Understand The Basics Of Microstrip Directional Couplers

This exposition on directional couplers offers designers the important theory, characteristics, and equations needed for microwave applications.

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DIRECTIONAL couplers play key roles in balanced and double-balanced mixers, balanced amplifiers, dividers, combiners, phase shifters, attenuators, modulators, discriminators, and feed networks in antenna arrays. A directional coupler is a reciprocal four-port device which provides two outputs when a signal is applied to its input. A hybrid network (or 3-dB directional coupler) is a special class of directional coupler in which the signals at the two output ports are equal.[1,2,3,4] The important characteristics of directional couplers are coupling, directivity, isolation, matching, insertion loss, phase and amplitude balance, power split, and bandwidth.

A directional coupler can have three different types of directivity (Fig.1).[4] In this article, port 1 is chosen as input to the coupler. An ideal directional coupler should have an infinite directivity. In an actual directional coupler, the isolated port is never completely isolated due to mismatching of terminations, losses, discontinues and manufacturing tolerances.

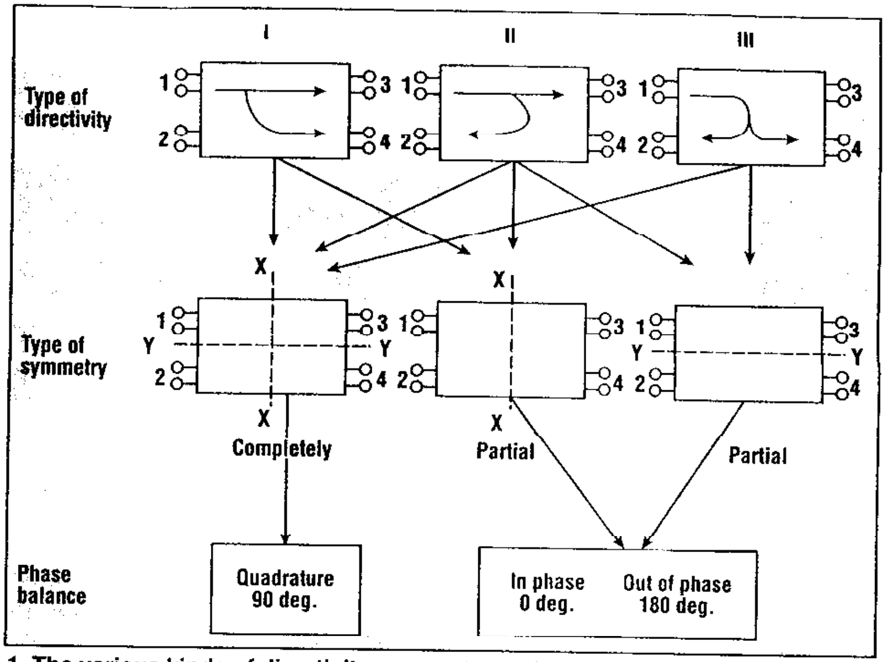
Phase balance is the relative phase difference of output waves. Quadrature (90 deg.) and in-phase-out-of-phase (0-deg. or 180-deg.) directional couplers are the most popular devices (Fig.1).

For the purposes of analysis and calculation, symmetry is a very important characteristic of directional couplers. There are three types of symmetry: complete (axes XX and YY), XX-axis partial symmetry, and YY-axis partial symmetry. Connections between types of symmetry, directivity and phase differences are shown in Fig 1.[4]

Table 1: Characteristics of ring and two-branch hybrids

Parameters	Hybrid ring	Two-branch hybrid
Input (port1) reflection coefficient	$-\dfrac{2\Gamma_2\Gamma_3\Gamma_4 + \Gamma_2 + \Gamma_3}{2 + \Gamma_4(\Gamma_2 + \Gamma_3)}$	$-\dfrac{2\Gamma_2\Gamma_3\Gamma_4 + \Gamma_3 - \Gamma_4}{2 + \Gamma_2(\Gamma_4 - \Gamma_3)}$
Insertion loss (dB)	$20\log\dfrac{\varphi_1}{\sqrt{2}(1+\Gamma_2)(1+\Gamma_3\Gamma_4)}$	$20\log\dfrac{\varphi_2}{\sqrt{2}(1+\Gamma_3)(1+\Gamma_2\Gamma_4)}$
Coupling (dB)	$20\log\dfrac{\varphi_1}{\sqrt{2}(1+\Gamma_3)(1+\Gamma_2\Gamma_4)}$	$20\log\dfrac{\varphi_2}{\sqrt{2}(1+\Gamma_4)(1-\Gamma_2\Gamma_3)}$
Isolation (dB)	$20\log\dfrac{\varphi_1}{(1+\Gamma_4)(\Gamma_2-\Gamma_3)}$	$20\log\dfrac{\varphi_2}{(1+\Gamma_2)(\Gamma_3+\Gamma_4)}$
Directivity (dB)	$20\log\dfrac{\sqrt{2}(1+\Gamma_3)(1+\Gamma_2\Gamma_4)}{(1+\Gamma_4)(\Gamma_2-\Gamma_3)}$	$20\log\dfrac{\sqrt{2}(1+\Gamma_4)(1-\Gamma_2\Gamma_3)}{(1+\Gamma_2)(\Gamma_3+\Gamma_4)}$

Parameters of actual directional couplers differ from the ideal due to the mismatching of termina-



1. The various kinds of directivity, symmetry and phase balance that a directional coupler can have are shown here.

tions, losses, discontinuities, as well as manufacture tolerances.

