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Microwave focusing and beam collimation using negative index of refraction lenses


Abstract: Negative index of refraction materials (NIMs) were first postulated by Veselago in 1968 and have recently been realised using structures formed with rings and wires deposited on printed circuit boards. The proof of the existence of negative index of refraction was established using a Snell’s law experiment with a wedge. The predicted and measured refraction angles were found to be consistent for a negative index material and in excellent agreement with the theoretical expectations. For microwave lenses NIMs have the advantage of being lighter, having better focusing properties and potentially lower aberrations. Simulation and experimental results on NIM configurations including gradient index of refraction and spherical 3D lenses are presented. Both focusing and beam collimating applications will be considered. These results will be compared to normal positive index of refraction material lenses.

1 Introduction

In 1968, Veselago [1] discussed the likelihood of a negative index of refraction material NIM. Recently, these NIMs were realised in practice by the appropriate combination of conductive elements deposited on a dielectric substrate [2]. In the microwave regime, NIMs are fabricated from metallic wires and rings assembled in a periodic cell structure. The rings are generally referred to as split ring resonators. NIMs have the property that the effective permittivity $\varepsilon_{\text{eff}}$ and permeability $\mu_{\text{eff}}$ are both negative resulting in an effective negative index of refraction, $n = \sqrt{-\varepsilon\mu}$. Initially there was some controversy regarding the existence of NIMs, but more recently the effect has been conclusively demonstrated [2, 3] using a Snell’s law experiment. Lenses built with NIMs reduced aberrations compared with an equivalent one made of a positive index of refraction material (PIM) [4]. Plano-concave NIM lenses have been built using NIM and photonic crystals with constant index of refraction, namely a cylindrical geometry lens at 14.7 GHz [5] and a similar lens at 9.265–9.490 GHz [6]. It was shown that an NIM lens in the RF regime can be easily manufactured with a gradient index of refraction (GRIN) [7] in the absence of any physical curved surfaces. Subsequently, a GRIN lens in cylindrical geometry has been built and tested [8, 9].

In this paper, we will discuss the performance of spherical NIM lenses with physical curvature and flat GRIN lenses. We will compare their performance to an equivalent PIM lens. Specifically we will address four major areas: (a) characterisation of the free space test setup used for our spherical lenses, (b) a plano–convex PIM lens, (c) a plano–concave NIM lens and (d) a plano–plano GRIN lens using an NIM medium. The PIM lens was modelled, designed and fabricated to observe the performance differences between lenses made of normal materials and NIMs. Also, we were interested in comparable GRIN lenses since they are easy to fabricate, are of uniform thickness and lighter than either the PIM or NIM lenses since they are effectively thinner. All of the lenses were designed to operate having the same nominal focal length (~12.7 cm) and aperture (~12.7 cm), giving an $F$ number approximately equal to 1.0. Since the dispersion curves are steep in the negative index region, the lenses only have the desired focal length at the designed frequency of ~14.8 GHz. The bandwidth of the negative index region was ~10%. The individual unit cells were designed by modelling the details of the rings, wires and substrates. However, for simulations of the entire NIM and GRIN lenses, an effective medium approximation was used, that is, the individual rings and wires were not modelled.

2 Characterisation of the empty aperture test setup

The experimental setup for the lens measurements incorporated a network analyser that was controlled by a computer running Labview. The setup utilises a source antenna illuminating a 6.20-cm radius aperture made of aluminium that is shielded by eccosorb to reduce reflections. Microwave studio (MWS) simulations of this setup were made for two cases: (a) the aperture illuminated by a plane wave and (b) the aperture illuminated by a small dipole on the optical axis approximating a point source. Line plot comparisons for the plane wave case of the MWS simulation and an analytical calculation were made. The analytical expression for the time averaged plane wave diffracted power per unit solid angle by a circular...
aperture is proportional to
\[ \frac{dP}{d\Omega} \propto \frac{2J_1(ka \sin \theta)}{ka \sin \theta}^2 \]  
(1.1)

