Interdigital Design Forms Low-Cost Bandpass Filters

A stripline-based filter-construction method provides compact size with high manufacturing repeatability.

Interdigital Filters, Part 1

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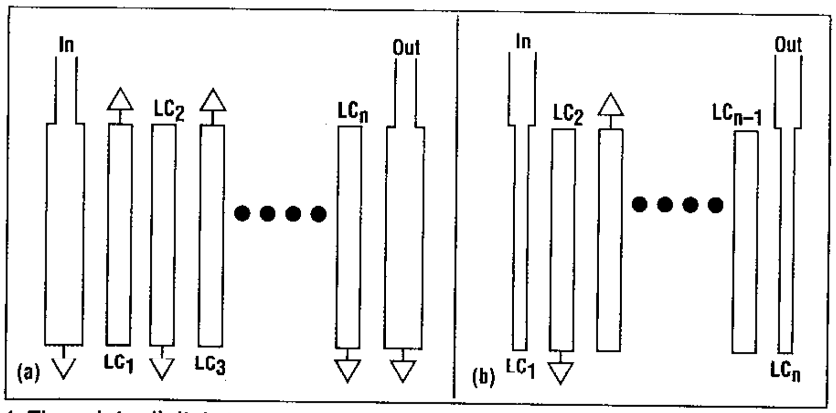
COMPETITIVE pressures continue to force engineers to seek low-cost solutions for the design of microwave components such as filters. One unconventional design approach provides low production cost by using an interdigital filter structure based on printed-circuit stripline fabrication. The filter design provides narrowband to wideband operation at frequencies from UHF to C-band. This first part of a two-part article describes the calculation of the filter's physical dimensions based on a thin-stripline approximation. In addition, measurement results are given for a narrowband interdigital filter designed with this approach.

Interdigital filters are based on an array of transverse-electromagnetic (TEM)-mode, quarter-wavelength stripline resonators. A typical interdigital filter is realized by suspending a quarter-wavelength stripline resonator array in an air-filled metal case.[1] The upper and lower metal covers act as reference ground planes for the stripline, while the input/output connector openings are machined out of the case. While this type of filter construction is very solid and reliable, it is expensive due to the required machining. Also, if the filter's center frequency is located in the lower L-band or UHF range, the physical dimensions of the filter will be excessively large since there is no dielectric scaling factor in the structure. To overcome this and other related problems (filter weight, bulk, etc.), a printed-circuit version of the interdigital filter has been designed and built. The printed-circuit design offers several advantages over its



1. These interdigital structures are used for filter designs with narrow-to-moderate (W ≤ 3) bandwidths (a) and wide (up to W = 0.7) bandwidths (b), respectively.

mechanical counterpart:

- It is economical, since the chemical etching process for a printed circuit (on a Teflon-based support material) is cheaper than any machining procedure (mill, lathe, etc.).
- The filter does not require calibration. After optimizing the response of the design prototype, circuit repeatability is provided by the chemical etching process.
- The structure's compact size allows the realization of high order filters (six or more poles) in a relatively-small space.

The first step in any filter design is the assessment of the filter requirements and specifications, such as:

- Center frequency.
- 3-dB bandwidth or equivalent.
- Number of poles or equivalent.

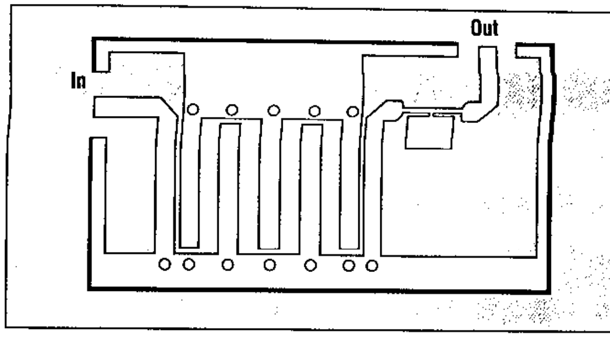
Other filter specifications which may be required are:

- Maximum in-band insertion loss.
- Minimum attenuation at given out-of-band frequencies.
- Ultimate rejection.
- Phase behavior or group delay (if any is required).

From these requirements, the following parameters must be synthesized:

- Filter characteristic (Butterworth, Chebyshev, etc.).
- Relative bandwidth (W).
- Filter complexity (in terms of the number of poles, given by n).

