Transmission Lines in
Computer Engineering
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Chapter

5

Crosstalk on Transmission Lines

5.1 Introduction

Crosstalk is the term used for signals coupled from one transmission line into another by time varying signals. The coupling, both capacitive and inductive, is generally due to the proximity of the two lines. This coupling is so undesirable that the strong terms *aggressor line* and *victim line* are used to describe the two interacting transmission-lines. This problem occurs on printed-circuit boards as well as on twisted-pair cables. An example of such coupling appears in the printed circuit board (PCB) representation of the two closely-spaced transmission-lines shown in figure 5.1.

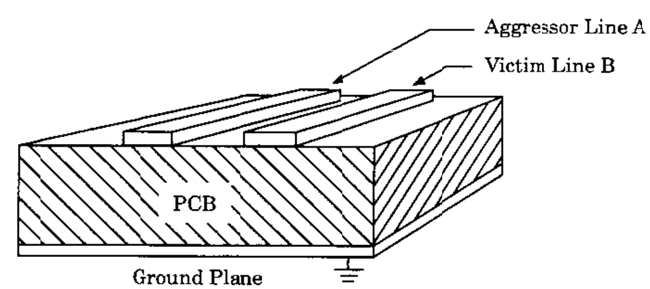

Figure 5.1 Two closely spaced PCB transmission lines.

5.2 Derivation of the Basic Crosstalk Relations

To simplify the analysis of this problem we will use the model shown in figure 5.2. Here we have shown two transmission lines of characteristic impedance Z_0 in close proximity. The two lines have a common ground. The top line, which is excited by a source, is the aggressor line. The bottom line, which is passive, is the victim line. The ungrounded conductors of both lines are sufficiently close to have mutual capacitive (electric) coupling of C_M farads/meter and mutual inductive (magnetic) coupling of L_M henries/meter. To keep the analysis simple, both lines are terminated in their characteristic impedance, so that any signals propagating in either direction on the lines cause no reflections. It is assumed that the coupling between the two lines is weak, so we will only concern ourselves with the crosstalk from line A induced in line B, but not with the crosstalk into line A due to the signals induced in line B.

As in all previous chapters, distance along the transmission line is expressed in terms of the variable z. The point on the line where crosstalk is taking place will be designated by the variable ζ. We will think of a signal propagating on aggressor line A arriving at the specific point $z = \zeta$ where it makes a differential contribution to the crosstalk signal on victim line B. The differential segment of line at $z = \zeta$ whose inductance and capacitance are responsible for the crosstalk has a length $\Delta\zeta$. To obtain the entire crosstalk signal it is necessary to sum the differential contributions for all values of ζ.

We will first derive the basic relation pertaining to capacitive crosstalk. We start the analysis by considering a section of both transmission lines, of width $\Delta\zeta$, which is located at the specific location $z = \zeta$, as shown in figure 5.3. The two lines are shown joined by a differential capacitance of value $C_M\Delta\zeta$. Transmission line B is assumed to be uncharged. A differential crosstalk current flows into

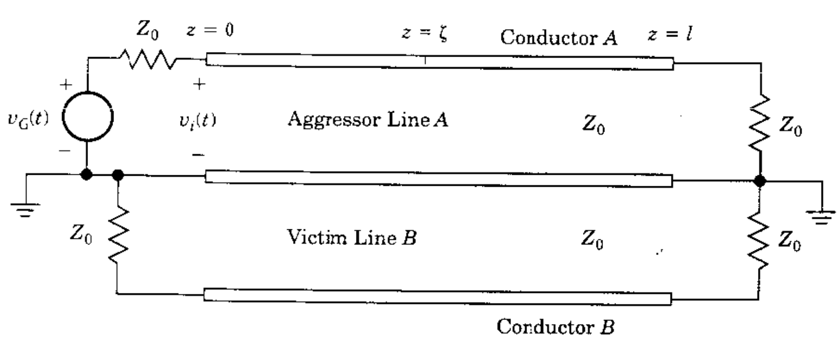

Figure 5.2 Model of two lines in close proximity.

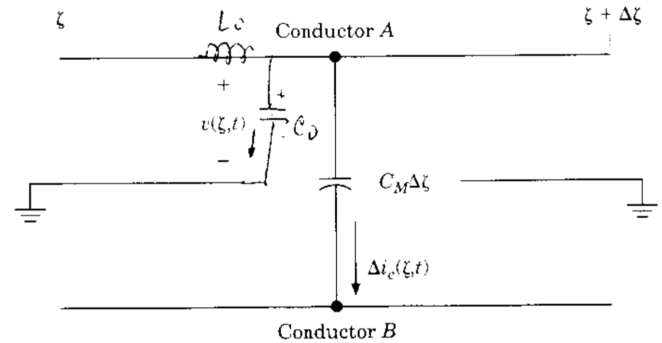

Figure 5.3 Schematic of a differential section of capacitively-coupled transmission-lines.

conductor B as a consequence of the voltage difference between the two conductors. Its value is

$$\Delta i_c(\zeta, t) = C_M \Delta\zeta \frac{\partial v(\zeta, t)}{\partial t} \tag{5.1}$$

This differential crosstalk-current is modeled by an ideal current source connected from the lower to the upper conductor of transmission line B, as shown in figure 5.4. Strictly speaking, the negative end of the current source should be shown attached to the upper conductor of transmission line A to be in agreement with figure 5.3. But the solution to the problem is not at all affected by showing the negative end of the current source attached to the ground conductor of transmission line B. The differential crosstalk-current at ζ sees the characteristic impedance Z_0 to *both* the left and right of ζ, hence it splits into two parts, with one half propagating to the right, and the other half propagating to the left. The two current waves, of value $\Delta i_c(\zeta, t)/2$, are accompanied by two voltage waves. The two voltage waves, which are of like polarity, are shown in figure 5.4 propagating away from the point $z = \zeta$. They are calculated by taking

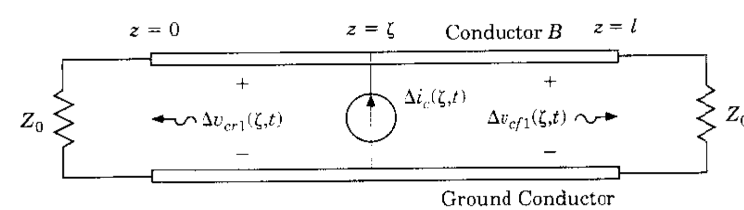

Figure 5.4 Differential crosstalk current due to capacitive coupling shown feeding into victim line B.

