{R_L' R_L}{X_L} = \frac{125 \times 100}{50} = 250\Omega$$

1b) Choose R_s & R_p

$R_s = 50\Omega$

$R_p = R_L' = 125\Omega$

2) Choose appropriate ckt for matching



(Refer notes for reason)

3) Calculate Q [Use only real part of load]

$$Q_s = Q_p = Q = \sqrt{\frac{R_L'}{R_s} - 1} = \sqrt{\frac{125}{50} - 1} = 1.58$$

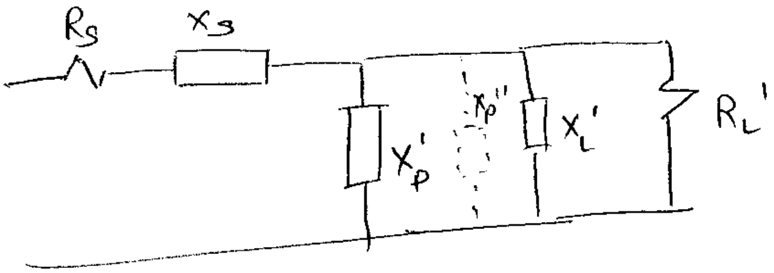
4) Calculate X_s & X_p

$$X_s = Q_s R_s = 1.58 \times 50 = 79\Omega$$

$$X_p' = \frac{R_p}{Q_p} = \frac{125}{1.58} = 79\Omega$$

contd

5) Draw ckt



6) Absorption method

[Defn - (Refer notes)]

$$X_p' = X_p'' \parallel X_L'$$

$$X_p' = \frac{X_p'' X_L'}{X_p'' + X_L'}$$

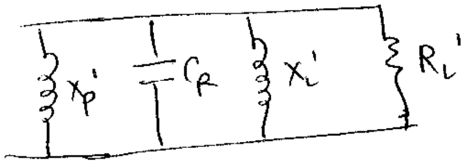
$$X_p' X_p'' + X_p' X_L' = X_p'' X_L'$$

$$X_p''(X_L' - X_p') = X_p' X_L'$$

$$X_p'' = \frac{X_p' X_L'}{(X_L' - X_p')} = 115\Omega$$

7) Resonance

[Defn - Refer notes]

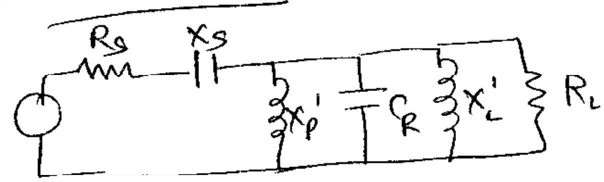


$$\omega^2 = \frac{1}{LC}$$

$$\omega L = X_L' = X_R = \frac{1}{\omega C_R}$$

$C_R = 0.6\,pF$

8) Draw ckt



(33 points) L matching

a. Design an L-matching network to connect a load that is 200 ohms to a 50 ohm line. Use a high-pass design.
b. For a frequency of 1 GHz, specify the C and L values. Sketch your final network.
c. Sketch a circuit to match a load that is 200 + j 100 ohms. (You may use either resonance or absorption, whichever is easier.)

a) & b)

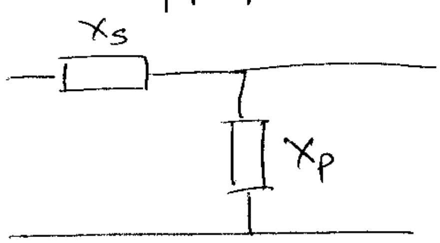
1) Choose R_p & R_s

[The impedance with greater value is R_p]

$R_p = 200\Omega$

$R_s = 50\Omega$

2] Choose appropriate ckt



(Refer notes for reason)