The two most popular ring directional couplers are of length 3/2Λ and Λ, where Λ is the guide wavelength. In the case of perfect input matching of the ring coupler of length 3/2 Λ (Fig. 2a),

$$Y_1^2 + Y_2^2 = 1 \qquad (1)$$

where:

$Y_1 = z_0/z_1$ and $Y_2 = z_0/z_2$ are normalized admittances of the ring coupler (Fig. 2a).

The scattering matrix of the ring coupler is:[5]

$$[S] = -i\begin{bmatrix} 0 & Y_1 & Y_2 & 0 \\ Y_1 & 0 & 0 & Y_2 \\ Y_2 & 0 & 0 & -Y_1 \\ 0 & Y_2 & -Y_1 & 0 \end{bmatrix} \qquad (2)$$

It follows from Eq. 2 that arg $S_{12}/S_{13} = 0$ and arg $S_{42}/S_{43} = \pi$, i.e., the ring is an in-phase-out-of-phase directional coupler. Since $S_{14} = S_{23} = S_{32} = S_{41} = 0$ and $|S_{11}| = |S_{22}| = |S_{33}| = |S_{44}| = 0$, the four-port network will have ideal directivity of type II (Fig.1) and perfect matching.

The simplest hybrid ring has equal segment admittances $Y_1 = Y_2$, which by substitution in Eq. 1, yields:

$$Y_1 = Y_2 = 1/\sqrt{2} \qquad (3)$$

Usually the ring coupler is loaded by terminations at ports 1, 2, 3, and 4 with reflection coefficients $\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4$. Characteristics of ring couplers versus reflection coefficients are shown in Table 1, where:

$$\phi_1 = 2(1 + \Gamma_2\Gamma_3\Gamma_4) + (1 + \Gamma_4)(\Gamma_2 + \Gamma_3) \qquad (4)$$

Graphs of operating parameters of the hybrid ring versus termination reflection coefficients are shown in Fig. 3. Absolute values of reflection coefficients of terminations connected to adjacent ports (with respect to the input) mainly affect the input matching, while the isolation is determined by relative values of reflection coefficients. It should be noted that the previously discussed characteristics of the ring coupler do not account for power dissipated as a result of conductor, dielectric, and radiation losses.

Define the resistive losses in terms of normalized attenuation:

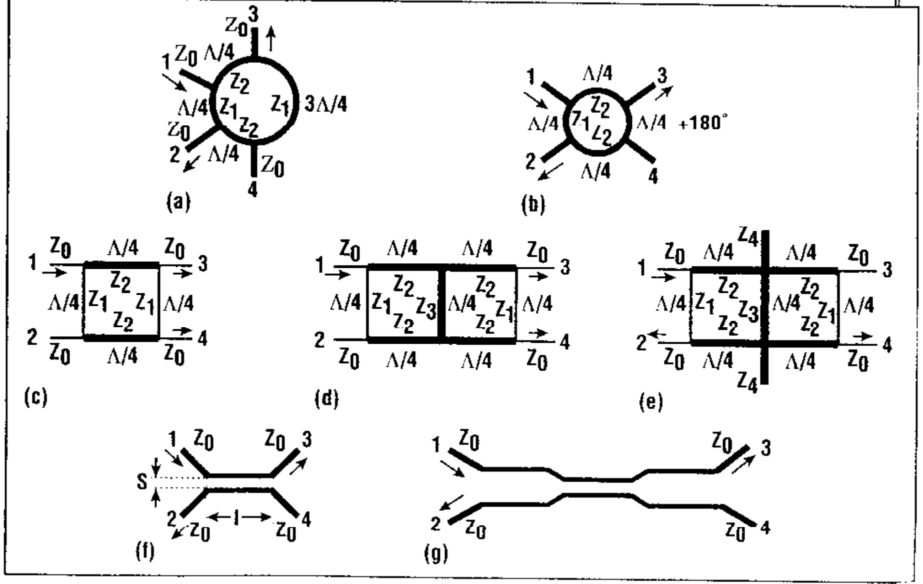
$$\alpha l = \frac{\pi}{Q\Lambda} l \ (Nepers) \qquad (5)$$

where:

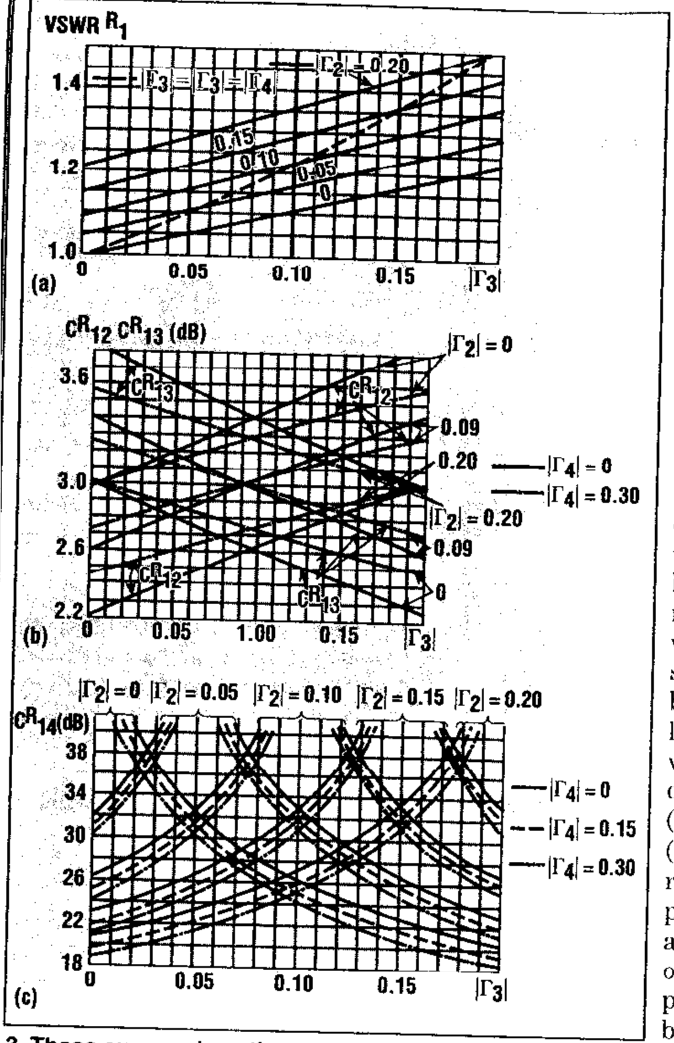
$Q = \beta/2\alpha$ is the quality factor of a 1/4-wave resonator which is shorted at one end and $\beta = 2\pi/\Lambda$ is the phase constant.

Characteristics of planar ring couplers depend on transmission line losses (Table 2).[4,7]

Consider the influence of fabrication tolerances on parameters of the hybrid ring. The results of calculations of the hybrid ring on microstrip line with a relative permittivity of substrate $\epsilon = 9.8$ are provided in



2. Directional couplers fall into three basic architectures: ring couplers (a and b), branch couplers with either two or three branches (c, d, and e), and coupled lines (f and g).



3. These curves show the matching (a), insertion loss (b), and isolation (c) of a hybrid-ring as a function of termination reflection coefficients ($|\Gamma_1$, $|\Gamma_2$, $|\Gamma_3$, and $|\Gamma_4$).

Fig. 4,[8]

$$\Delta X = \Delta W/\Delta h$$

where:

ΔW = the width of the microstrip line, and

h – the thickness of the substrate.

In the ring directional coupler, it is often necessary to account for the influence of discontinuities where the coupler connects to the input and output lines. This is most important for the high-frequency range when the size of the discontinuities is more than 1/10 the height of the guide wavelength. This type of discontinuity is equivalent to a T-connection. The characteristics of isolation, VSWR, and insertion losses of the hybrid ring with discontinuities and without discontinuities are shown in Fig. 5.[6,8]

The ring coupler of length $3/2\Lambda_0$ has the disadvantage of a narrow bandwidth (approximately 20 percent) due to the increased length of the 3/4-wavelength section (Fig. 2a). Also, it occupies a large circuit area. There are modifications of this coupler in which a 1/2-wavelength-long segment of the 3/4-wavelength section is replaced by a 1/4-wavelength section with a fixed 180-deg. phase shifter (phase inverter (Fig. 2b).[9] This Λ ring coupler has perfect isolation and is independent of frequency. The phase difference between the two output ports (0 deg. or 180 deg.) is also independent of frequency. In practice, bandwidth is equal to one octave.

The branch-line coupler (Fig. 2c to e) consists of a main line which is coupled to a secondary line by $\Lambda_0/4$-long branches spaced by $\Lambda/_0/4$.[10,11] The bandwidth of the branch-line coupler can be enlarged by increasing the number of branches.

Most commonly used is the two-branch coupler (Fig. 2c). In the case of perfect matching of input port 1, element S_{11} must be equal to zero which yields:

$$Y_1^2 = Y_2^2 - 1 \qquad (6)$$

where:

$Y_1 = 1/Z_1$, and $Y_2 = 1/Z_2$ are admittances normalized with respect to the input admittance Y_0.

A two-branch coupler is ideally matched if Eq. 6 and its scattering matrix is:

$$[S] = -\frac{1}{\sqrt{1+Y_1^2}} \begin{bmatrix} 0 & 0 & i & Y_1 \\ 0 & 0 & Y_1 & i \\ i & Y_1 & 0 & 0 \\ Y_1 & i & 0 & 0 \end{bmatrix} \qquad (7)$$

It follows from Eq. 7 that:

$$S_{12} = S_{21} = 0$$
$$arg\left(S_{13}/S_{14}\right) = \pi/2 \qquad (8)$$

which means that the two-branch coupler has an ideal directivity of Type 1 (Fig. 1) and that there exists an inherent 90-deg. phase difference between the output ports.

