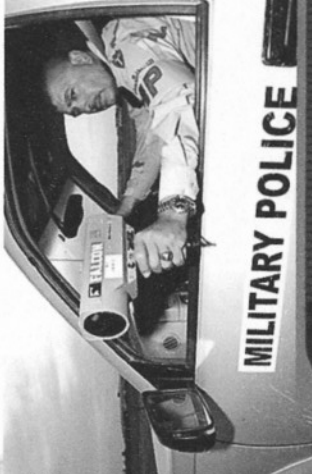
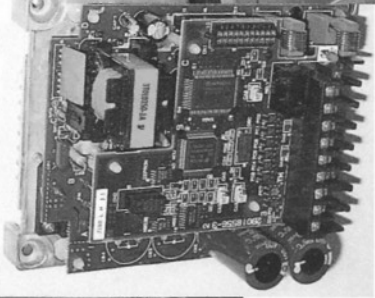


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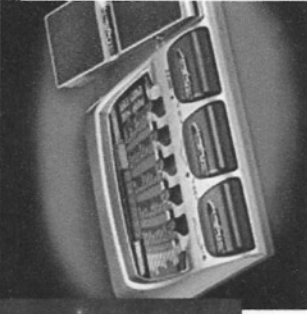
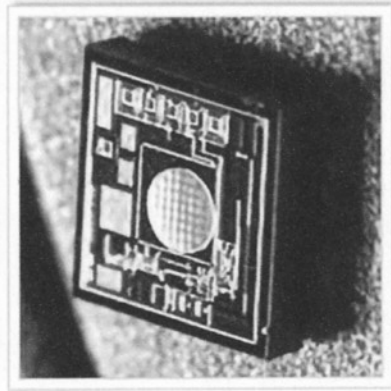
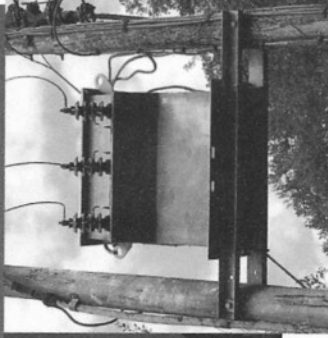
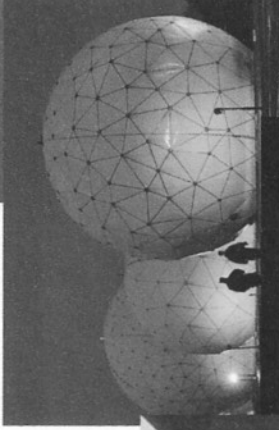
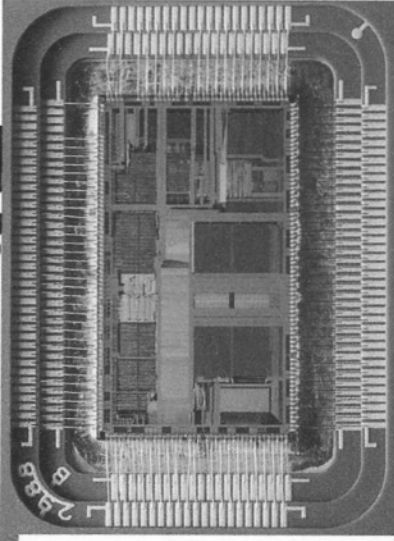
# Introduction to Electrical and Computer Engineering

*Professor Angela Rasmussen*

# What is an Electrical Engineer



- Communications
- Computer Engineer
- Control Engineer
- Electronics
- Instrumentation Engineering
- Microelectronics
- Power Engineering
- Signal Processing



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**UNIVERSITY OF UTAH**  
**Department of Electrical and Computer Engineering**  
**ECE 1270 - Introduction to Electrical and Computer Engineering**

**Instructor:** Dr. Angela Rasmussen  
Office: MEB 3254 Office hours: Wed. 10:35-11:35am, by appt.  
Phone: 971-1096 (cell phone)  
Email: ece1270@comcast.net

**Class Website:** <http://www.ece.utah.edu/~ece1270>

**Prerequisite:** MATH 1210 or 1270

**Co-requisite:** ECE 1020, MATH 1220 or 1280, and PHYCS 2210

**Required Text:** *Electric Circuits, 8<sup>th</sup> Edition*  
James W. Nilsson and Susan A. Riedel  
Prentice Hall: Upper Saddle River, NJ

**Required Packets:** ECE 1270 Study Guide  
(or available on web) Carl H. Durney and Neil E. Cotter  
Available from Campus Copy Center in Union Building

ECE 1270 Conceptual Tools  
Neil E. Cotter et al.  
Available at Copy Center

**Homework:** Due **before** class Wednesday's or on day indicated in syllabus.  
No late HW accepted.  
Turn in to locker on 3rd floor of MEB near southeast stairway.