3] Calculate Q

$$Q = \sqrt{\frac{R_p}{R_s} - 1} = \sqrt{\frac{200}{50} - 1} = 1.73$$

$Q_s = Q_p = Q = 1.73$

4) Calculate X_s & X_p

$X_s = Q_s R_s = 86.6\Omega$

$X_p = \frac{R_p}{Q_p} = \frac{200}{1.73} = 115.6\Omega$

5) Calculate components L & C

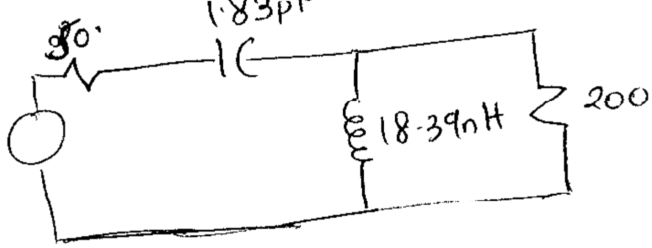
$$X_s = \frac{1}{\omega C} \Rightarrow C = \frac{1}{\omega X_s} = \frac{1}{2\times\pi\times1\times10^9\times86.6}$$

$$= 0.001837\times10^{-9}$$

$$= 1.83\,pF$$

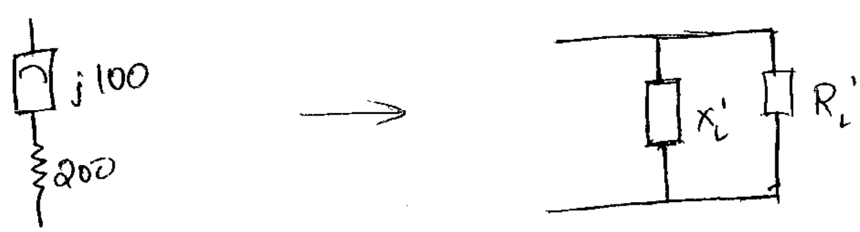
$$L = \frac{X_p}{\omega} \Rightarrow L = \frac{115.6}{2\pi\times1\times10^9} = 18.39\,nH$$

6) Draw the ckt with calculated values



c) if the Load is $\underline{200 + j100}$
(Absorption)

a) Convert series to parallel



$$R_L' = \frac{R_L^2 + X_L^2}{R_L} = \frac{(200)^2 + (100)^2}{200} = 250\Omega$$

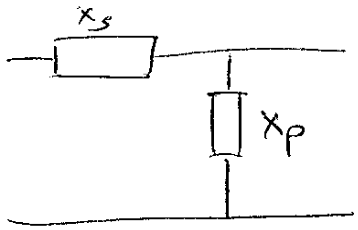
$$X_L' = \frac{R_L' R_L}{X_L} = \frac{R_L' \times R_L}{X_L} = \frac{250\times200}{100} = 500\Omega$$

b) Choose R_s & R_p

$R_s = 50\Omega$

$R_p = R_L' = 250\Omega$

c) Choose appropriate ckt



d) Calculate Q

$$Q_s = Q_p = Q = \sqrt{\frac{R_L'}{R_s} - 1} = \sqrt{\frac{250}{50} - 1} = 2$$

e) Calculate X_s & X_p

$X_s = Q_s R_s = 100\Omega$

$X_p' = \frac{R_p}{Q_p} = \frac{250}{2} = 125\Omega$

f) Calculate L & C

$$X_s = \frac{1}{\omega C} \Rightarrow C = \frac{1}{\omega X_s} = \frac{1}{2\times\pi\times1\times10^9\times 100}$$

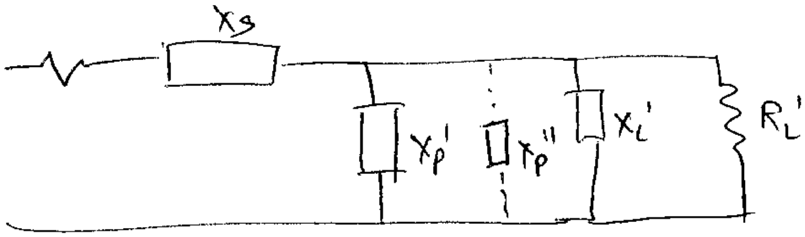
$$C = \frac{1}{2\pi\times10^{11}} = 0.159\times10^{-11}$$

$$= 1.59\,pF$$

$$L = \frac{X_p'}{\omega} = \frac{125}{2\pi\times10^9} = 19.89\,nH$$

– Don't waste time Calculating L & C skip to step (g)

