Interferometric direction finding with a metamaterial detector
Suresh Venkatesh, David Shrekenhamer, Wangren Xu, Sameer Sonkusale, Willie Padilla, and David Schurig

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The metamaterial concept enables the implementation of structures with desired material properties through the design of the elemental units that comprise them. The collection of elemental units, or unit cells, exhibits an average behavior which can be understood through effective medium theory. A very powerful feature of this concept is the wide range of length scales at which one can design the unit cells, within the requirement that the simplest (and most useful) average behavior will arise when they are small compared to the operational wavelength. At gigahertz frequencies, where the unit cells can be on the millimeter scale, one can bring to bear the arsenal of micro-electronics within the unit cell itself. One can then design a medium with desired material properties, but with the added ability to sense, excite, or tune the individual unit cells in an addressable manner. In a traditional material, such microscopically local interactivity is generally not practical. This convenience of scale opens up a unique design methodology wherein the medium is the device. One can always implement the desired functionality for a device previously used as a focal device. For both amplitude and phase sensing mediums, a desirable base is the perfect absorber, i.e., a thin or two dimensional medium that is perfectly matched to free space, resulting in minimal reflection over a range of incidence angles. Reflection minimization helps maximize the detected power and minimize the device’s observability at operational frequencies. To incorporate sensing into the medium, the unit cells are adapted so that the power absorbed by the medium is dissipated in the input resistance of a low noise amplifier mounted within the unit cell area. Additional electronics implement either an intensity or vector measurement.

In this article we describe direction finding measurements performed with our existing device medium, where the vector measurements are performed off-board with a lock-in amplifier, and unit-cell (pixel) addressing is achieved by manually connecting coaxial cables to ports on the unit-cell back-side. We also discuss the performance of an integrated device, with in-unit-cell mixing. This avoids the routing of high frequency signals from the detector to the off board back-end electronics, as these high frequency signals are more prone to amplitude and phase distortions. Here, performance measures are determined using the existing mediums geometry and matching capability, together with proposed integrated measurement electronics, and with signals processed using the well known MUltiple SIgnal Classification (MUSIC) algorithm.

The detector consists of 11 × 11 electrically coupled LC resonators (ELC) arranged in a square lattice. These ELCs form the front end elements and are designed to have a surface impedance that matches free space, thus acting as a perfect absorber. Instead of absorbing the incident RF radiation and dissipating it in the metamaterial structure, one can tap electromagnetic energy through vias and process it with conventional electronics as described later in the article. The array of ELCs was patterned on Rogers 4003 substrate; Fig. 1 shows one of the pixels and its corresponding back-end electronics. Free space measurements of the center pixel were performed with our existing device medium, where the vector measurements are performed off-board with a lock-in amplifier, and unit-cell (pixel) addressing is achieved by manually connecting coaxial cables to ports on the unit-cell back-side. We also discuss the performance of an integrated device, with in-unit-cell mixing. This avoids the routing of high frequency signals from the detector to the off board back-end electronics, as these high frequency signals are more prone to amplitude and phase distortions. Here, performance measures are determined using the existing mediums geometry and matching capability, together with proposed integrated measurement electronics, and with signals processed using the well known MUltiple SIgnal Classification (MUSIC) algorithm.

The array of ELCs was patterned on Rogers 4003 substrate; Fig. 1 shows one of the pixels and its corresponding back-end electronics. Free space measurements of the center pixel were carried out in an anechoic chamber to measure reflection
this experiment.

The rest of the circuitry presented in our earlier article.

A detailed discussion on the interferometric receiver described in the previous paragraph, correlations of the central horizontal strip of pixels with the phase reference pixel were manually recorded. Fig. 2(b) shows the measured phase variation by making eight correlation measurements along with the linear regression fit to the measured data. The measurements of the edge pixels were justifiably removed from the data, and did not match the slope of the remaining points. This deviation is likely due to diffraction effects at the edges and the absence of a complete set of nearest neighbors. 11 For the plane wave $E = E_0 e^{i \mathbf{kr}} = E_0 e^{i \mathbf{k} \mathbf{r}}$, the phase variation $\psi$ and the direction of arrival (DOA) of the wavefront $\phi$ are related by

$$\psi (\mathbf{r}) = \mathbf{k} \cdot \mathbf{r} + \psi_0, \quad (1)$$

$$\psi_m = \sin (\phi) k a + \psi_0, \quad (2)$$

$$\Delta \psi = \psi_{m+1} - \psi_m = \sin (\phi) k a, \quad (3)$$

where $\mathbf{k}$ is the wave vector with $k = 2\pi/\lambda$, $\lambda$ is the operating wavelength = 0.12 m, $m$ is the pixel number on the detector, and $a$ is the unit-cell dimension = 27.3 mm. From linear

from any two pixels in the array. The RF signals from the two pixels are directly tapped after the balun and fed to the receiver. The primary channel (blue) in the receiver always acts as a phase reference. The signal from the center pixel was chosen for this purpose. Only the signal from the primary/phase reference channel is amplified with a high gain low noise RF amplifier. Later, both the signals are directly down converted to baseband (DC - 100 kHz) using passive RF mixers with a common local oscillator (LO). (A common LO is essential in order to maintain coherence between the two signals). These down converted signals are then filtered with low pass filters (LPF) and fed to the lock-in amplifier which acts as an analog correlator. Correlation coefficients with the center pixel and other test pixels are recorded by manually connecting the secondary channel (red) to the respective test pixel. Fig. 2(a) shows the dual channel receiver.