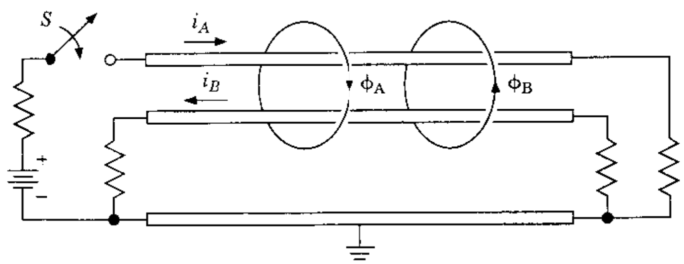

Figure 5.5 Model for reviewing Lenz's law.

(5.1), dividing by 2, and multiplying by Z_0. At $z = \zeta$ their values are

$$\Delta v_{cr1}(\zeta,t) = \Delta v_{cf1}(\zeta,t) = \frac{C_M \Delta\zeta}{2}\frac{\partial v(\zeta,t)}{\partial t} Z_0 \qquad (5.2)$$

Before we proceed with the analysis of inductively coupled crosstalk, we will first review Lenz's law.

In figure 5.5 are shown two transmission lines in close proximity. When switch S is closed, the direct-voltage source on the left side tries to establish the current i_A in the top conductor. This current is accompanied by a rising magnetic flux ϕ_A. Lenz's law states that the lower conductor will respond in such a way as to counteract the changes caused by the buildup of current in the top conductor. The lower conductor will therefore have a current induced in it with a polarity appropriate for the magnetic flux ϕ_B, which will attempt to cancel out the magnetic flux ϕ_A. The current i_B, and its polarity, are shown in figure 5.5. With this thumbnail review out of the way we can now return to the problem of inductively induced crosstalk.

As we did for the capacitively coupled analysis, we start by considering a section of both transmission lines, of width $\Delta\zeta$, which is located at $z = \zeta$, as shown in figure 5.6.

The two lines shown are magnetically coupled with a mutual inductance of value $L_M \Delta\zeta$. The dot orientation shown is in accordance with Lenz's law and is justified as follows. A rising current in line A will cause the upper inductor to assume a positive voltage polarity at the dot. This will cause the induced voltage in conductor B also to have positive polarity at the dot. The resultant current flow in line B will then try to cancel the magnetic field that conductor A is trying to establish.

Transmission line B is assumed to be deenergized, hence there is no previous voltage or current on line B. A differential crosstalk-voltage is induced in line B with the polarity shown. Its value is

$$\Delta v_c(\zeta,t) = L_M \Delta\zeta \frac{\partial i(\zeta,t)}{\partial t} \qquad (5.3)$$

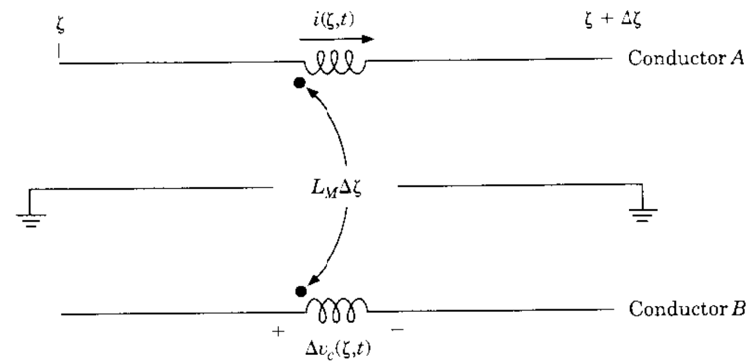

Figure 5.6 Schematic of a differential section of inductively-coupled transmission-lines.

This differential crosstalk-voltage is modeled by a voltage source as shown in figure 5.7. The differential crosstalk-voltage at ζ is connected at each end to a transmission line of characteristic impedance Z_0. Hence it encounters a total impedance of $2Z_0$. The current that flows at ζ is therefore the voltage of (5.3) divided by $2Z_0$. This local current creates the two current waves, as shown in figure 5.7, propagating away from $z = \zeta$. At $z = \zeta$ their values are

$$\Delta i_{cr2}(\zeta,t) = -\Delta i_{cf2}(\zeta,t) = \frac{1}{2Z_0} L_M \Delta\zeta \frac{\partial i(\zeta,t)}{\partial t} \qquad (5.4)$$

Note that $\Delta i_{cf2}(\zeta,t)$ is in fact negative since its direction of flow is contrary to that indicated in figure 5.7.

Since propagating voltage and current waves on transmission lines are related by the characteristic impedance Z_0, we find the values of the two voltage waves by simply multiplying (5.4) by the characteristic

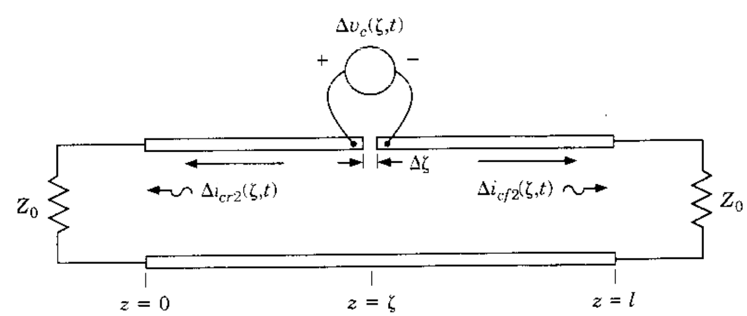

Figure 5.7 Differential crosstalk voltage due to inductive coupling shown exciting victim-line B.

impedance of transmission line B, which in our model is equal to Z_0, to obtain

$$\Delta v_{cr2}(\zeta,t) = -\Delta v_{cf2}(\zeta,t) = \frac{1}{2}L_M\Delta\zeta\frac{\partial i(\zeta,t)}{\partial t} \tag{5.5}$$