Two types of filter structures are considered. The first structure (Fig. 1a) is best suited for narrow- or moderate-bandwidth filters ($W \leq 0.3$), while the second configuration (Fig. 1b) is useful for large-bandwidth filters (up to $W = 0.7$). The design formulas must be solved with the appropriate W and n values as well as the set of filter coefficients (given by g_i) corresponding to the given filter characteristic. The relative dielectric constant (ϵ_r) of the support medium must also be provided in order to obtain the set of normalized (per unit length) self and mutual capacitances (given by C_k/ϵ and $\Delta C_{k,k+1}/\epsilon$, respectively) for the resonators.



3. This printed-circuit interdigital filter uses a five-pole design while providing a center frequency of 2870 MHz.

A set of formulas has been developed to obtain normalized multiple coupled-stripline elements' self and mutual capacitances.[1] These capacitance values are the starting point for filter dimensioning, since they are needed to determine the resonator widths and spacings.

Given a fractional bandwidth of $W = BW_3/f_0$ (where BW_3 is the 3-dB bandwidth and f_0 is the center frequency), prototype filter coefficients of g_k (where k = 1, 2, ..., n), and a filter input/output admittance of Y_a, the interdigital-filter phase parameter (θ) is computed as:

$$\theta = (\pi/2)(1 - W/2) \quad (1)$$

For the narrow-bandwidth structure of Fig. 1a, the $J_{k,k+1}$ and $N_{k,k+1}$ parameters are defined as:

$$J_{k,k+1}/Y_a = 1/(g_k g_{k+1})^{0.5} \quad \text{for } k = 1, \ldots, n \quad (2)$$

$$N_{k,k+1} = [(J_{k,k+1}/Y_a)^2 + \tan^2(\theta/4)]^{0.5} \quad (3)$$

The following parameters can then be defined:

$$M_1 = Y_a(J_{01}h^{0.5}/Y_a + 1) \quad (4)$$

$$M_n = Y_a(J_{n,n+1}h^{0.5}/Y_a + 1) \quad (5)$$

where h is an arbitrarily-chosen scale factor whose value can be adjusted in order to maximize the resonator's unloaded quality factor (Q) and scale the resonator widths and spacings to achieve manufacturability (e.g., to prevent infinitesimally-small widths and gaps which can lead to poor fabrication precision).

The normalized stripline elements' self capacitances are computed as:

$$C_0/\varepsilon = 120\pi(2Y_a - M_1)/\varepsilon_r^{0.5} \quad (6)$$

$$C_1/\varepsilon = (120\pi/\varepsilon_r)[Y_a - M_1 + hY_a[\tan\theta/2 + (J_{01}/Y_a)^2 + N_{12} - J_{12}/Y_a]] \quad (7)$$

$$C_k/\varepsilon = (120\pi h Y_a)[(N - J/Y_a)_{k-1,k} + (N - J/Y_a)_{k,k+1}] \quad \text{for } k = 2, \ldots, n-1 \quad (8)$$

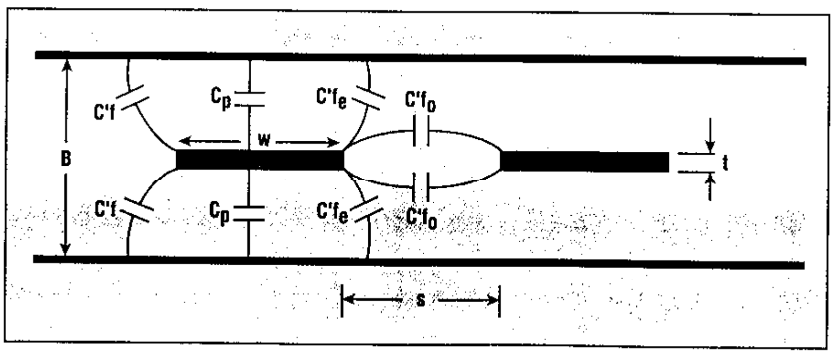
$$C_n/\varepsilon = (120\pi/\varepsilon_r^{0.5})[Y_a - M_n + hY_a[\tan\theta/2 + (J_{n,n+1}/Y_a)^2 + N_{n-1,n} - J_{n-1,n}/Y_a]]^{0.5} \quad (9)$$

$$C_{n+1}/\varepsilon = 120\pi \times (2Y_a - M_n)/\varepsilon_r^{0.5} \quad (10)$$

where:

ε = the dielectric constant.

The mutual capacitances of the normalized stripline elements are given by:



2. The mutual and self capacitances of the coupled-stripline elements must be determined before the physical dimensions are calculated.