For a demonstration of direction finding, the detector was tilted with respect to the horn antenna, so that the horn appeared to be a source at an angle of 45° to the detector normal. By this approach a uniform phase gradient was established along the detector in the horizontal direction. Using the interferometric receiver described in the previous paragraph, correlations of the central horizontal strip of pixels with the phase reference pixel were manually recorded. Fig. 2(b) shows the measured phase variation by making eight correlation measurements along with the linear regression fit to the measured data. The measurements of the edge pixels were justifiably removed from the data, and did not match the slope of the remaining points. This deviation is likely due to diffraction effects at the edges and the absence of a complete set of nearest neighbors. For the plane wave $E = E_0 e^{i \mathbf{kr}} = E_0 e^{i \mathbf{k} \mathbf{r}}$, the phase variation $\psi$ and the direction of arrival (DOA) of the wavefront $\phi$ are related by

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where $\mathbf{k}$ is the wave vector with $k = 2\pi/\lambda$, $\lambda$ is the operating wavelength = 0.12 m, $m$ is the pixel number on the detector, and $a$ is the unit-cell dimension = 27.3 mm. From linear
regression, the phase interval $\Delta \psi$ is $58.5^\circ \pm 1.1^\circ$, and the predicted source direction is $45.6^\circ \pm 1.1^\circ$, in agreement with the experimentally configured angle. The uncertainty in the DOA $\Delta \phi$ depends inversely on the signal to noise ratio (SNR), the square root of number of correlations, and the effective area of the detector.

The MUSIC algorithm is a DOA estimation technique first proposed by Schmidt. The algorithm provides an unbiased estimate of the number of signals, their DOA, and their strength, by exploiting the noise subspace of the input covariance matrix. Conventional interferometry is a special case of MUSIC. The mathematical model of the 2D MUSIC algorithm is explained below.

Consider an $N \times N$ planar array in the $zy$ plane, where $x = r \sin(\theta) \cos(\phi)$, $y = r \sin(\theta) \sin(\phi)$, $z = r \cos(\theta)$ and $(r, \theta, \phi)$ denotes the usual spherical coordinate system with $r \geq 0$, $0^\circ \leq \theta \leq 180^\circ$, and $-180^\circ \leq \phi \leq 180^\circ$. Let $K$ signals $[S_1(t), S_2(t), ..., S_K(t)]$ be incident on the array at angles $[(\phi_1, \theta_1), (\phi_2, \theta_2), ..., (\phi_K, \theta_K)]$ where $(\phi_m, \theta_m)$ is the azimuth and elevation angle of the $m$th signal. Let $S_k(t) = R e^{|e^{j2\pi f_0 t}}$, where each $S_k(t)$ contains at least $2f_c \tau = P$ independent samples, $f_c$ is the frequency of the signal, and $\tau$ is the system integration time.

The received signal at the $m$th pixel, at time $t$ can be written as

$$X_m(t) = \sum_{k=1}^{K} A_m(\phi_k, \theta_k) S_k(t) + \nu(t), \quad (4)$$

$$X = [X_1(n), X_2(n), ..., X_N^2(n)]^T, \quad (5)$$

where $A_m(\phi_k, \theta_k)$ is the time delay or phase factor for a signal incident at an angle $(\phi_k, \theta_k)$ on the $m$th pixel, $\nu(t)$ is the noise function with zero mean and variance $= \sigma^2$, $n = [0, 1, ..., P-1]$ is the discrete time index, and $X$ is a data matrix of size $N^2 \times P$.

The input covariance matrix $R$ can be constructed as

$$R = XX^T, \quad (6)$$

where $R$ is a square matrix of size $N^2 \times N^2$. Eigen value decomposition of $R$ leads to

$$R = E_n A E_n^T + E_s A E_s^T, \quad (7)$$

where $E_n = [e_1, e_2, ..., e_K]$ is the eigen signal space, $E_s = [e_{K+1}, e_{K+2}, ..., e_{N^2}]$ is the eigen noise space, and $A = [A_1, A_2, ..., A_{N^2}]$ are the eigenvalues. The full rank of the matrix $R$ is $K$, where $K$ is the number of eigenvalues that fall above certain threshold ($K$ is also the number of incident signals at the array). These are called the signal eigenvalues, and the remaining eigenvalues are called the noise eigenvalues. One needs to note that in case of incident signals that are coherent, the rank of the matrix $R = 1 < K$, leading to the deteriorated performance of the MUSIC algorithm. One can overcome this problem by non-uniformly sampling at each pixel or by spatial smoothing techniques.