**Cheating:** Any form of cheating will result in an "E" grade. Students are encouraged to discuss assignments, but each student must do all their own work on assignments.

**Equal Access:** The University of Utah seeks to provide equal access to its programs, services and activities for people with disabilities. If you will need accommodations in this class, reasonable prior notice needs to be given to the instructor and to the Center for Disability Services, 162 Olpin Union Building, 518-5020 (V/TDD) to make arrangements for accommodations.

All written information in this course can be made available in alternative format with prior notification.

Any questions of concerns about the above information may be directed to:

Olga Nadeau	Julene Persinger
Director, CDS	ADA Coord/Assoc Director, OEO/ER
162 Olpin Union Building	135 Park Building
581-5020	581-8365

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Week	Date	Topic
1	Jan. 7	Course Procedures, Intro, Basic DC Circuits: Algebra, Passive Sign Convention, Units, Voltage $v$ , Current $i$ , Power $p$ , Sources, Kirchoff's Laws, Ohm's Law
	Jan. 9	Passive Sign Convention, Sources-voltage, current, independent and dependent, Kirchoff's Laws, Ohm's Law
	Jan. 11	Kirchoff's Laws, Ohm's Law (cont)
2	Jan. 14	Circuits: Resistor Networks (parallel/series), voltage and current dividers, power dissipation, Op Amps
<b>HW1</b>	Jan. 16	<b>HW1 due</b> Op Amps: Ideal Amplifiers
	Jan. 18	Op Amps: Ideal Amplifiers
3	Jan. 21	Martin Luther King Day
<b>HW2</b>	Jan. 23	<b>HW2 due</b> Review
	Jan. 25	<b>Midterm #1</b>
4	Jan. 28	Node Voltage Method
	Jan. 30	Node Voltage, Mesh Current Method
	Feb. 1	Mesh Current, Thevenin Equivalent
5	Feb. 4	Thevenin Equivalent
<b>HW3</b>	Feb. 6	<b>HW 3 due</b> Review
<b>HW4</b>	Feb. 8	<b>HW 4 due</b> Review
6	Feb. 11	<b>Midterm #2</b>
	Feb. 13	RC/RL Circuits: C(Capacitor Equations), L (Inductor Equations), General Solution
	Feb. 15	RC/RL Circuits: C(Capacitor Equations), L (Inductor Equations), General Solution (Cont.)
7	Feb. 18	President's Day
	Feb. 20	RC/RL Examples
<b>HW5</b>	Feb. 22	<b>HW5 due</b> Examples
8	Feb. 25	Maximum Power Transfer, Superposition
<b>HW6</b>	Feb. 27	<b>HW6 due</b> Examples
	Feb. 29	Review
9	March 3	Review
	March 5	<b>Midterm #3</b>
	March 7	Complex Analysis – Basic Math (properties of complex number, rationalization, add, subtract)
10	March 10	Complex Analysis – Basic Math (properties of complex number, rationalization, add, subtract) (Cont.)
	March 12	Complex Analysis - Impedance, Phasors, Ohms Law Impedance Circuits – Kirchoff's Laws, Node-Voltage Method, Thevenin Equivalent
	March 14	Impedance Circuits – Superposition
11	March 17-22	Spring Break
12	March 24	Examples
<b>HW7</b>	March 26	<b>HW7 due</b> Examples
	March 28	Examples
13	March 31	Examples
<b>HW8</b>	April 2	<b>HW8 due</b> Examples
	April 4	Examples
14	April 7	Review
	April 9	<b>Midterm #4</b>
	April 11	Review
15	April 14	Review
<b>HW9</b>	April 16	<b>HW9 due</b>
	April 18	Review
16	April 21	<b>HW10 due</b>
	April 23	Review
<b>Wed</b>	<b>April 30</b>	<b>Final Exam, 8-10am (Comprehensive)</b>