Table 1: Calculated capacitances for the narrowband filter

K	$J_{k,k+1}$	$N_{k,k+1}$	C_k/ε	$C_{k,k+1}/\varepsilon$
0	0.933	0	4.014	0.922
1	0.797	12.499	4.023	0.157
2	0.607	12.488	4.656	0.120
3	0.607	12.488	4.691	0.120
4	0.797	12.499	4.656	0.157
5	0.933	0	4.023	0.922
6	0	0	4.014	0

$$C_{01}/\varepsilon = 120\pi \times (M_1 - Y_a)/\varepsilon_r^{0.5} \quad (11)$$

$$C_{k,k+1}/\varepsilon = (120\pi h Y_a) \times (J_{k,k+1}/Y_a) \; for\ k = 1, ..., n-1 \quad (12)$$

$$C_{n,n+1}/\varepsilon = 120\pi \times (M_n - Y_a)/\varepsilon_r^{0.5} \quad (13)$$

For the wide-bandwidth filter configuration of Fig. 1b, the $J_{k,k+1}$ and $N_{k,k+1}$ parameters are given by:

$$\frac{J_{k,k+1}}{Y_a} = \frac{g_2}{g_0(g_k g_{k+1})^{0.5}} \; for\ k = 2, ..., n-3 \quad (14)$$

$$\frac{J_{n-2,n-1}}{Y_a} = \frac{1}{g_0}\left(\frac{g_2 g_0}{g_{n-2} g_{n-1}}\right)^{0.5} \quad (15)$$

$$N_{k,k+1} = [(J_{k,k+1}/Y_a)^2 + (g_2 \tan\theta/2g_0)^2]^{0.5} \; for\ k = 2, ..., n-3 \quad (16)$$

The element impedances and admittances are:

$$Z_1/Z_a = g_0 g_1 \tan\theta \quad (17)$$

$$Y_2/Y_a = (g_2/2g_0)\tan\theta + N_{23} - J_{23}/Y_a \quad (18)$$

$$Y_k/Y_a = (N - J/Y_a)_{k-1,k} + (N - J/Y_a)_{k,k+1} \; for\ k = 3, ..., n-2 \quad (19)$$

$$\frac{Y_{n-1}}{Y_a} = \frac{(2g_0 g_{n-1} - g_2 g_{n+1})\tan\theta}{2g_0 g_{n+1}} + N_{n-2,n-1} - J_{n-2,n-1}/Y_a \quad (20)$$

$$Z_n/Z_a = g_n g_{n+1} \tan\theta \quad (21)$$

These values can then be used to obtain the self capacitances of the normalized stripline elements:

$$C_1/\varepsilon = (120\pi Y_a/\varepsilon_r^{0.5}) \times [(1 - h^{0.5})/(Z_1/Z_a)] \quad (22)$$

$$C_2/\varepsilon = (120\pi Y_a/\varepsilon_r^{0.5})h \times (Y_2/Y_a) - h^{0.5}C_1/\varepsilon \quad (23)$$

$$C_k/\varepsilon = (120\pi Y_a/\varepsilon_r^{0.5})h(Y_k/Y_a) \; for\ k = 3, ..., n-2 \quad (24)$$

$$C_{n-1}/\varepsilon = (120\pi Y_a/\varepsilon_r^{0.5})h \times (Y_{n-1}/Y_a) - h^{0.5}C_n/\varepsilon \quad (25)$$

$$C_n/\varepsilon = (120\pi Y_a/\varepsilon_r^{0.5}) \times [(1 - h^{0.5})/(Z_n/Z_a)] \quad (26)$$

The mutual capacitances of the normalized stripline elements are given by:

$$C_{12}/\varepsilon = (120\pi Y_a/\varepsilon_r^{0.5}) \times [h^{0.5}/(Z_1/Z_a)] \quad (27)$$

$$C_{k,k+1}/\varepsilon = (120\pi Y_a/\varepsilon_r^{0.5}) \times h(J_{k,k+1}/Y_a) \; for\ k = 2, ..., n-2 \quad (28)$$

$$C_{n-1,n}/\varepsilon = (120\pi Y_a/\varepsilon_r^{0.5}) \times [h^{0.5}/(Z_n/Z_a)] \quad (29)$$