But on transmission line A, voltage and current waves are also related by the characteristic impedance, which in our model is Z_0. This allows us to replace $i(\zeta,t)$ with $v(\zeta,t)/Z_0$, for the final result

$$\Delta v_{cr2}(\zeta,t) = -\Delta v_{cf2}(\zeta,t) = \frac{1}{2Z_0}L_M\Delta\zeta\frac{\partial v(\zeta,t)}{\partial t} \tag{5.6}$$

Summing the capacitively and inductively induced voltages from (5.2) and (5.6), we get for the forward and reverse crosstalk voltages at some point on the line $z = \zeta$

$$\Delta v_{cf}(\zeta,t) = \frac{1}{2}\left(C_M Z_0 - \frac{L_M}{Z_0}\right)\frac{\partial v(\zeta,t)}{\partial t}\Delta\zeta \tag{5.7}$$

$$\Delta v_{cr}(\zeta,t) = \frac{1}{2}\left(C_M Z_0 + \frac{L_M}{Z_0}\right)\frac{\partial v(\zeta,t)}{\partial t}\Delta\zeta \tag{5.8}$$

We define the forward and reverse crosstalk coefficients as

$$K_{cf} \equiv \frac{1}{2}\left(C_M Z_0 - \frac{L_M}{Z_0}\right) \quad \text{(seconds/meter)} \tag{5.9}$$

$$K_{cr} \equiv \frac{\nu}{4}\left(C_M Z_0 + \frac{L_M}{Z_0}\right) \quad \text{(dimensionless)} \tag{5.10}$$

and we observe that K_{cf} can be positive, negative, and even zero. The ν appearing in the last equation represents the speed of signal propagation on the transmission lines. K_{cr} is defined in a seemingly peculiar manner, but it will become clear later that this is done in order to simplify the final crosstalk equations.

Using the above definitions and also the notation

$$v'(\zeta,t) \equiv \frac{\partial v(\zeta,t)}{\partial t} \tag{5.11}$$

in (5.7) and (5.8) we rewrite the two *basic crosstalk relations* in the more succinct form

$$\left.\begin{aligned}\Delta v_{cf}(\zeta,t) &= K_{cf}v'(\zeta,t)\Delta\zeta \qquad (5.12)\\ \Delta v_{cr}(\zeta,t) &= \frac{2}{\nu}K_{cr}v'(\zeta,t)\Delta\zeta \qquad (5.13)\end{aligned}\right\}$$

The above are the basic crosstalk equations, and are by no means the final solutions to the crosstalk problem. The derivation of the forward crosstalk equation will be undertaken in the next section and that for the reverse crosstalk will be taken up in the section that follows.

5.3 The Forward Crosstalk Equation

Assume that we have an incident wave $v_i(t - z/\nu)$ traveling from left to right on aggressor line A, as is shown in figure 5.8. On transmission line A this wave produces at $z = \zeta$ a time waveform described by $v_i(t - \zeta/\nu)$. According to (5.12), we have a forward crosstalk waveform on victim line B, at ζ, given by

$$\Delta v_{cf}(\zeta,t) = K_{cf}v_i'\left(t - \frac{\zeta}{\nu}\right)\Delta\zeta \tag{5.14}$$

When this waveform reaches an observer standing on line B, at a point z which is to the right of ζ, it is delayed by $(z - \zeta)/\nu$ seconds. Hence the argument in (5.14) has to be corrected by this amount. We get

$$\Delta v_{cf}(z,t) = K_{cf}v_i'\left(t - \frac{\zeta}{\nu} - \frac{z-\zeta}{\nu}\right)\Delta\zeta \tag{5.15}$$

Again it is reiterated that the term $(z - \zeta)/\nu$ represents the additional delay required for the signal to propagate from its point of origination ζ, to the observer's location at z. The above simplifies to

$$\Delta v_{cf}(z,t) = K_{cf}v_i'\left(t - \frac{z}{\nu}\right)\Delta\zeta \tag{5.16}$$

which in the limit as $\Delta\zeta$ goes to zero produces the integral

$$v_{cf}(z,t) = K_{cf}\int_0^z v_i'\left(t - \frac{z}{\nu}\right)d\zeta \tag{5.17}$$

Although aggressor line A can extend to the left of $z = 0$, this is the leftmost coordinate on victim line B. Hence the mutual coupling

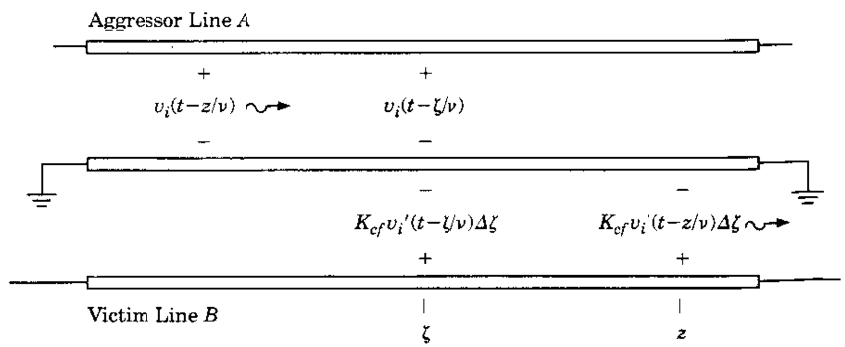

Figure 5.8 The creation of the forward crosstalk voltage.

between lines A and B begins at $z = 0$ which is the justification for the lower limit of integration. The upper limit of integration corresponds to the point z at which the observer is located. The waveform $v_i(\cdot)$ on line A makes no further contribution to the forward crosstalk after it has passed the observer who is located at point z. Once the waveform $v_i(\cdot)$ passes the observer at z it still continues to make crosstalk contributions on line B which propagate to the right, away from the observer, hence they cannot be sensed by the observer.