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## Text Reading and Example Problems with Answers

### Unit 1:

Ch. 1 Units, Voltage, Current, Power, Passive Sign Convention (*Summary pg. 17*)

Ch. 2 Analysis of circuits - Dependent sources, power, Kirchhoff's Law's, Ohm's Law (*Summary pg. 47*)

*Assessments:* (pg. 28, 32, 42, 46) 2.1-2.5, 2.8-2.10

*Back of Chapter Problems:* (pg. 48-53) Pr. 2.2-2.3, 2.6, 2.8, 2.14, 2.17-2.19, 2.24, 2.28

Ch. 3 (3.1-3.4) Resistors in parallel and series, Voltage Divider, Current Divider

*Assessments:* (pg. 62, 65, 67) 3.1-3.4

*Back of Chapter Problems:* (pg. 80-84) Pr. 3.1, 3.2, 3.5, 3.6, 3.13, 3.15, 3.21-3.23

Ch. 5 (5.1-5.2) Op Amps

*Assessments:* (pg. 161) 5.1

*Back of Chapter Problems:* (pg. 176) Pr. 5.1-5.3

*Examples:* 2.1, 2.2, 2.6-2.8, 2.10-2.11, 3.1-3.4

### Unit 2:

Ch. 4 (4.1-4.11) Definitions, Node Voltage, Supernode, Mesh current, Supermesh, Thevenin Equivalent

*Assessments:* (pg. 99-101, 104, 107, 109, 112, 116, 123, 125) 4.1-4.14, 4.16, 4.18-4.20

*Back of Chapter Problems:* (pg. 139-149) Pr. 4.6, 4.9, 4.10, 4.12, 4.13, 4.19-4.21, 4.26, 4.27, 4.31, 4.32, 4.37, 4.38, 4.41, 4.42, 4.47, 4.50, 4.54-4.56, 4.63, 4.67a, 4.71, 4.77

*Examples:* 4.1-4.3, 4.4-4.7, 4.10, 4.11

### Unit 3:

Ch. 6 (6.1.6.3) – Inductors, Capacitors (*Summary pg. 217*)

Ch. 7 (7.1-7.4) - 1st order response, RL switch, RC switch, General Solution

*Assessments:* (pg. 195, 199, 203, 236, 240, 245, 248) 6.1-6.5, 7.1(a-d), 7.2a, 7.3(a-d), 7.4-7.6

*Back of Chapter Problems:* (pg. 218, 220-222, 265-273) Pr. 6.1, 6.3, 6.14, 6.15, 6.21, 6.22, 6.26, 6.27, 7.1(a-c), 7.2, 7.3, 7.21, 7.24, 7.33-7.35, 7.50, 7.51

Sections 4.12, 4.13 Maximum Power Transfer, Superposition

*Assessments:* (pg. 129) 4.12-4.22;

*Back of Chapter Problems:* (pg. 149) Pr. 4.79, 4.80

*Examples:* 7.1-7.3, 7.4-7.6, 7.7-7.9, 4.12, 4.13

### Unit 4:

Ch. 9 Complex Numbers, Sinusoidal Signals, Phasors, Frequency Domain Analysis

*Assessments:* (pg. 342, 346, 348, 349, 352, 358, 360, 361) 9.1-9.8, 9.11-9.13

*Back of Chapter Problems:* (pg. 377-384) Pr. 9.12, 9.13, 9.15-9.16, 9.21, 9.26-9.28, 9.51, 9.56, 9.58, 9.61

*Examples:* 9.1-9.3, 9.5b, 9.6, 9.7, 9.10-9.12, 9.15

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# Lecture #1

Dec. 2007  
Angela Rasmussen

## UNITS

Current	Amps	A
Voltage	Volts	V
Resistance	Ohms	$\Omega$
Capacitance	Farads	F
Inductance	Henrys	H
power	watts	W

## Values

G	Giga	$10^9$
M	Mega	$10^6$
K	Kilo	$10^3$
m	milli	$10^{-3}$
$\mu$	micro	$10^{-6}$
n	nano	$10^{-9}$
P	pico	$10^{-12}$
f	femto	$10^{-15}$