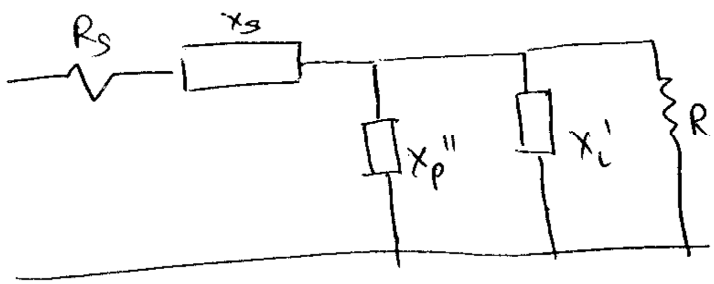
g) Draw the ckt



h) Calculate X_p''

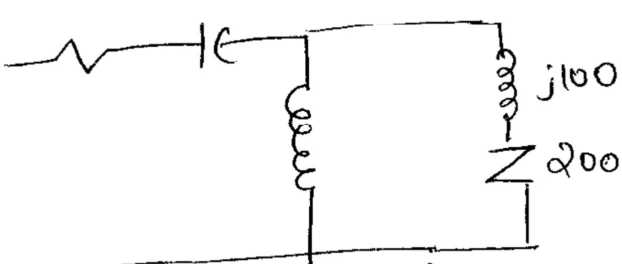
$$X_p'' = \frac{X_p' X_L'}{(X_L' - X_p')} = \frac{125 \times 500}{(500-125)} = 166.67$$

i) Draw final ckt

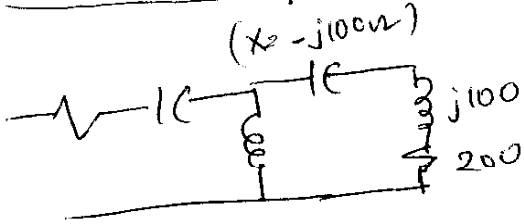


if we use resonance

a)



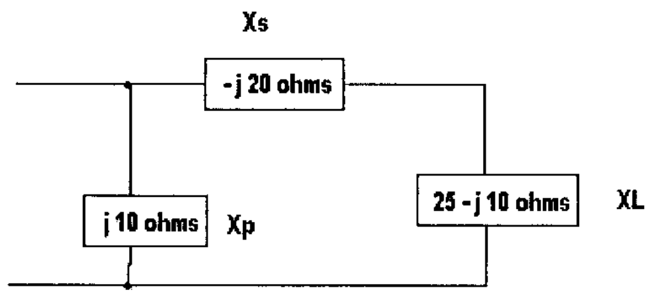
b)



[Add capacitance in series if it is inductive load & viceversa (Ha Ha Ha so simple)]

33 points) **L matching**

a. Design an L-matching network to connect a load that is 75 ohms to a 50 ohm line. Use a high-pass design.
b. For a frequency of 1 GHz, specify the C and L values.
c. The circuit shown below was designed to match a load of 25 ohms to the generator. Specify C and L components for proper matching when the load is 25 – j 10 ohms. If you cannot absorb the load, describe in detail how to redo the problem using resonance (you do not need to do the actual calculations, but do specify each step).



a)

1) Choose R_p & R_s

2) Choose appropriate ckt

3) Calculate Q

4) Calculate X_s & X_p

ii) Calculate L & C values
(b)

iii) The trick [Hint :- Use resonance]