The integrand in (5.17) is not a function of the variable of integration ζ, hence it can be taken outside the integral. The final result of the integration is the forward-crosstalk equation

$$v_{cf}(z,t) = z\,K_{cf}\,v_i'\!\left(t - \frac{z}{\nu}\right) \tag{5.18}$$

where, as mentioned in (5.11), the prime in the expression $v_i'(\cdot)$ denotes differentiation with respect to time.

Example 5.1 We have the same arrangement as shown in figure 5.2. The source $v_G(t)$ has the appropriate waveform to create $v_i(t)$, the voltage at the input to the transmission line shown in figure 5.9a, and its derivative is shown in figure 5.9b. It is assumed (to simplify the drawings) that the signal transition time a is smaller than the transmission-line transit-time T and that K_{cf} is positive. We wish to plot the forward crosstalk waveforms at $z = \frac{1}{4}l$, $z = \frac{1}{2}l$, and $z = \frac{3}{4}l$.

To solve this problem we simply evaluate (5.18) for the three desired values of z to obtain

$$v_{cf}\left(\frac{1}{4}\,l,t\right) = \frac{1}{4}\,lK_{cf}v_i'\left(t - \frac{1}{4}\,T\right) \tag{5.19}$$

$$v_{cf}\left(\frac{1}{2}\,l,t\right) = \frac{1}{2}\,lK_{cf}v_i'\left(t - \frac{1}{2}\,T\right) \tag{5.20}$$

$$v_{cf}\left(\frac{3}{4}\,l,t\right) = \frac{3}{4}\,lK_{cf}v_i'\left(t - \frac{3}{4}\,T\right) \tag{5.21}$$

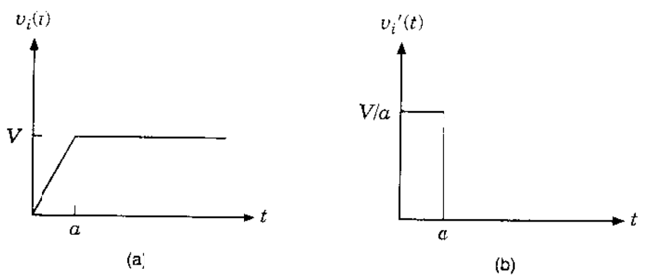

Figure 5.9 (a) The input voltage on aggressor line A (b) and its time derivative.

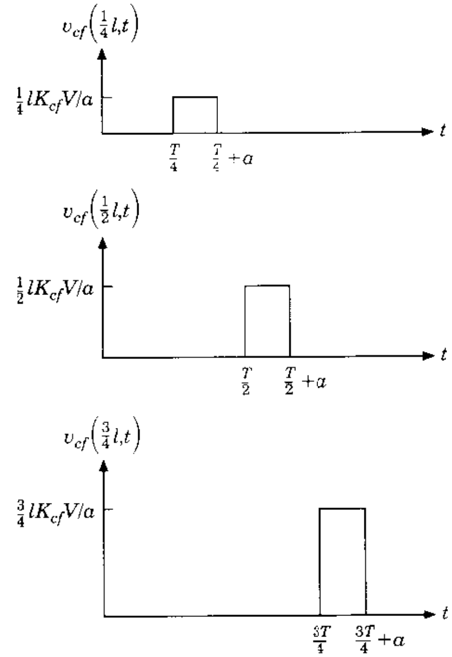

Figure 5.10 The forward crosstalk waveforms at $z = \frac{1}{4}\,l$, $z = \frac{1}{2}\,l$, and $z = \frac{3}{4}\,l$.

All that is left to be done is to use the waveform $v_i'(t)$ (shown in figure 5.9b) in the above equations to obtain the solutions shown in figure 5.10. ■

5.4 The Reverse Crosstalk Equation

Assume that we have an incident wave $v_i(t - z/\nu)$ traveling from left to right on aggressor line A, as is shown in figure 5.11. On transmission line A this wave produces a time waveform at ζ described by $v_i(t - \zeta/\nu)$. According to (5.13), we have a reverse crosstalk waveform on victim line B, at $z = \zeta$, given by

$$\Delta v_{cr}(\zeta,t) = \frac{2}{\nu}K_{cr}v_i'\left(t - \frac{\zeta}{\nu}\right)\Delta\zeta \tag{5.22}$$

The reverse crosstalk waveform propagates in the direction of decreasing z. When this waveform reaches an observer standing on line B, at a point z which is to the left of ζ, it takes on the form

$$\Delta v_{cr}(z,t) = \frac{2}{\nu}K_{cr}v_i'\left(t - \frac{\zeta}{\nu} - \frac{\zeta - z}{\nu}\right)\Delta\zeta \tag{5.23}$$

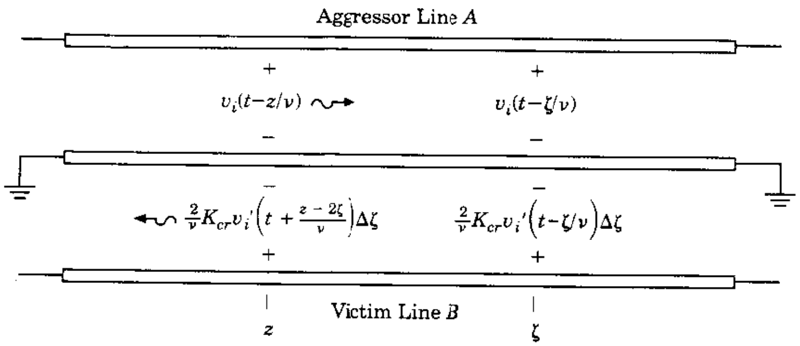

Figure 5.11 The creation of the reverse crosstalk voltage.