## Conversions:

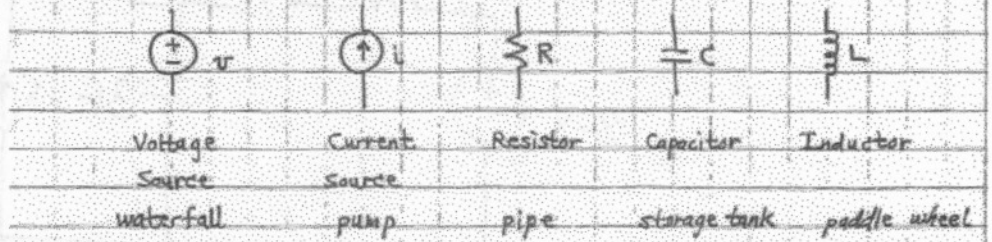
$$1F \equiv 1 \frac{C}{V}$$

$$1A \equiv 1 \frac{C}{sec.}$$

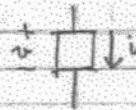
$$\Omega \equiv \frac{V}{A}$$

$$1H \equiv \frac{V \cdot S}{A}$$

## Circuit Elements

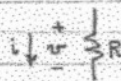


## Passive Sign Convention



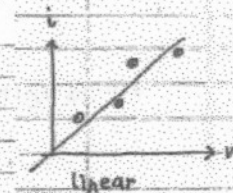
Current arrow always pts from plus to minus.

## Ohms Law



$$v = iR \quad \text{or} \quad i = \frac{v}{R}$$

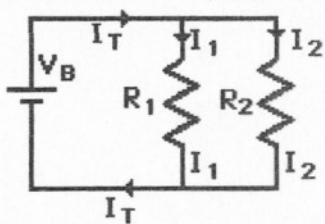
Linear device  $2x$  current  $\Rightarrow 2x$  voltage



## Kirchoff's Current Law (KCL):

- If you have water flowing into and out of a junction of several pipes, water flowing into the junction must equal water flowing out.
- The same applies to electric currents.

The algebraic sum of the currents into a node at any instant is zero



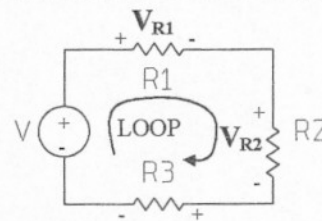
$$I_T - I_1 - I_2 = 0$$

$$-I_T + I_1 + I_2 = 0$$

$$I_T = I_1 + I_2$$

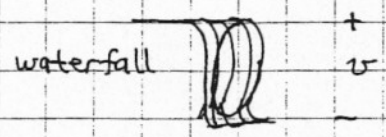
## Kirchoff's Voltage Law (KVL):

The algebraic sum of the voltages around any closed path is zero



$$-v + v_{R1} + v_{R2} + v_{R3} = 0$$

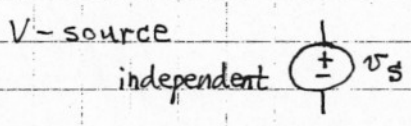
Voltage



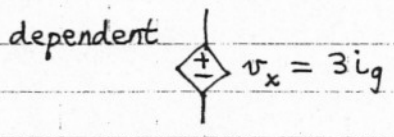
Voltage ↔ altitude  
Volts 10V typ 1mV small  
V

Lift charge  $\Delta q$   
change energy  $\Delta W$

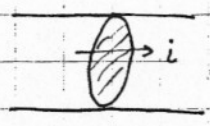
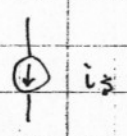
$$V = \lim_{\Delta q \rightarrow 0} \frac{\Delta W}{\Delta q} = \frac{dW}{dq}$$



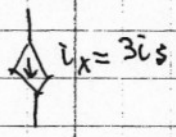
± indicates polarity of measurement



Current

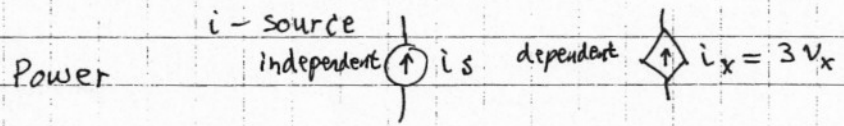


Current ↔ flow rate  
Amps 1A large 1mA typ  
A

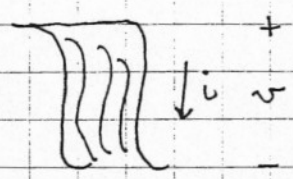


move charge  $\Delta q$   
in time  $\Delta t$

$$i = \lim_{\Delta t \rightarrow 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt}$$