For the 3-dB coupler (hybrid), the normalized admittances of two segment lines are provided by:

$$Y_1 = 1,\ Y_2 = \sqrt{2} \qquad (9)$$

Isolation of the hybrid ring has wider bandwidth than the isolation of the two-branch hybrid. Two-branch hybrid isolation is usable only over approximately a 10-percent bandwidth.

Consider the parameters of a two-branch hybrid where the terminations have reflection coefficients Γ_1, Γ_2, Γ_3, Γ_4.[4] The main parameters of the two-branch hybrid are shown in Table 1, where:

$$\phi_2 = 2(1 + \Gamma_2\Gamma_3\Gamma_4) + (1 - \Gamma_2)(\Gamma_3 - \Gamma_4) \qquad (10)$$

The graphs of two-branch hybrid parameters versus modules of termination reflection coefficients are shown in Fig. 6.

Comparing parameters of the two-branch coupler with those of the ring coupler with mismatched terminations leads to the following conclusions. In the ring coupler, isolation is better, but matching is worse than in the two-branch coupler. If the output ports have identical terminations,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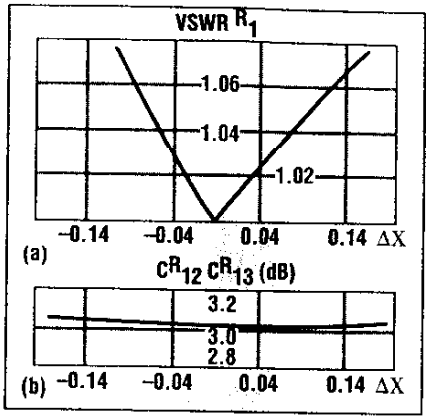

4. Fabrication tolerances can affect the parameters of a hybrid ring as shown here by curves of dimensional changes and their effect on matching (a) and insertion loss (b).

the ring coupler isolation is ideal, but input matching is not, while for the two-branch coupler, matching is perfect, but its isolation is not.

Consider the parameters of a branch hybrid with losses. The two-branch hybrid with ideal matching of all ports has the characteristics shown in Table 1.[4] The power split in the two-branch hybrid, (C^B_{13}–C^B_{14} = 0), unlike the hybrid-ring power split (see Table 2), does not depend on losses. This is generally characteristic of full symmetry couplers.

Figure 7 shows the graphs of microstrip two-branch hybrid parameters versus $\Delta X = \Delta W/\Delta h$ for a relative substrate dielectric constant of 9.8. Figures 4 and 7 show that the two-branch hybrid is more sensitive to production tolerances than the hybrid ring.

PARAMETERS OF ACTUAL DIRECTIONAL COUPLERS DIFFER FROM THE IDEAL DUE TO MISMATCHING OF TERMINATIONS, LOSSES, DISCONTINUITIES, AND MANUFACTURING TOLERANCES.

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Table 2: Characteristics of planar couplers vs. transmission-line losses

Parameters	Hybrid ring	Two-branch hybrid	Coupled-line coupler
VSWR	$\frac{13\sqrt{2}\alpha l + 4}{11\sqrt{2}\alpha l + 4}$	$\frac{1 + 3.62\alpha l}{1 + 1.21\alpha l}$	–
Insertion loss (dB)	$20\log\frac{6\sqrt{2}\alpha l + 2}{4\alpha l + \sqrt{2}}$	$3.01 + 20\log(1 + 2.414\alpha l)$	$10\log\frac{1}{1-K^2} \times \left(\frac{8\alpha l + Z_{0e} + Z_{0o}}{4\alpha l + Z_{0e} + Z_{0o}}\right)$
Coupling (dB)	$20\log\frac{6\sqrt{2}\alpha l + 2}{3\alpha l + \sqrt{2}}$	$3.01 + 20\log(1 + 2.414\alpha l)$	$10\log\frac{1}{K^2} \times \left(\frac{8\alpha l + Z_{0e} + Z_{0o}}{4\alpha l + Z_{0c} + Z_{0o}}\right)$
Isolation (dB)	$20\log\frac{12\sqrt{2}\alpha l + 4}{\sqrt{2}\alpha l}$	$6.02 + 20\log\left(1 + \frac{0.414}{\alpha l}\right)$	–

Consider the effect of discontinuities which arise in T-connections between branches, connecting lines, and input/output (I/O) lines. In general, all I/O lines at these T-connections have different characteristic impedances. Analysis of the effects of discontinuities shows that working parameters of the two-branch coupler (Fig. 8) are more sensitive to them than ring-coupler characteristics (Fig. 5).

The branch coupler has the advantage of adjacent output ports which permit combining them in the planar design. For example, a balanced mixer that has the two-branch coupler and two diodes in output ports 3 and 4 can have one planar output intermediate-frequency (IF) port.

Consider the three-branch coupler whose circuit view is shown in Fig. 2d. Similar to the two-branch coupler, the three-branch coupler has the ideal directivity of type I.