THIN STRIPLINE

Figure 2 shows a typical coupled-stripline configuration. If the dielectric thickness (B) and metallization thickness (t) satisfy the limitation $t/B \leq 0.1$, then the approximate formulas for even- and odd-mode impedances and capacitances on thin stripline can be applied. This holds true for any practical printed-circuit stripline realization. If the further limitation $w/B \geq 0.35$ is satisfied, the approximate formulas for the even- and odd-mode impedances (Z_{oe} and Z_{oo}, respectively) can be defined as:[2,3]

$$Z_{oo} = [30\pi(B-t)]/[\varepsilon_r^{0.5}(w + BC_fA_o/2\pi)] \quad (30)$$

$$Z_{oe} = [30\pi(B-t)]/[\varepsilon_r^{0.5}(w + BC_fA_e/2\pi)] \quad (31)$$

with:

$A_o = [1 + \ln(1 + \coth\theta)]/\ln 2$,
$A_e = [1 + \ln(1 + \tanh\theta)]/\ln 2$, and:

$$Cf(t/B) = 2\ln\left(\frac{2B-t}{B-t}\right) - \frac{t}{B}\ln\left[\frac{t(2B-t)}{2(B-t)}\right] \quad (32)$$

where:

$\theta = \pi s/2B$, and
s = the spacing between filter elements.

These formulas can be used in two ways:

(1) To calculate the Z_{oe} and Z_{oo} values from the physical parameters of the coupled stripline (w and s); or

(2) To synthesize the practical dimensions of the coupled stripline (w, s) from Z_{oe} and Z_{oo}.

In both cases, the t and B values must be assessed before starting the computation. From a practical point of view, this means that ϵ_r, B, and t must be chosen. Note that the B value is calculated as the sum of the lower and upper printed-circuit-board (PCB) dielectric thicknesses. For the interdigital filter design, it is necessary to deduce physical dimensions w and s from previously-computed C_k/ϵ and $C_{k,k+1}/\epsilon$ values.

It can be shown that the normalized mutual capacitances per unit length may be expressed in terms of t/B and s/B only. The total even- and odd-mode normalized capacitances per unit length of the coupled lines are given by:[1]

$$C_{oe}/\varepsilon = 120\pi/Z_{oe}\varepsilon_r^{0.5} \quad (33)$$

$$C_{oo}/\varepsilon = 120\pi/Z_{oo}\varepsilon_r^{0.5} \quad (34)$$

Note that:

$$C_{oe}/\varepsilon = 2(C_p/\varepsilon + C'f_e/\varepsilon + C'f/\varepsilon) \quad (35)$$

$$C_{oo}/\varepsilon = 2(C_p/\varepsilon + C'f_o/\varepsilon + C'f/\varepsilon) \quad (36)$$

Consequently, $\Delta C/\epsilon$ can be obtained by subtracting Eq. 36 from Eq. 35 and substituting the resulting values into Eqs. 33 and 34:

$$\Delta C/\varepsilon = C'f_o/\varepsilon - C'f_e/\varepsilon = \frac{60\pi}{\varepsilon_r^{0.5}}\left(\frac{1}{Z_{oo}} - \frac{1}{Z_{oe}}\right) \quad (37)$$

Subsituting the results of Eqs. 30 and 31 into Eq. 37 yields the following formula:

$$\frac{\Delta C}{\varepsilon} = \frac{Cf}{\pi(1-t/B)\ln 2} \times \ln(\cot g\theta) \quad (38)$$

Note that the dependence on w no longer appears. Normalized inter-strip spacing s/B is readily obtained from the previous expression as:

$$\frac{s}{B} = \frac{1}{\pi}\ln\left(\frac{K2+1}{K2-1}\right) \quad (39)$$

where:

$K1 = Cf/[\pi(1 - t/B)\ln 2]$, and
$K2 = \exp[(\Delta C/\epsilon)/K1)$.

In order to determine the normalized stripline width (w/B), the following expression for Z_{oe} is compared with Eq. 31:

$$Z_{oe} = 60\pi/[\varepsilon_r^{0.5}(C_p/\varepsilon + C'f_e/\varepsilon + C'f/\varepsilon)] \quad (40)$$

Substitution of the above formulas for Cf and A_e in Eq. 31 yields:

$$Z_{oe} = 60\pi/\{\theta_r^{0.5}[w/B + (Cf \times A_e)/2\pi][2B/(B-t)]\} = 60\pi/(\theta_r^{0.5}\{2w/[B(1-t/B)]\} + \{[Cf \times \ln(1+\tanh\theta)]/[\pi(1-t/B)\ln 2]\} + Cf/[\pi(1-t/B)]) \quad (41)$$

A comparison with Eq. 40 provides the following results:

$$\frac{C_p}{\varepsilon} = \frac{2w}{B(1-t/B)} \quad (42)$$

$$C'f_e/\varepsilon = [Cf \times ln(1+tanh\theta)]/[\pi(1-t/B)ln2] \quad (43)$$

$$\frac{C'f}{\varepsilon} = \frac{Cf}{\pi(1-t/B)} \quad (44)$$

where:

C'fe/ϵ = the normalized even-mode interstrip fringing capacitance, and

C'f/ϵ = the external fringing capacitance of the stripline.