Power



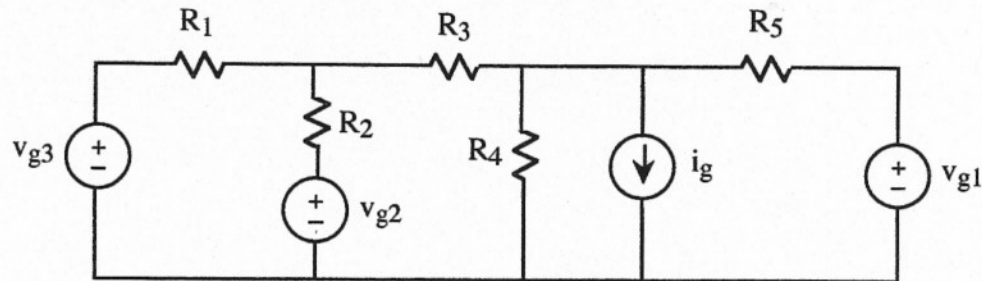
Power ↔ hydroelectric power  
Watts 1/4 W typ 100 W med  
W

change energy by  $\Delta W$   
in time  $\Delta t$

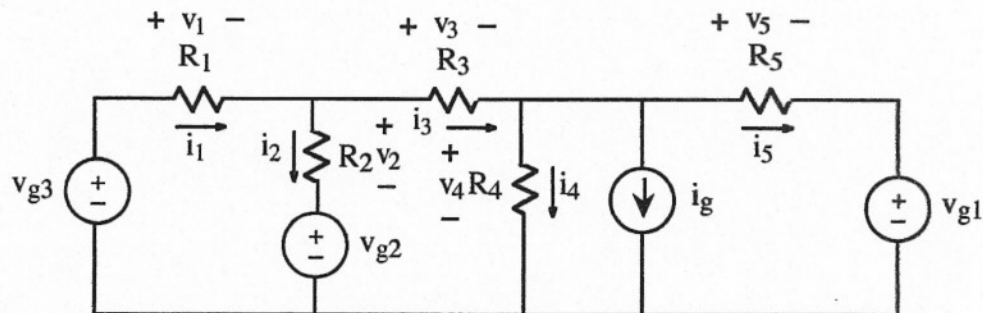
$$P = \lim_{\Delta t \rightarrow 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

chain rule  $P = \frac{dW}{dq} \frac{dq}{dt}$  tot pow in circuit = 0

**EX:** In the circuit below, label currents in each element and then label potential differences according to the passive sign convention.



**ANS:**



(Any labeling with current measurement arrows pointing from + signs to - signs of voltage measurements is valid.)

**SOL'N:** The answer is not unique. We may choose either direction for the + and - of the voltage measurement for each resistor and then label the current measurement with an arrow pointing from the + sign to the - sign. Or we may choose either direction for the current measurement arrow for each resistor and then label the voltage measurement with the + and - sign such that the current measurement arrow points toward the - sign. What matters is the consistency of voltage and current measurements with each other, not the actual direction of current flow or the sign of actual voltage.

When we deal with a source, we normally leave it as is and avoid adding labels to it. Sometimes, however, we may wish to calculate the power dissipation for a source. In that case, we take the + and - sign or the arrow as the direction of one measurement, and we make the other measurement consistent with it.

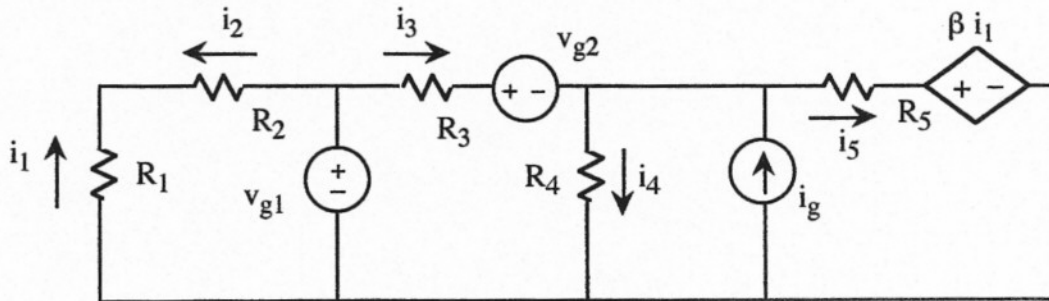


Thus, if we were to measure current in the voltage sources for the above circuit, the arrows would all point down. If we were to measure voltage in the current source for the above circuit, the + sign would be on top and the - sign would be on the bottom.