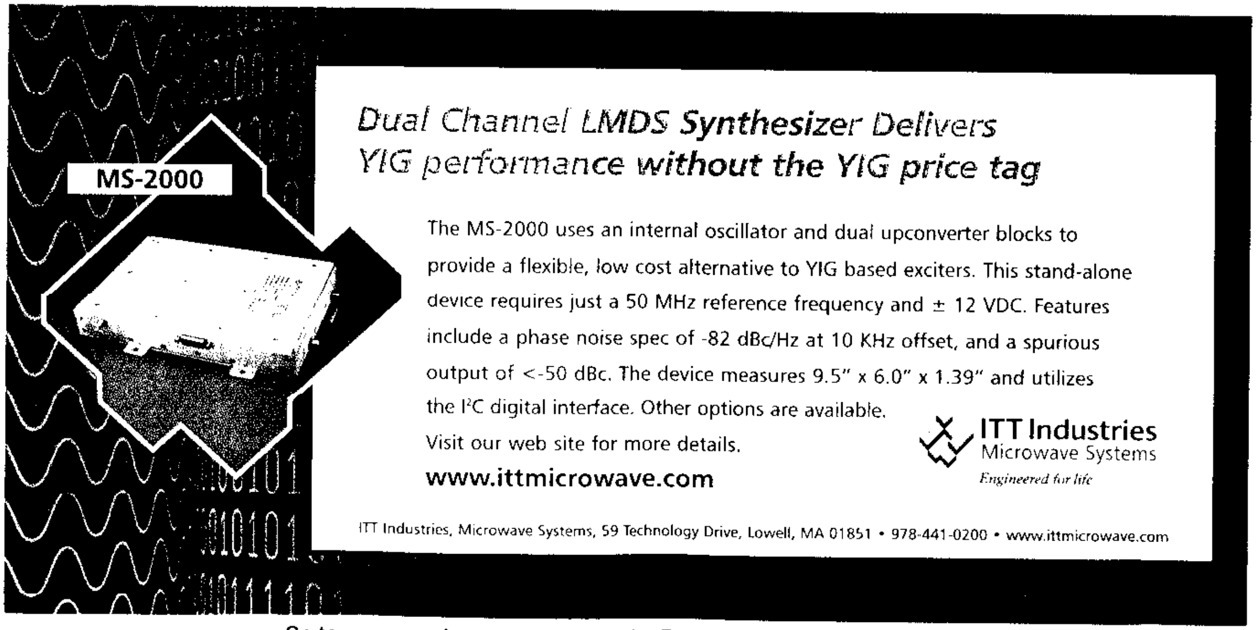
$$S_{12} = S_{21} = 0 \qquad (11)$$

Thus, it is evident that port 2 is isolated and waves reaching port 3 and port 4 have a differential 90-deg. phase shift:

$$\arg {}^{S_{14}}\!/_{S_{13}} = {}^{\pi}\!/_{2} \qquad (12)$$

Several remarks on the three-branch coupler are in order. The bandwidth of the three-branch coupler is similar to that of the ring coupler, however, its parameters, to a great extent, depend on discontinuities and tolerances of line dimensions.

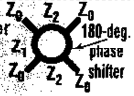
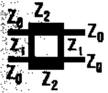
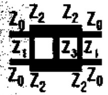
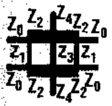
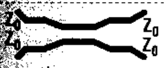
The three-branch coupler has a larger bandwidth than the two-branch coupler. Additional branches can expand the bandwidth even further. However, couplers with more than four branches are difficult to fabricate in microstrip because the end branches require impedances which reach the upper limits of practical realization.

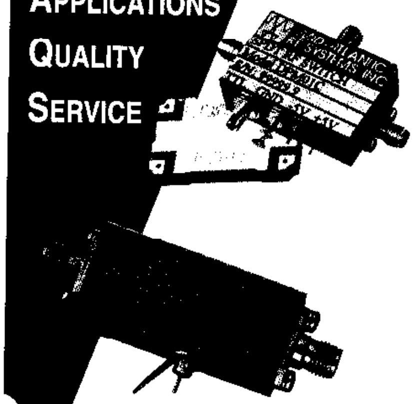


The three-branch coupler with power-split regulation is shown in Fig. 2e.[12] Two open or short lines are connected to the center branch. The power split between port 2 and port 4 depends on the length of these stubs. This directional coupler has Type III directivity (Fig. 1).

One of the most useful structures of directional couplers is the four-port network formed by two coupled lines that are close enough to each other so they are coupled through electric and magnetic fields (Fig. 2f and g).