The term $(\zeta - z)/\nu$, which is positive since z is to the left of ζ, represents the additional delay required for the signal to propagate from ζ to the observer's location at z. The above reduces to

$$\Delta v_{cr}(z,t) = \frac{2}{\nu}K_{cr}v_i'\left(t + \frac{z-2\zeta}{\nu}\right)\Delta\zeta \tag{5.24}$$

which in the limit as $\Delta\zeta$ goes to zero produces the integral

$$v_{cr}(z,t) = \frac{2}{\nu}K_{cr}\int_z^l v_i'\left(t + \frac{z-2\zeta}{\nu}\right)d\zeta \tag{5.25}$$

The lower limit of integration corresponds to the point z at which the observer is located. The waveform $v_i(\cdot)$ on line A makes no contribution to the reverse crosstalk at point z until it has reached that point. The signal $v_i(\cdot)$ continues to make contributions to the reverse crosstalk waveform until it finally goes out of existence when it reaches the end of the transmission line at $z = l$. The result of the integration is

$$v_{cr}(z,t) = -K_{cr}v_i\left(t + \frac{z-2\zeta}{\nu}\right)\Bigg|_z^l \tag{5.26}$$

which, when evaluated at the limits becomes

$$v_{cr}(z,t) = K_{cr}\left[v_i\left(t - \frac{z}{\nu}\right) - v_i\left(t - 2T + \frac{z}{\nu}\right)\right] \tag{5.27}$$

Example 5.2 We have the same problem as we considered in example 5.1, but this time we are interested in finding the *reverse crosstalk* waveforms at $z = \frac{1}{4}l$, $z = \frac{1}{2}l$, and $z = \frac{3}{4}l$

Using (5.27) we readily obtain the three expressions

$$v_{cr}\left(\frac{1}{4}l,t\right) = K_{cr}\left[v_i\left(t - \frac{1}{4}T\right) - v_i\left(t - \frac{7}{4}T\right)\right] \tag{5.28}$$

$$v_{cr}\left(\frac{1}{2}l,t\right) = K_{cr}\left[v_i\left(t - \frac{1}{2}T\right) - v_i\left(t - \frac{3}{2}T\right)\right] \tag{5.29}$$

$$v_{cr}\left(\frac{3}{4}l,t\right) = K_{cr}\left[v_i\left(t - \frac{3}{4}T\right) - v_i\left(t - \frac{5}{4}T\right)\right] \tag{5.30}$$

These three waveforms are shown in figure 5.12. ■

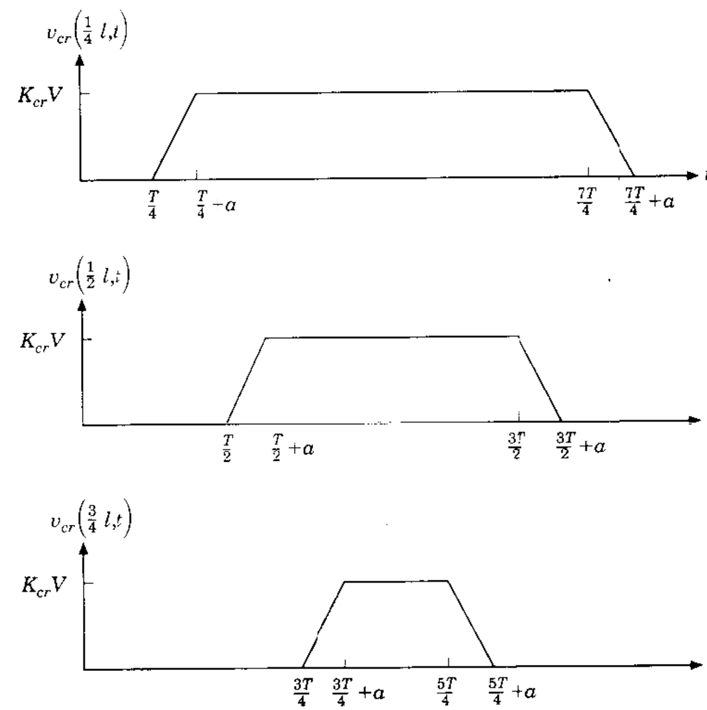

Figure 5.12 The reverse crosstalk waveforms at $z = \frac{1}{4}l$, $z = \frac{1}{2}l$, and $z = \frac{3}{4}l$.

5.5 Unmatched Aggressor Lines

The forward crosstalk equation (5.18) and the reverse crosstalk equation (5.27) were derived for the case of matched aggressor lines. The solutions can be readily extended to cover unmatched aggressor lines. We will assume that in figure (5.2) the resistors of value Z_0 terminating aggressor line A are replaced with some other resistors, so that the reflection coefficient at the right end of line A is ρ_L and the reflection coefficient on the left end is ρ_S. In this case it is best to consider separately the crosstalk due to incident waves and that due to reflected waves.

In the preceding two sections we plotted forward and reverse crosstalk along various positions on the line. In most practical cases it is easiest to observe the waveforms at either end of the victim line, a task that can be difficult to carry out at other locations. As a consequence we will henceforth confine our attention to the crosstalk waveforms at $z = 0$ and $z = l$. An example will best illustrate the method.

Example 5.3 Assume that in figure 5.2 line A is series terminated, hence the right end is terminated in an open circuit while the terminating impedance on the left end of the line is Z_0. Line A is excited by the waveform shown in figure 5.9 and line B is matched at both ends. We wish to determine the crosstalk on line B at $z = 0$ and at $z = l$.

The crosstalk due to the incident wave was discussed in examples 5.1 and 5.2. In this case we are interested in the solutions at both ends of the line. We simply substitute $z = 0$ and $z = l$ into (5.18) and (5.27) to produce the equations

$$v_c(l,t) = lK_{cf}\,v_i'(t - T) \tag{5.31}$$

$$v_c(0,t) = K_{cr}[v_i(t) - v_i(t - 2T)] \tag{5.32}$$

Substitution of the incident waveform $v_i(t)$ shown in figure 5.9 into the above equations produces the crosstalk result shown in figure 5.13.

Since the reflection coefficient at the right end of aggressor line A is unity, we know that the reflected wave will be identical to the incident wave except that it will occur T seconds later. Since the reflected wave propagates from right to left the idea of forward and reverse crosstalk on line B is reversed. As a consequence the response that previously occurred at $z = 0$ will now occur at $z = l$ and vice versa. Hence figure 5.14 was obtained from figure 5.13 by simply interchanging the two graphs, and delaying the response by T seconds.