A newly graduated engineer with antennas background was put on a test project. The supervisor gave the engineer one of the antennas the firm had developed and asked him to set up a circuitry which will match it to a source with source impedance 75 ohms. . The new kid ran through his antennas notes and found the impedance to be 60+j80 ohms. Try and help the engineer to solve his woes by designing a optimal circuitry using simple circuit components available.(The design should be performed at 1 MHz) (Try this)

— ENJOY —

Pi Network

- Pi Network is designed using Two L-Matching Networks
- We introduce a virtual resistance R between 2 L-Nets

eg.- Design a Pi Network to match a 100Ω source to 1000Ω load

1) Calculate Q

$$Q = \sqrt{\frac{\max(R_g, R_L)}{R} - 1}$$

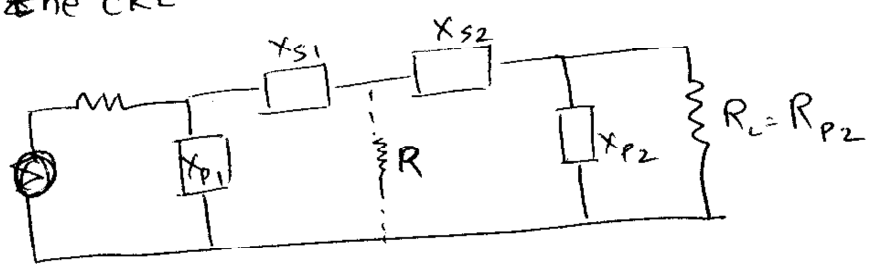
$$R = \frac{\max(R_g, R_L)}{Q^2 + 1}$$

eg

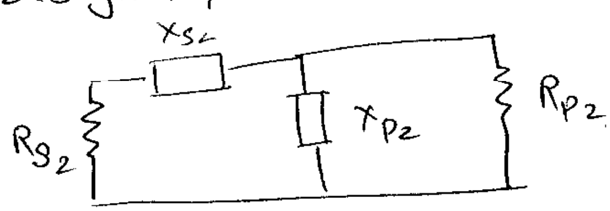
$Q = 15$

$$R = \frac{1000}{15^2+1} = 4.42\Omega$$

2) Draw the ckt



3) Design N/w 2



a) $X_{p2} = \frac{R_{p2}}{Q} = \frac{1000}{15} = 66.7\Omega$

b) $X_{s2} = Q R_{s2}$

$= 15 \times 4.42$

$= 66.3\Omega$

4) Design N/w(1)

a) Q of the 1st N/w is defined as

$$Q_1 = \sqrt{\frac{R_S}{R} - 1}$$

$$= \sqrt{\frac{100}{4.42} - 1} = 4.6$$

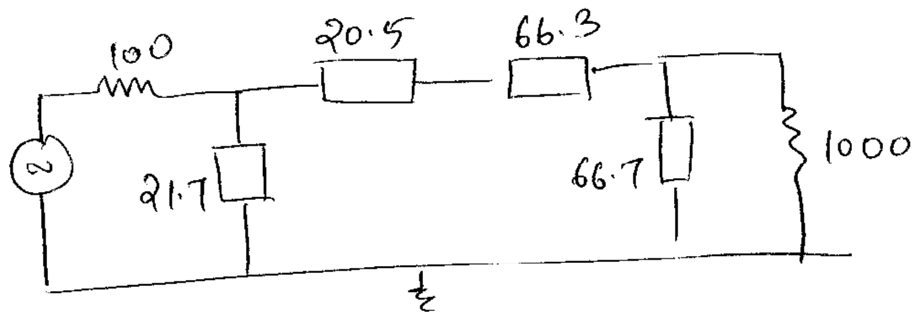
b) $X_{P_1} = \frac{R_S}{Q_1} = \frac{100}{4.6} = 21.7\Omega$

c) $X_{S_1} = Q_1 R$

$= 4.6 \times 4.46$

$= 20.51\Omega$

5) Draw the final ckt diagram



(NOTE)

If you had complex Load then
Step1 is converting series to parallel & then follow
Step1 to 5 above. Once you get to Step5 above
then use either absorption or resonance to match the
Load.