Table 3: Characteristics of print directional couplers

Directional coupler view	Type of directivity	Phase balance (deg.)	Coupling (dB)	BW (percent)	Ideal matching conditions	Hybrid equations	Independence from mismatched terminations			Independence from losses
							VSWR	Isolation	Coupling	
Ring coupler length 3λ/2 	2	0 to 180	3 to 6	20	$Y_1^2 + Y_2^2 = 1$	$Y_1 = Y_2 = 1/\sqrt{2}$	2	1	1	2
Ring coupler length λ 	2	0 to 180	3	Octave	$Y_1^2 + Y_2^2 = 1$	$Y_1 = Y_2 = 1/\sqrt{2}$				
Two-branch coupler 	1	90	3 to 6	15	$Y_1^2 = Y_2^2 - 1$	$Y_1 = 1; Y_2 = \sqrt{2}$	1	2	2	3
Three-branch coupler 	1	90	3	20	$Y_3 = 2Y_2^2Y_1/(1+Y_1^2)$	$Y_1 = 1;$ $Y_2^2 - Y_3 = 0$				
Three-branch coupler with power-split regulation 	3	90	Variable	20	$Y_3 = 2Y_2^2Y_1/(1+Y_1^2)$	$Y_1 = 1;$ $Y_2^2 - Y_3 = 0$				
Coupled-line coupler 	2	90	3 to 30	Octave	$Z_{0e}Z_{0o} = 1$		1	2	2	1
 Three-stage coupled-line coupler	2	90	10 to 30	Two octave	$Z_{0e1}Z_{0o1} = Z_{0e2}Z_{0o2} = Z_{0e3}Z_{0o} = 1$					



A coupler can be represented by independent even and odd modes. The final results are obtained by superposition of the two modes. In the even-mode case, currents in both lines are equal and co-directional. In the odd-mode case, currents are equal and opposite. According to the mirror-reflection method, it is possible to calculate homogeneous coupled lines with normalized characteristic impedances Z_{0e} (even mode) and Z_{0o} (odd mode) [the last suffix identifies the mode].

Perfect matching ($S_{11} = 0$) occurs when:

$$Z_{0e} Z_{0o} = 1 \qquad (13)$$

The scattering matrix of the ideally matched coupler and its frequency characteristics can be derived from:[4]

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{13} \\ S_{13} & 0 & 0 & S_{12} \\ 0 & S_{13} & S_{12} & 0 \end{bmatrix} \qquad (14)$$

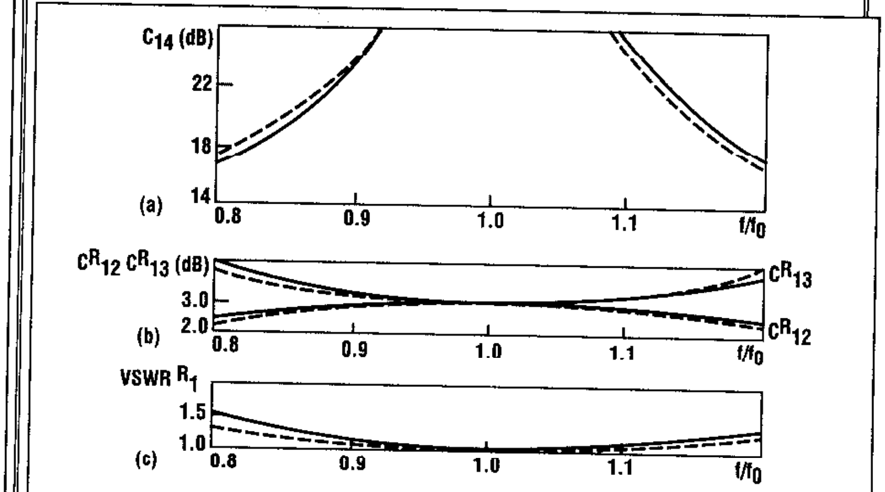
where:

$$S_{12} = i(Z_{0e} - Z_{0o}) \sin \Theta / 2 \cos \Theta + i(Z_{0e} + Z_{0o}) \sin \Theta \qquad (15)$$

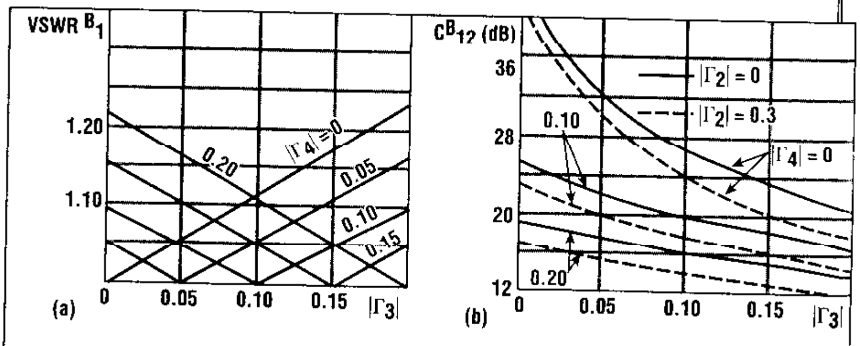
$$S_{13} = 2 / 2 \cos \Theta + i(Z_{0e} + Z_{0o}) \sin \Theta \qquad (16)$$

where:

the suffix "o" denotes the midband operating frequency, $\Theta = 2\pi l/\Lambda$ = the electrical length of the coupled lines, and



5. Discontinuities have an effect on a hybrid ring as illustrated by curves of isolation (a), insertion losses (b), and VSWR (c) [solid lines]. The dashed lines are plots without discontinuities.