Since the reflection coefficient on the left side of the aggressor line is zero, there are no further reflections and the problem is solved. It only remains to add the solutions at $z = 0$ and $z = l$. ■

We can readily generalize the above analysis to cover additional reflections on line A. We have to simply be aware of the direction of propagation of reflected waves in order to associate the proper response with the appropriate end of the line. It is also important to

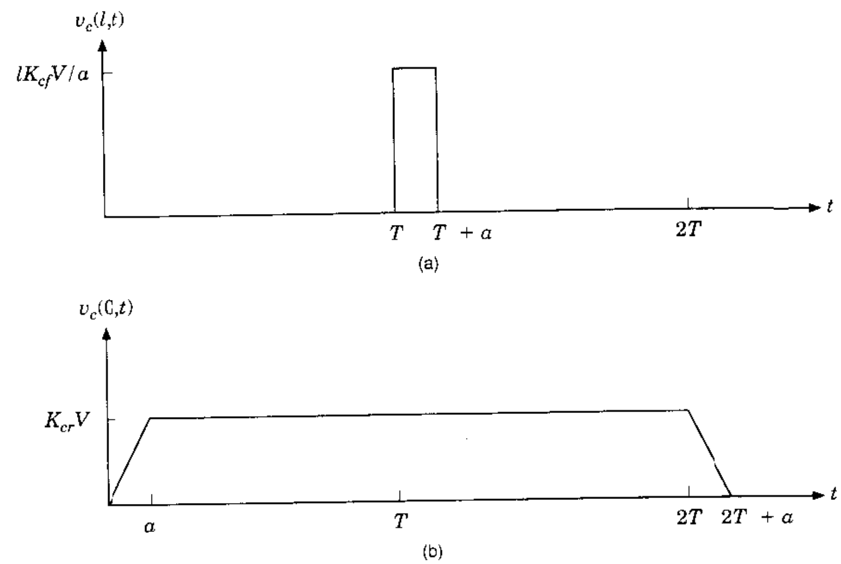

Figure 5.13 The solution due to the incident wave for the crosstalk problem considered in example 5.3.

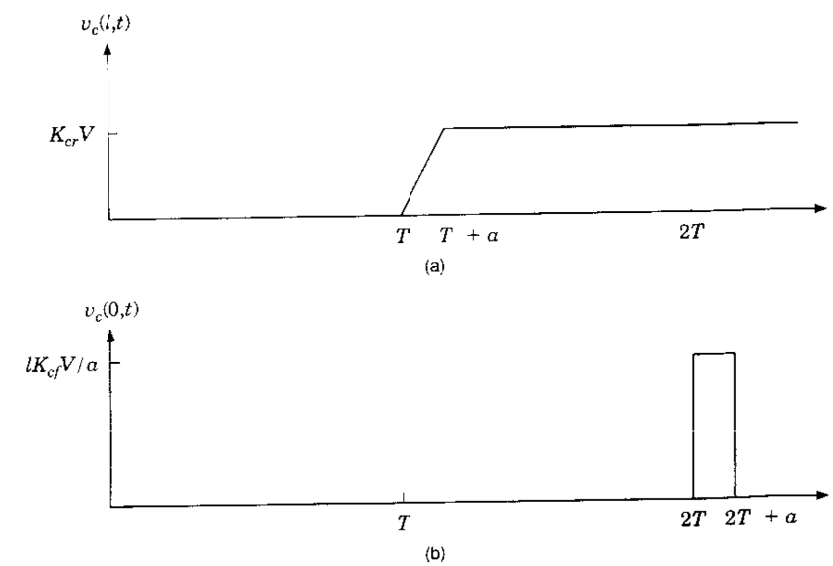

Figure 5.14 The solution due to the reflected wave for the crosstalk problem considered in example 5.3.

account for additional delays in integer multiples of T corresponding to the time that the reflection begins.

5.6 Unmatched Victim Lines

In the last section we considered the case of the unmatched aggressor line. Now we want to consider crosstalk for the case of the unmatched victim line. To facilitate the analysis we will confine ourselves to the case in which one end of the transmission line is terminated in a matched impedance, while the other end has a non-zero reflection coefficient. The method is best presented through an example.

Example 5.4 We will assume that in figure 5.2 the victim line is series terminated, so that its left-hand side has a terminating resistor Z_0, whereas the right side is terminated in an open circuit. Find and plot the crosstalk voltage at $z = 0$ on line B. As in the previous examples, it will be assumed that K_{cf} is positive.

In the discussion that follows it is necessary to refer constantly to figure 5.15. The forward crosstalk waveform increases in size as it progresses from left to right, as was illustrated in example 5.1, and takes on a maximum at $z = l$. The waveform $v_{cf}(l,t)$ can be obtained as the logical continuation of example 5.1 and is illustrated in the top diagram of figure 5.15. The right-hand side of line B is terminated in an open circuit, where the reflection coefficient is unity, hence the incident forward crosstalk voltage $v_{cf}(l,t)$ gives rise to the reflected voltage which is designated $v_{cf-}(l,t)$. This voltage propagates in the direction of decreasing z, and does not change in amplitude, since it is a direct consequence of reflection (and only an indirect consequence of crosstalk). It reaches the left end of line B after a time delay T. This is $v_{cf-}(0,t)$, the third waveform shown in figure 5.15.

From example 5.2 it is clear that the reverse crosstalk waveform $v_{cr}(z,t)$ is zero at the right termination, hence it is not at all affected by the open circuit termination of line B. The waveform, $v_{cr}(0,t)$ shown in figure 5.15, is the reverse crosstalk voltage at $z = 0$ and can be readily obtained by taking example 5.2 to its logical conclusion. The final result shown is $v_{tot}(0,t)$ and was obtained by summing $v_{cf-}(0,t)$ with $v_{cr}(0,t)$. ■

Other problems pertaining to unmatched victim lines can be solved in a manner similar to that used in the above example.

5.7 Measurement of Crosstalk Coefficients

To measure the crosstalk coefficients it is best to try to rely on methods that are very simple in that they do not require very complicated equipment. Sine-wave generators and conventional (non-storage) oscilloscopes are common equipment in most laboratories. The method described below is based on the use of this kind of equipment.