The MUSIC pseudospectrum is calculated as

$$P_M(\phi, \theta) = \frac{1}{\vert A(\phi, \theta)^H E_n E_s^T A(\phi, \theta) \vert}, \quad (8)$$

where $A(\phi, \theta)$ is the array factor for any given $(\phi, \theta)$ and any significant peaks in the pseudospectrum denote the DOA of incident signals.

We demonstrate the working of this algorithm for the current metamaterial detector properties: $N = 11$ with uniform pixel weighting, pixel spacing of 27.3 mm, and frequency of operation $f_c = 2.5$ GHz. Fig. 3(a) shows the pseudospectrum of MUSIC algorithm for eight signals with arbitrarily chosen directions of incidence. The “+” markers in the figure show the actual DOA of eight signals. Each signal has an input SNR = 0 dB and is sampled at the Nyquist rate $(f_s = 2f_c)$ with $P = 512$ samples. The noticeable peaks of the pseudospectrum coincide with the markers, demonstrating the working of this algorithm for the current detector parameters.

Fig. 3(b) shows the performance of conventional beamforming method for the same input parameters as above. In beam-forming, one constructs a primary narrow beam from individual pixels of the array operating in interferometric mode. This single narrow beam can be spatially scanned (weighting each pixel with appropriate delay and summing) to measure the power received from each direction. The direction at which the received power is maximum is the DOA estimate. The received power pattern can be calculated as

$$P_B(\phi, \theta) = \vert A(\phi, \theta)^H R A(\phi, \theta) \vert, \quad (9)$$

where $R$ is the input covariance matrix in Eq. (6). The primary $-3$ dB beamwidth using this technique is $\sim \lambda/D$ which is the conventional Abbe’s limit, where $\lambda$ is the operating wavelength and $D$ is the largest baseline. The beam-forming method has substandard performance when compared with the MUSIC algorithm.

FIG. 3. (a) The normalized MUSIC pseudospectrum. (b) and (c) The conventional beam-forming method and signal space method on the same incident signals (8 incident signals with the following (azimuth, elevation) angles ($-60^\circ$, 40°), ($-40^\circ$, 60°), ($-30^\circ$, 140°), ($-15^\circ$, 90°), ($-5^\circ$, 120°), (30°, 155°), (45°, 100°), (60°, 30°); each signal SNR = 0 dB, $P = 512$ samples).
Instead of constructing the pseudospectrum based on the noise space, one can do the inverse by constructing the pseudospectrum based on the signal space. The pseudospectrum based on the signal space is given by

$$P_S(\phi, \theta) = |A(\phi, \theta)^\dagger E_x E_y^\dagger A(\phi, \theta)|,$$

(10)

The MUSIC algorithm based on the noise space again performs better in terms of the resolving power as compared with the signal space method shown in Fig. 3(c).

To implement DOA estimation using MUSIC with the current metamaterial detector configuration, requires discretized time domain samples from each pixel. These samples should be of high fidelity and also have high SNR. Fig. 4(a) shows a system level architecture for the back-end. The back-end electronics consist of on-board down converting stage for each pixel and an off-board sampling stage, comprised of commercial off-the-shelf components. The on-board system consists of a matching stage and a heterodyne stage. The heterodyne stage has a high-gain-low noise S-band amplifier whose output is filtered by a narrow bandpass surface acoustic wave filter. Then the filtered RF signal is down converted by an active mixer to baseband (DC - 8 MHz). Footprints of all these components are small enough to be mounted on the back plane within each pixel. The down converted signal is then sampled by an off-board sampling stage which consists of an LPF and an analog to digital converter (ADC). The ADC is chosen to have a sampling rate of 16 Msamples/s and an integration time of 1 ms, therefore collecting 16 k samples from each pixel. The sampling of each pixel output happens in a time division multiplexed (TDM) fashion in order to reduce the complexity. The digital data from each pixel should be phase adjusted for the multiplexing time delay and processed to form the MUSIC pseudospectrum. For this configuration, the pseudospectrum can be generated at frame rates of ~100 ms. The calculated overall noise figure of this receiver chain is about 2.5 dB.

The device’s resolving power with the proposed back-end architecture was analyzed. The azimuthal separation between two sources (of equal amplitude and near normal incidence $\theta = 90^\circ$) was decreased until no local minimum of the MUSIC pseudospectrum was observed between them. Fig. 4(b) shows the angular resolution for a given incident radiation density at the device (blue curve). Sources with power outputs typical of mobile/wireless devices (about +24 dBm) can potentially be resolved at a kilometer distance away with sub-degree resolution. This metric was also analyzed for a traditional $\lambda/2$ spaced antenna array with the same aperture area (Fig. 4(b) black dashed curve). The metamaterial detector outperforms the traditional array at low SNRs. Such RF direction finders can find potential applications in detecting RF interference sources in radio astronomy, remote sensing, radio monitoring, and imaging applications.

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5. See supplementary material at http://dx.doi.org/10.1063/1.4851936 for the simulation on unit cells with and without neighboring pixels.