where \( J_1 \) is a Bessel function of first order, \( k \) the wave vector, \( a \) the aperture radius and \( \theta \) the angle with respect to the propagation direction. We found that the far field comparison between the FDTD and analytical methods was in good agreement. We estimate the far field approximation to be valid if
\[ Z_{ff} = \frac{2D^2}{\lambda} = \frac{2(2 \times 6.19 \text{ cm})^2}{(3 \times 10 \text{ cm s}^{-1}/15 \text{e9 s}^{-1})} = 153 \text{ cm}, \]
where \( D \) is the diameter of the aperture.

The electric near field downstream from the aperture was measured. The comparisons of the MWS FDTD simulation to the experimental results are shown in Fig. 1. These experimental results are in excellent agreement with the simulations and indicate that we are able to adequately model the setup used to test our lenses. We also measured surface plots in the far field at 2.0 m from the empty aperture illuminated by a small dipole source placed at the approximate focal spot of the lenses to be tested. These measurements were made to determine the gain of the lenses in the far field as compared to the empty aperture.

3 Design and characterisation of PIM lens

A PIM lens was fabricated to compare the NIM and GRIN lenses to be discussed later. This lens was made of Rexolite having \( n = 2.53 \) \( (\alpha = 1.59) \). The designed focal length of the lens was 12.7 cm. This lens along with a simple ray tracing analysis is shown in Fig. 2. Note that for the full aperture not all of the rays intersect at the same point due to aberration effects that cause the focal spot to be elongated.

A comparison of the experimental and simulated electric near field focusing characteristics for the PIM lens is shown in Fig. 3. The line plots in this figure show excellent agreement between the experiment and simulation. The oscillations along the \( z \) propagation direction for the experimental data are due to interference between the receiving antenna and the mounting fixture. Also, the experimental profile is broader than the simulated profile due to the finite dimensions of the receiving antenna.

We also measured the far field pattern/gain with respect to the empty aperture. We define gain with respect to the empty aperture as \( G(\text{dB}) = 10 \log \left( \frac{E_{\text{PIM}}}{E_{\text{APT}}} \right) \). The gain for the PIM lens is approximately 10 dB above the empty aperture.

4 Design and characterisation of NIM lens

We have fabricated an NIM lens with a designed focal length of 12.2 cm and a 24.4 cm radius of curvature. This design assumes that the NIM index of refraction \( n = -1.0 \) at an operating frequency of ~14.8 GHz. The smooth spherical lens surface is replaced by a stepwise approximation with steps the size of the unit cell dimension at 0.254 cm as shown in Fig. 4.

The unit cell dimensions for the 2E2H NIM (2E2H indicates that the structure couples with two polarisations of the E and H fields) are shown in Fig. 5. Note that this unit cell is indefinite so that \( \varepsilon = \mu = (-1.0, -1.0, 1.0) \) Our ray tracing and effective media simulations have shown that this indefinite unit cell will perform nearly as well as a full 3E3H unit cell having \( \varepsilon = \mu = (-1.0, -1.0, -1.0) \) [9]. The 2E2H unit cell lends itself to simpler fabrication techniques and reduces the need for cut wires in the propagation direction. Impedance matching with \( Z = 1.0 \) is achieved by designing \( \varepsilon = \mu \) at the operating frequency. A comparison of the simulated and measured S21 for the Pathfinder 2E2H unit cell is also shown in Fig. 5. Note that at the operating frequency the experimental transmission is ~0.90 for a single unit cell due to dissipative losses and impedance mismatch of the material. This unit cell loss has an impact on the lens performance where multiple unit cells are required in the propagation direction.