The normalized stripline width can be found from Eq. 42 as:

$$\frac{w}{B} = \frac{1}{2}\left(1-\frac{t}{B}\right)\frac{C_p}{\varepsilon} = \frac{1}{2}\left(1-\frac{t}{B}\right)\times\left(\frac{C_{oe}}{2\varepsilon} - \frac{C'f_e}{\varepsilon} - \frac{C'f}{\varepsilon}\right) \quad (45)$$

For the multiple parallel-coupled stripline sections, the previous formula must be written as follows for internal lines (i.e., lines which have neighboring elements on both sides):

$$\frac{w_k}{B} = \frac{1}{2}\left(1-\frac{t}{B}\right)\times\left(\frac{C_k}{2\varepsilon} - \frac{C'fe_{k-1,k}}{\varepsilon} - \frac{C'fe_{k,k+1}}{\varepsilon}\right) \quad (46)$$

For the first and the last lines (which have only one neighboring element), the normalized stripline width is obtained as:

$$\frac{w_0}{B} = \frac{1}{2}\left(1-\frac{t}{B}\right)\times\left(\frac{C_0}{2\varepsilon} - \frac{C'fe_{01}}{\varepsilon} - \frac{C'f_e}{\varepsilon}\right) \quad (47)$$

All interdigital-filter dimensions are obtained by evaluating Eqs. 39, 46, and 47, substituting the previously-computed values for C_k/ϵ and $\Delta C_{K,K+1}/\epsilon$. From these equations, the s_k/B and w_k/B parameters are obtained.

The resonator's length must be a quarter wavelength at f_0. This length is computed by taking into account the stripline's effective dielectric constant:

$$L = \frac{1}{4} \times \frac{c}{f_0} \times \frac{1}{\varepsilon_r^{0.5}} \quad (48)$$

APPROXIMATION CHECK

The result given by Eq. 39 for s/B is only applicable for the limit t/B → 0 with w/b ≥ 0.35. While the first limitation holds true in any printed-circuit realization, the second limitation refers to the width of the resonator strip and should be checked a posteriori (i.e., after completion of the resonator dimensioning). However, for very narrow resonators, the interaction between the fringing fields is no longer negligible and the result given by Eq. 39 could be inaccurate.

Practical computation of Eq. 39 has shown that the error introduced is very small and is, in fact, negligible for $\Delta C/\epsilon \leq 0.8$. For $\Delta C/\epsilon \geq 1.5$, the equation is no longer useful and the exact solution for thick-bar stripline must be considered instead of Eq. 39.

In these situations, s/B can be computed by using the Getsinger curves[4] in conjunction with the $\Delta C/\epsilon$ values computed from the self/mutual-capacitance formulas given previously (which also provide the corresponding C'fe/ϵ value). The C'fe/ϵ result should be used in Eqs. 46 and 47 for the stripline-width computation.

Table 2: Computed physical dimensions for the narrowband filter

K	s_k/B	$c'fe_{k,k+1}$	w_k/B	s_k (mm)	w_k (mm)
Before Getsinger curve					
0	0.172	0.162	0.667	0.623	2.149
1	0.693	0.406	0.706	2.232	2.274
2	0.779	0.423	0.733	2.509	2.362
3	0.779	0.423	0.734	2.509	2.363
4	0.693	0.406	0.733	2.232	2.362
5	0.172	0.162	0.706	0.623	2.274
6	0	0	0.667	0	2.149
Using Getsinger curve					
0	0.195	0.173	0.665	0.623	2.141
1	0.603	0.406	0.700	2.232	2.256
2	0.779	0.423	0.733	2.509	2.362
3	0.779	0.423	0.734	2.509	2.363
4	0.693	0.406	0.733	2.232	2.362
5	0.172	0.162	0.700	0.623	2.256
6	0	0	0.665	0	2.141

Note: The first set of results generated a $\Delta C_{01}/\epsilon$ value that was too close to unity. The Getsinger curve was then applied to obtain the second set of results.