In the setup in figure 5.2, for the source $v_G(t)$ we select a sinusoidal voltage generator whose output is a sine wave of radian frequency

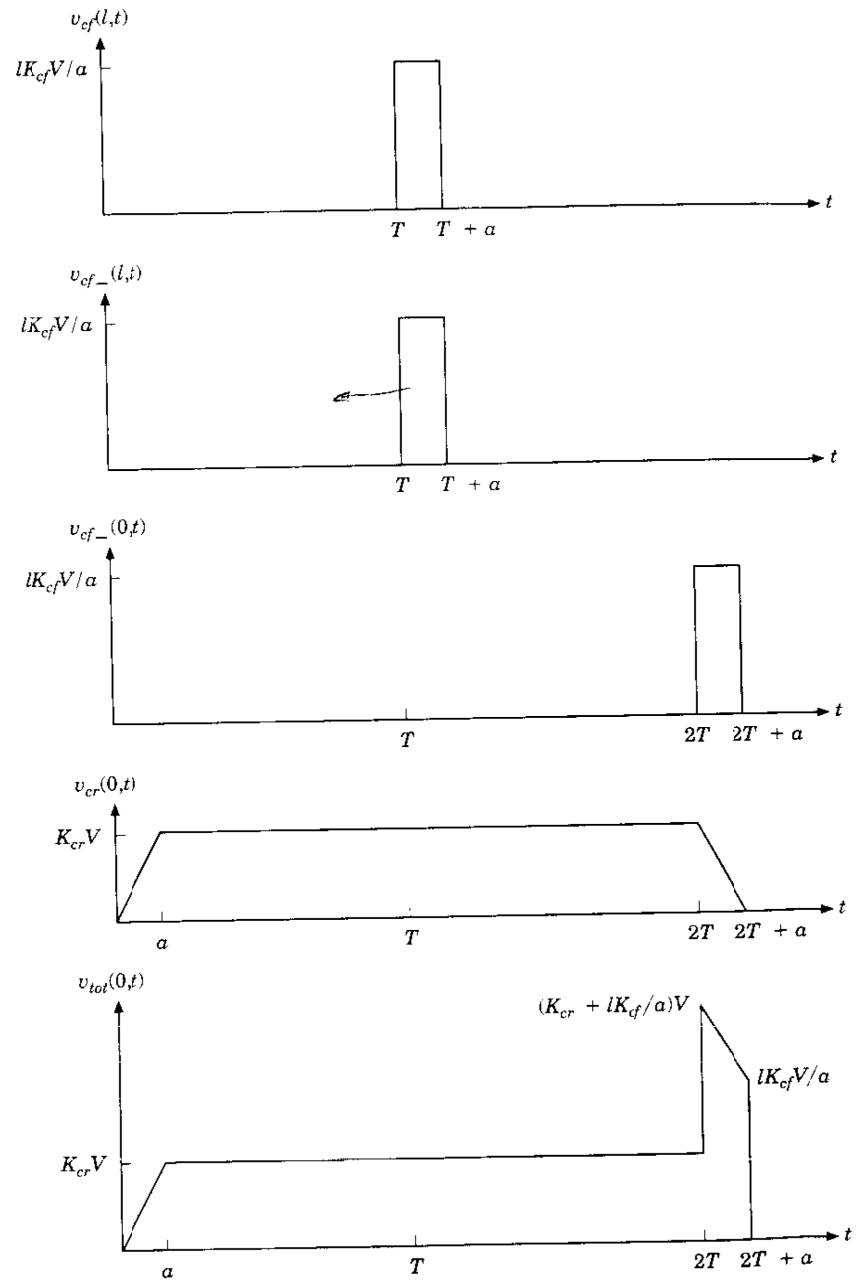

Figure 5.15 The progressive solution to the crosstalk problem considered in example 5.4.

$\omega_0 = 2\pi f_0$. As a consequence, the voltage input to the aggressor line A which is seen on an oscilloscope screen is

$$v_i(t) = V \sin \omega_0 t \tag{5.33}$$

whose peak-to-peak value is given by

$$|v_i(t)|_{pp} = 2V \tag{5.34}$$

Using (5.33) in (5.27), we find that the reverse crosstalk voltage observed on an oscilloscope at the left end of victim line B will be

$$v_{cr}(0,t) = K_{cr}\{V \sin \omega_0 t - V \sin[\omega_0(t - 2T)]\} \tag{5.35}$$

We wish to avoid having to determine accurately the product $\omega_0(2T)$ needed in the above expression, and at the same time to improve the observability of the reverse crosstalk waveform. Accordingly we adjust the frequency of the sine-wave generator $v_G(t)$ so that waveform $v_{cr}(0,t)$, which is displayed on the oscilloscope, is maximized. This occurs when

(sin $\omega_0 t$ cos $\omega_0 2T$ − cos $\omega_0 t$ sin $\omega_0 2T$)

$$\omega_0(2T) = \pi \tag{5.36}$$

or

$$f_0 = \frac{1}{4T} \tag{5.37}$$

When the above condition obtains, the observed reverse crosstalk is given by

$$v_{cr}(0,t) = 2K_{cr} V \sin \omega_0 t \tag{5.38}$$

whose peak-to-peak voltage is

$$|v_{cr}(0,t)|_{pp} = 4K_{cr} V \tag{5.39}$$

The forward crosstalk voltage is observed on an oscilloscope at the right end of the victim line. Substituting (5.33) into (5.18) we find the expression for this voltage to be

$$v_{cf}(l,t) = l\,\omega_0 K_{cf} V \cos[\omega_0(t - T)] \tag{5.40}$$

Using the condition from (5.36) we reduce the above to

$$v_{cf}(l,t) = l\,\omega_0 K_{cf} V \sin \omega_0 t \tag{5.41}$$

which has a peak-to-peak value given by

$$|v_{cf}(l,t)|_{pp} = 2l\,\omega_0 |K_{cf}| V \tag{5.42}$$

The absolute magnitude of K_{cf} is used above because the forward crosstalk coefficient can be either positive or negative, as is readily apparent from (5.9).