![Fig. 1 Experimental (top left) and MWS simulated (top right) electric near field amplitude of empty aperture at 14.6 GHz](image)

Line plots (bottom) of the experimental (dash) and MWS simulated (solid) fields are in excellent agreement.
A detailed comparison between the MWS near field focusing simulation and experimental data is shown in Fig. 6 for the NIM lens. The line plots in this figure show excellent agreement between the experiment and simulation. Note that for the simulation, the effective \( \varepsilon \) and \( \mu \) were adjusted to \((2.0, 2.0, 1.0)\) which is within our fabrication and experimental error. As mentioned previously, the oscillations along the \( z \) propagation direction for the experimental data are due to interference between the receiving antenna and the lens-mounting fixture. Also, the experimental profile is broader than the simulated due to the finite dimensions of the receiving antenna.

The electric far field pattern/gain with respect to the empty aperture was also experimentally determined. The gain for the NIM lens was \( \sim 5.0 \text{ dB} \) above the empty aperture.

5 Design and characterisation of GRIN lens

We have fabricated a GRIN lens having a designed focal length of 12.7 cm as shown in Fig. 7. The principle advantage of a GRIN lens is its uniform thickness, which on average is thinner than a physically curved NIM lens as characterised above. The GRIN lens is constructed by using an NIM with a variable index of refraction in the radial direction, perpendicular to the direction of propagation \( z \). To design the lens we used a ray-tracing calculation based on an anisotropic eikonal equation [9]. The gradient required for the 12.7 cm focal length lens is \( \varepsilon = \mu = -1.1 - 0.0501 r^2 + 0.0001 r^4 \). The unit cell stepwise approximation to this smooth gradient is also shown in Fig. 7. The GRIN lens has 5 NIM unit cells in the propagation direction. Each unit cell is 0.20 cm in width for a total thickness of 1.0 cm. The number of unit cells designed was 19, labelled A–S as indicated in the figure. The details of the A–S unit cell designs are shown in Fig. 8. Note that this geometry corresponds to a 1E1H unit cell (1E1H indicates that only one polarisation for the electric and magnetic fields are coupled by the structure), that is, \( \varepsilon = (-1.0, 1.0, 1.0) \) and \( \mu = (1.0, -1.0, 1.0) \). The index of refraction and impedance for the A–S type cells were calculated from simulated \( S \) parameters for normal incidence of the electromagnetic wave. The unit cells were designed such that

![Fig. 2 Rexolite PIM lens (top) having dimensions shown in ray tracing diagram (bottom)](image)

![Fig. 3 Experimental (top left) and MWS simulated (top right) focusing electric near field amplitude of PIM lens at 14.6 GHz](image)

Line plots (bottom) of the experimental (dash) and MWS simulated (solid) fields are in excellent agreement. Focal spot for this lens is at \( \sim 9.0 \text{ cm} \)
impedance was matched to free space at the operating frequency.

The MWS near field effective medium focusing simulation and experimental data for a 1E1H unit cell GRIN lens are compared in Fig. 9. This analysis indicates that the 1E1H unit cell performs well for this GRIN application. The 1E1H unit cell has the distinct advantage of being simple to fabricate and assemble into a GRIN lens. Note that the index gradient for the simulation was adjusted (i.e. \( e = \mu = -1.1 - 0.03 r^2 + 0.0001 r^4 \)) to better match the experimental data. This effectivly reduces the index gradient in the lens so that at the centre A type unit cell the refractive index is \(~1.1\) and at the edges the S type unit cell refractive index is \(~2.4\). This indicates that for the fabricated lens the gradient achieved was somewhat lower than the design goal as shown in Fig. 7.

The measured electric far field pattern/gain with respect to the empty aperture indicated that the gain for the GRIN lens was \(~10.0\) dB above the empty aperture.