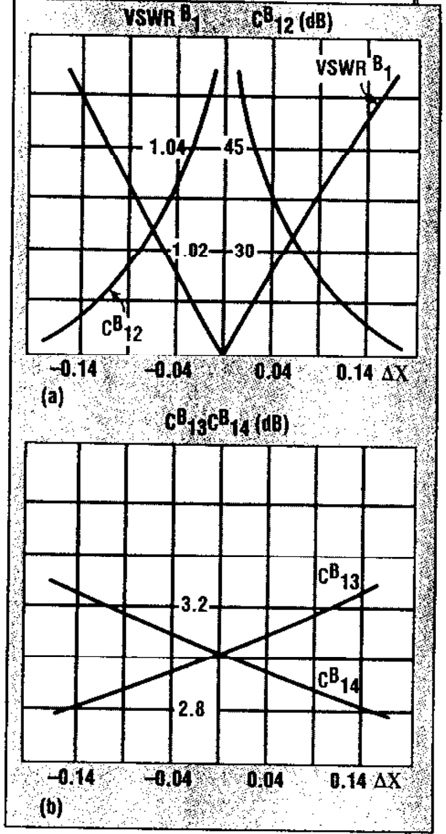
6. These plots are of matching (a) and isolation (b) versus reflection coefficients for a two-branch hybrid.

Λ = the guide wavelength.

The real directional coupler coupling (C_{12}) and insertion loss (C_{13}) will be a combination of coupling loss, conductor loss, dielectric loss, mismatch loss, discontinuities loss, along with loss due to production tolerances.

The characteristics of coupled-line couplers will be investigated where terminations have reflection coefficients Γ_1, Γ_2, Γ_3, Γ_4 that are connected correspondingly to ports 1, 2, 3, and 4.

The plot of the $VSWR_1$ as a function of reflection coefficients $|\Gamma_2|$ and $|\Gamma_4|$ of output terminations is shown in Fig. 9a.[4] As illustrated, the best matching of the 3-dB coupler is realized when the reflection coefficients of loads in ports adjacent to the input are equal.



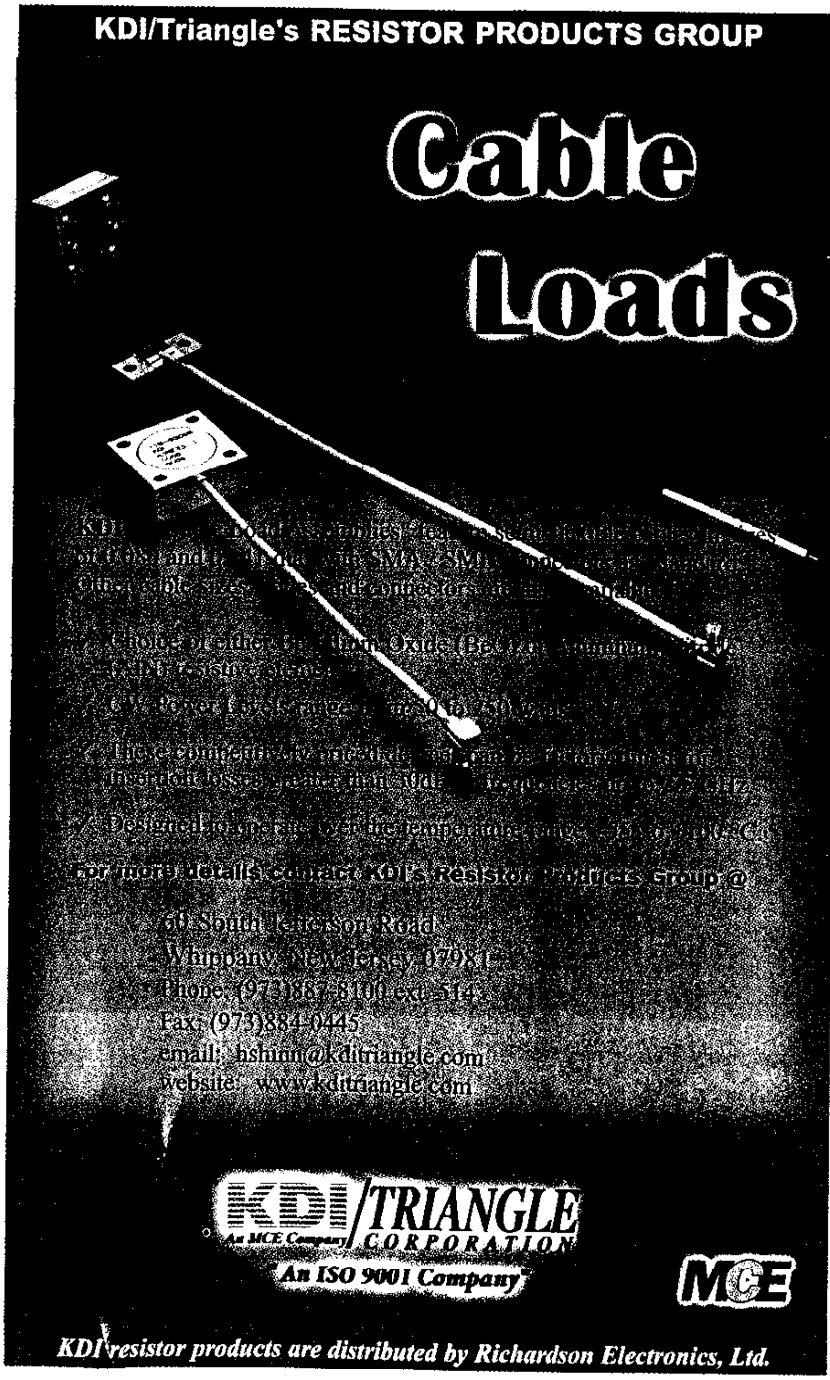
7. The two-branch hybrid's graph of matching and isolation (a) and insertion loss (b) as a function of fabrication tolerances is shown here.