T-Network

- The T-Network is also designed using Two L-Matching Networks
- We introduce virtual resistance R between 2 L-Nets

eg:- Design a T-Network to match 10-ohm source to 50 Ω load.

$Q = 10$

1) Calculate Q, R

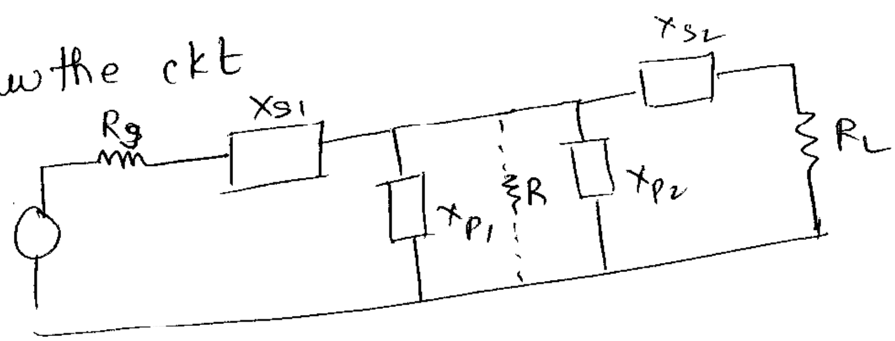
$$Q = \sqrt{\frac{R}{\min(R_g, R_L)} - 1}$$

$$R = R_g (Q^2 + 1)$$

$$= 10 (101)$$

$$= 1010 \Omega$$

2) Draw the ckt



3) Design N/w ① first

a) $X_{s_1} = Q R_s = 10(10) = 100 \Omega$

b) $X_{p_1} = \frac{R}{Q} = \frac{1010}{10} = 101 \Omega$

4) Design N/w (2)

a) $Q_2 = \sqrt{\frac{R}{R_L} - 1}$

$= \sqrt{\frac{1010}{50} - 1}$

$= 4.4$

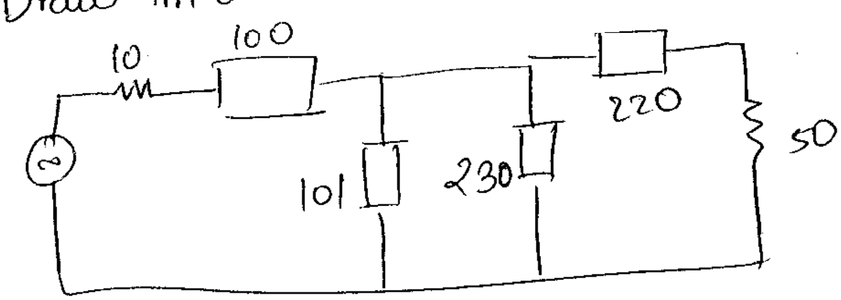
b) $X_{P_2} = \frac{R}{Q_2}$

$= \frac{1010}{4.4}$

$= 230\Omega$

c) $X_{S_2} = Q_2 R_L = 4.4 \times 50 = 220\Omega$

5) Draw final ckt



NOTE

If you have complex load follow step 1 to step 5 & then use resonance to match the load.

Design a pi-Network and T-Network to match a 100+j50 ohm load to a 50 ohm source ($Q = 12$)

π-Network

1) Convert series to shunt

2) Calculate Q, R

3) Draw ckt

4) Design N/w 2

a)

b)

5) Design N/w 1

a)

b)

c)

6) Do absorption or resonance

7) Draw final ckt

T- Network

1) Calculate Q, R

2) Draw ckt

3) Design N|w ①

a)

b)

4) Design N|w ②

a)

b)

c)

5) Use Resonance to match ckt

6) Draw final ckt with all components

BEST OF LUCK