6 Comparison of experimental data for aperture, PIM, NIM and GRIN lenses

We have compared the performance of our PIM, NIM and GRIN lenses to each other at the approximate design frequency (14.8 ± 0.3 GHz) as well as below (13.1 GHz) and above (16.3 GHz) the design point. As shown in Fig. 5 (S21 magnitude against frequency for NIM unit cell), we expect the NIM lens to have little transmission at 13.1 GHz, perform best at \(~14.8\) GHz (where unit cell is matched to free space) and have degraded performance at 16.3 GHz (where the NIM has good transmission but is not matched to free space). Indeed, this is what we observed.
in our experimental measurements as shown in Fig. 10. In this figure, we plot the electric field values in the far field for the empty aperture PIM, NIM and GRIN lenses at 13.1, 14.8 and 16.3 GHz, respectively. The aperture and lenses were illuminated by a small dipole point source placed at the focal spot for each lens and the corresponding location for the empty aperture. Note that the electric field value for the NIM is \(~50\%\) of the value for the PIM lens.

Fig. 6 Experimental (top left) and MWS simulated (top right) focusing electric near field amplitude of NIM lens at 14.6 GHz. Line plots (bottom) of the experimental (dash) and MWS simulated (solid) fields are in excellent agreement. Focal spot for this lens is at \(~11.0\) cm.

Fig. 7 Fabricated GRIN lens (top), location of various cell types (bottom left) and step wise approximation to smooth \(n = \mu = -1.1 - 0.0501 r^2 + 0.0001 r^4\) gradient (bottom right) needed for 12.7 cm focal length. Note that each unit cell A–S has a different index of refraction ranging from \(n = -1.1\) to \(-3.2\).
This is due to losses in the NIM unit cell. If the transmission is \( \sim 0.90 \) for a single unit cell at the design frequency, then the transmission through the \( \sim 5 \) unit cell thick NIM lens is \( S_{21,NIM} \sim (0.9)^5 = 0.59 \). For the PIM lens the transmission is higher. For this case \( S_{11,PIM} = E_r/E_i \sim (n-1)/(n+1) = (1.59 - 1.00)/(1.59 + 1.00) = 0.23 \), where \( E_i \) is the incident field, \( E_r \) the reflected field and \( n \) the refractive index of Rexolite. For one interface \( S_{21,PIM} = (1 - S_{11}^2)^{1/2} = (1 - 0.23^2)^{1/2} = 0.97 \), that becomes 0.94 for two interfaces, so we expect that \( S_{21,NIM}/S_{21,PIM} \sim 0.59/0.94 = 0.63 \) which is approximately what we observe experimentally. The experimental value (\( \sim 0.50 \)) is probably lower due to fabrication tolerances and the resulting increased impedance mismatch. We note, however, that the FWHM of the NIM lens (\( \sim 12.5^\circ \)) is smaller than the GRIN lens (\( \sim 14^\circ \)) and PIM lens (\( \sim 15^\circ \)), which was expected based on an aberration analysis [9].

The GRIN lens exhibits a normalised electric far field value that is \( \sim 75\% \) that of the PIM lens. This is higher than the NIM lens due to the lower losses of the 1E1H unit cell structure. The GRIN lens FWHM maximum (\( \sim 14^\circ \)) is smaller than the PIM lens but larger than the NIM lens.

Fig. 8 1E1H NIM unit cell design for 19 step GRIN lens

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<th>Cell Label</th>
<th>Average (cm)</th>
<th>Index (n)</th>
<th>Impedance (Z)</th>
<th>w (mm)</th>
<th>( g_o ) (mm)</th>
<th>( g_i ) (mm)</th>
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7 Conclusion

We have simulated, designed, fabricated and tested a variety of lenses operating in the microwave region. These lenses include PIM, NIM and GRIN lenses. The test setup in which these lenses were measured was also characterised. Our results indicate that in general the lenses made of NIMs are lighter than the normal PIM lenses. This is due to the honeycomb-like techniques used for NIM fabrication. The NIM lens exhibited a gain less than the GRIN and PIM lenses due to the inherent higher material losses. The GRIN lens, however, had a gain comparable to the PIM lens. On the basis of these results, further development and testing of lenses made of NIMs is warranted.

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9 References