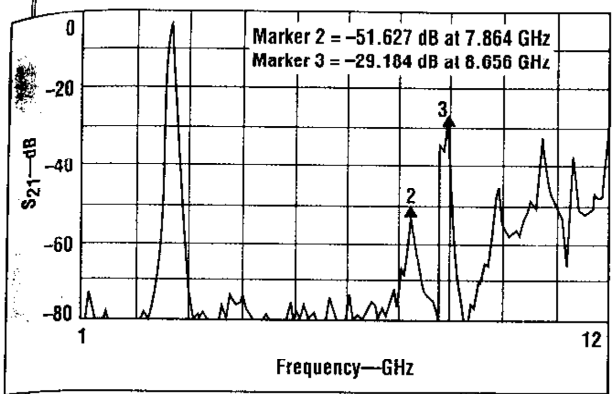
EXPERIMENTAL RESULTS

The above computational procedure is illustrated using two interdigital-filter examples, including experimental results for fabricated structures. As a general rule, all interdigital filters exhibit a second passband response at $3f_0$, thus a lowpass filter must be cascaded to the interdigital structure in order to attenuate this unwanted response. The lowpass filter must provide a good matching characteristic (in the passband) in order to prevent distortion or rippling of the interdigital-filter response.

The first filter example has the following specifications:

- Chebyshev response with 0.1-dB maximum ripple.
- Five poles.
- f_0 = 2870 MHz.
- 3-dB bandwidth = 147 MHz.
- Input/output impedance = 50 Ω.

The narrowband filter's fractional



4. The measured S_{21} response was obtained for the 2870-MHz interdigital-filter design.

bandwidth is W = 147/2870 ≈ 0.051. Due to the narrow W value, the narrowband-filter model is used for this example. Thus, the resulting filter is composed of seven elements—five resonators and two impedance-transforming elements.

The filter is realized on a 0.062-in. (1.57-mm)-thick 5870 RT Duroid (ϵ_r = 2.33) substrate that is cladded on both sides with 1-oz. copper so that the stripline structure dimensions are t = 0.035 mm and B = (1.57 × 2) + t ≈3.22 mm.

In order to plate through the resonator ground reference holes, an extra copper layer is added (in the metallization process) to the 35-μm basic copper layer, so that the total copper clad thickness to be considered in the formulas is 0.07 mm (or 70 μm). By introducing the g_i parameters of the 0.1-dB-ripple Chebyshev family in the self/mutual-capacitance equations, a set of values are obtained for C_k/ϵ and $\Delta C_{k,k+1}/\epsilon$ (Table 1).

An admittance factor of h = 0.04 was chosen as a tentative value. The validity of this choice will be checked when the stripline widths and spacings are calculated. By introducing the computed C_k/ϵ and $\Delta C_{k,k+1}/\epsilon$ values in Eqs. 39, 46, and 47, the filter's physical dimensions (s_k and w_k) were obtained (Table 2).

The $\Delta C_{01}/\epsilon$ value obtained is too close to unity. Consequently, a comparison with the Getsinger curve must be made in order to find more-reasonable values for s/B and $C'fe_{01}$. The graphically-obtained parameters are s/B = 0.195 and $C'fe_{01}$ = 0.173. The physical dimensions are then recalculated using these s/B and $C'fe_{01}$ values (also shown in Table 2).

As can be seen, only minor changes have been introduced in the s_0/B, w_1/B, and w_2/B values, while all other dimensions remain unchanged. The required lowpass filter was realized on the stripline structure by scaling a three-element elliptic prototype with a cutoff frequency of f_c = 3.5 GHz, following the procedure given by Howe.[5] This lowpass filter type proved to be the easiest to fabricate on the chosen dielectric support material.

Figure 3 shows the complete narrowband-filter structure. The plated through holes are required as ground references for the interdigital filter's resonator elements.

Figure 4 presents a plot of the S_{21} characteristics for the realized filter. The in-band insertion loss of 3 dB is due to dielectric and metal losses. The maximum in-band return loss is 10 dB, which indicates a fairly-good impedance match. Due to the presence of the lowpass filter, the second passband response is attenuated by approximately 30 dB.

The second part of this two-part article will continue the examination of interdigital filters by presenting a wideband-filter example (W > 0.5). The calculation of mutual/self inductances and physical parameters will be described, and measurement results will be presented. ••

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