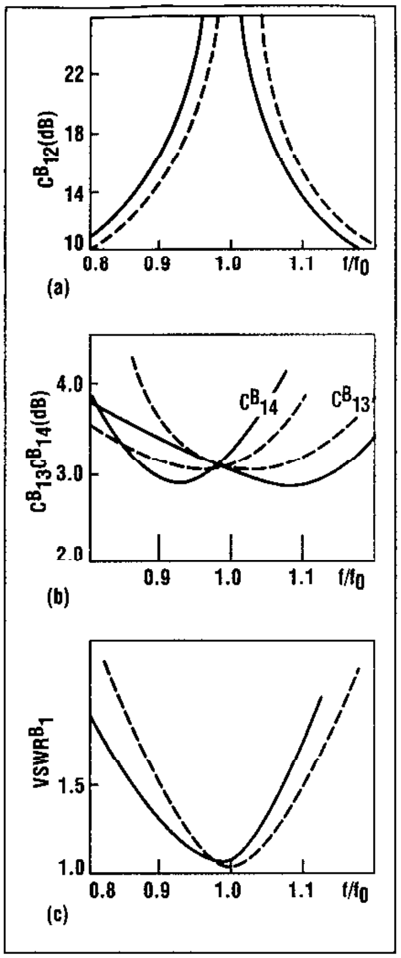
Figure 9b shows the dependence of parameters $\Delta C_{12} = C_{12} - C^0_{12}$ (solid lines) and $\Delta C_{13} = C_{13} - C^0_{13}$ (dashed lines) from coefficients $|\Gamma_2|$ and $|\Gamma_3|$ for coupling of C^0_{12} = 3 dB and C^0_{12} = 20 dB.

Characteristics of directivity versus reflection coefficients $|\Gamma_2|$ and $|\Gamma_3|$ are plotted in Fig. 9c. For a particular C^0_{12}, curves corresponding to different values of $|\Gamma_2|$ diverge in a fan-like pattern as $|\Gamma_3|$ decreases.[4]

Table 2 defines the main performance parameters of directional couplers, taking losses in lines into account. The line losses of the two-branch hybrid and the coupled-line coupler vary identically with the losses in the lines.

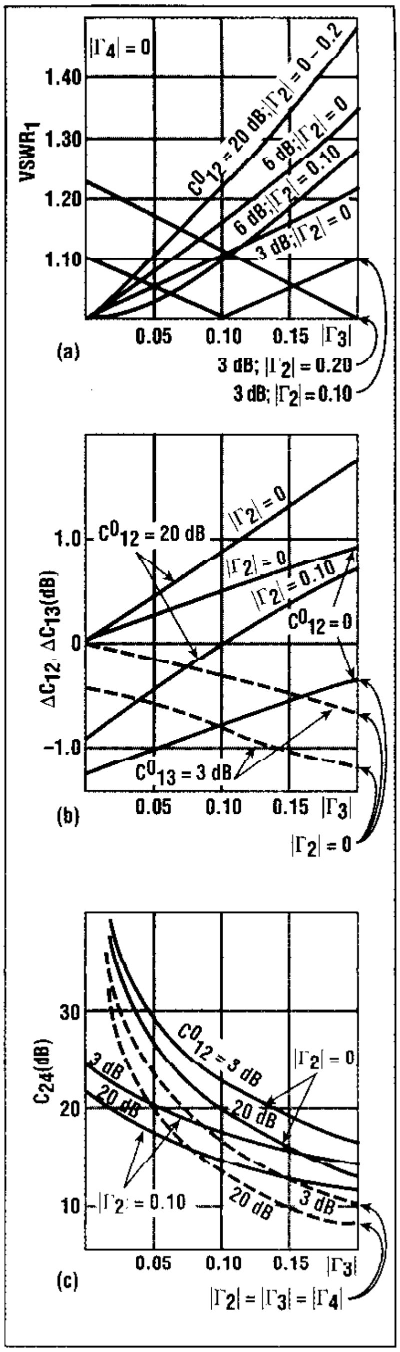
Table 3 illustrates the performance, matching conditions, equations, and other characteristics of the





8. These three curves illustrate the effect of discontinuities (solid lines) on isolation (a), insertion loss (b), and matching (c) as a function of frequency. The dashed lines show the effects on these parameters without discontinuities.

print directional couplers that are shown in Fig. 2. ●●



9. The coupled-line directional coupler's VSWR (a), deviation of coupling and insertion loss (b), and directivity (c) as a function of reflection coefficients at the coupler's output ports are provided by these graphs.

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