Taking suitable ratios, we find that the crosstalk coefficients can be determined using

$$K_{cr} = \frac{|v_{cr}(0,t)|_{pp}}{2|v_i(t)|_{pp}} \tag{5.43}$$

and

$$|K_{cf}| = \frac{|v_{cf}(l,t)|_{pp}}{l\,\omega_0 |v_i(t)|_{pp}} \tag{5.44}$$

To determine the sign of K_{cf} we observe that $v_{cf}(l,t)$ in (5.41) is in phase with $v_i(t)$ in (5.33) if K_{cf} is positive. If the two voltages are 180° out of phase, then this indicates that K_{cf} is negative.

Example 5.5 Two closely-spaced transmission-lines have a common length $l = 10$ m and a one-way delay $T = 50$ ns. From (5.37) we find that the proper frequency for performing the test is $f_0 = 5$ MHz. Accordingly the voltage generator is adjusted, as accurately as possible, to produce a sine wave of 5 MHz. The reverse crosstalk voltage observed at the left end of the victim line B is maximized by fine tuning the frequency adjusting dial on the sine-wave generator.

It is found, by observing $v_i(t)$ on a well-calibrated oscilloscope, that indeed $f_0 = 5$ MHz. It is also found that $|v_i(t)|_{pp} = 10$ V. In addition it is observed that $|v_{cr}(0,t)|_{pp} = 0.3$ V and $|v_{cf}(l,t)|_{pp} = 0.0315$ V. Furthermore, it is found that $v_{cf}(l,t)$ is 180° out of phase with $v_i(t)$ which indicates immediately that K_{cf} is negative.

From the data given we readily find by using (5.43) and (5.44) that $K_{cr} = 0.015$ and $K_{cf} = -0.01$ ns/m. ■

5.8 Conclusion

Crosstalk is of great concern in the design of interconnections in digital computer applications. One might be tempted to try to keep signal rise time small, in order to minimize forward crosstalk, but this choice does not always exist. Usually the drive signal is determined by the choice of the type of digital logic-devices. This specifies the drive signal, which determines the forms of $v_i(\cdot)$ and $v_i'(\cdot)$. To keep crosstalk coupling to a minimum, it is necessary to observe some precautions.

- The length of printed circuit board traces over which adjacent signal carrying lines are parallel should be kept to a minimum.
- If lines must run parallel, then they should be well separated so that the side-to-side capacitance of the lines will be small.
- Adjacent signal layers in printed circuit boards should be separated by ground planes.
- Adjacent signal-carrying conductors should be separated with conductors grounded at both ends. This is particularly feasible in the case of flat parallel cables. An example of this is the cable used to connect the parallel ports of personal computers to printers.
- Use of twisted-pair transmission-lines confines the electromagnetic field largely to the two wires and has a tendency to cancel signals induced from other lines, because both wires pick up essentially the same signal.
- Where possible use coaxial cable. Coaxial cable confines almost the entire electromagnetic field to the region between the two conductors. The field radiated to the space outside the cable is minimal and conversely the coaxial cable picks up very little signal from other circuits.

Bibliography

1. B. Hart, *Digital Signal Transmission — Line Circuit Technology* (Chapter 6), Van Nostrand Reinhold Co. Ltd., Wokingham, U.K., 1988.
2. A. Feller, H. R. Kaupp, and J. J. Digiacomo, "Crosstalk and Reflections in High-Speed Digital Systems, *Proceedings — Fall Joint Computer Conference*, 1965, pp. 511–525.
3. *High Speed Digital Symposium*, held at Hewlett-Packard, Paramus, New Jersey, August 8, 1990. Valuable notes were made available. Of particular interest were those by Dr. Edward P. Sayre of North East Systems Associates, 256 Great Road—Suite 13, Littleton, Massachusetts.

Problems

P5.1 In the paragraph above (5.2) the statement is made that "Strictly speaking, the negative end of the current source should be shown attached to the upper conductor of transmission line A to be in agreement with figure 5.3. But the solution to the problem is not at all affected by showing the negative end of the current source attached to the ground conductor of transmission line B." Convince yourself by examples that this is indeed the case.

P5.2 How can (5.9) and (5.10) be modified to account for the fact that the two interacting transmission-lines possess different characteristic-impedances?

Hint: The solution to this problem requires a re-examination of the deriving (preceding) equations.

P5.3 For example 5.1 find the expression and plot the forward crosstalk at the receiving (right) end of the line.

P5.4 For example 5.2 find the expression and plot the reverse crosstalk at the sending (left) end of the line.

P5.5 Determine the following:

(a) Where is the point on the line where we have reverse crosstalk exclusively?

(b) Where is the point on the line where we have forward crosstalk exclusively?

(c) Consider the answers to parts (a) and (b). Can a setup as shown in figure 5.2 be used to measure the forward and reverse crosstalk coefficients by using the waveform of figure 5.9 for excitation?

P5.6 A raised cosine pulse, described by

$$v_i(t) = V(1 - \cos 2\pi f_0 t), \qquad \text{for } 0 \le t \le 1/f_0$$

is introduced on the left end of the aggressor line shown in figure 5.2. Assume that K_{cf} is positive and $T \ge 1/(2f_0)$. Plot $v_{cf}(l,t)$ and $v_{cr}(0,t)$ which appear at the ends of the victim line.

P5.7 Suppose in figure 5.2 the victim line has length l but the aggressor line is longer than l. In what way, if any, would that affect the results?

P5.8 Suppose in figure 5.2 the aggressor line has length l but the victim line is longer than l. In what way, if any, would that affect the results?

P5.9 In example 5.3, find the total voltages $v_{tot}(0,t)$ and $v_{tot}(l,t)$ from the waveforms plotted in figures 5.13 and 5.14.

P5.10 Suppose in figure 5.2 the aggressor line is terminated in a short circuit on the right and is matched with a resistance Z_0 on the left end. Reanalyze example 5.3.

P5.11 Suppose in example 5.4 the victim line is terminated in a short circuit on the right and in Z_0 on the left. Reanalyze example 5.4 to find and plot $v_{tot}(0,t)$.

P5.12 Repeat example 5.4 assuming that line B is parallel terminated, namely the right-hand side is terminated in Z_0, whereas the left-hand side is terminated in a short circuit. Solve for $v_{tot}(l,t)$.