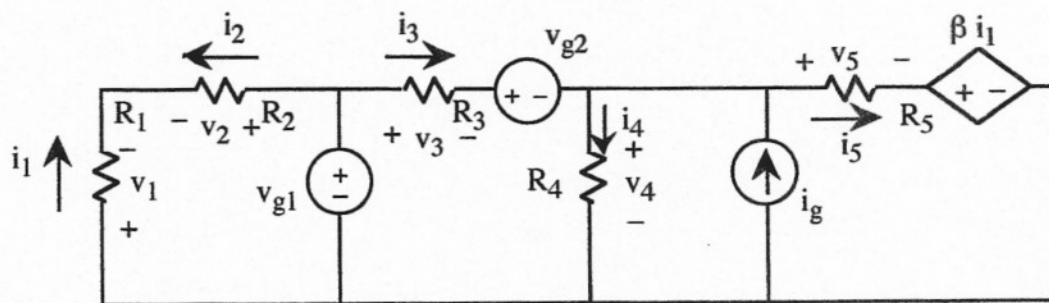
It often turns out to be convenient to label resistors with the current measurement arrow pointing to the right or pointing down. This often matches the direction of the physical current flow and is often the direction we would measure the output voltage for a circuit. Later on, we will usually designate the bottom wire on a circuit diagram as the "reference" which is where the minus sign of every voltage measurement will be located.

Note: we normally add voltage and current measurement labels for only the resistors in a circuit.

**EX:** In the circuit below, the currents in each element are labeled as shown. Using the passive sign convention, label the potential difference across each element and show its polarity.



**ANS:**



(Any labeling with current measurement arrows pointing from + signs to - signs of voltage measurements is valid.)

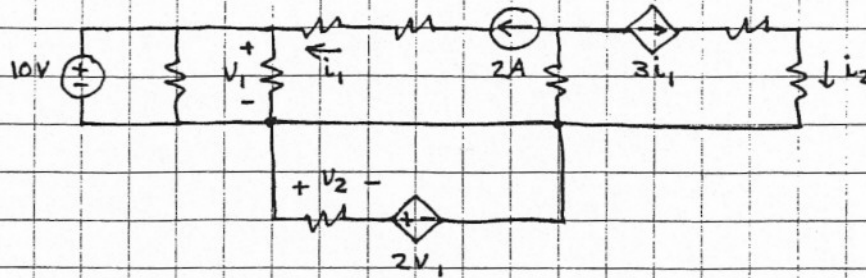
**SOL'N:** The passive sign convention dictates that the current measurement arrow always points away from the + sign and toward the - sign of the voltage measurement. Note that the current arrow and + and - sign for voltage indicate only the Polarity Of current and voltage Measurements—NOT the direction of the physical flow of current or the sign of the actual voltage. In other words, the arrows and + and - sign tell us how to connect the leads of a multimeter to make a measurement. We do not know, (nor do we have to know), in advance which direction current is flowing or what the actual sign of the voltage will be.

- DEF:** A loop is any continuous path (that may even cross gaps across open space) that ends where it starts.
- NOTE:** The goal of writing current summation and voltage loop equations is to obtain  $n$  equations in  $n$  unknowns that we can solve to find all the currents and voltages in a circuit.
- TOOL:** Loops that cross over themselves may always be treated as two smaller loops. (Smaller loops yield simpler equations.)
- NOTE:** We may proceed in either direction around a loop when we write a voltage loop equation, (but we must continue in the same direction all the way around the loop).
- TOOL:** We set the sum of voltage drops around a loop to zero. If we exit a circuit element from the + sign of the voltage measurement as we proceed around the loop, then that voltage appears with a plus sign in the loop equation. If we exit a circuit element from the - sign of the voltage measurement as we proceed around the loop, then that voltage appears with a minus sign in the loop equation.
- NOTE:** Using a + sign for a voltage term if we enter a circuit element from the + sign of the voltage measurement, and using a - sign for a voltage term if we enter a circuit element from the - sign of the voltage measurement yields an equation equivalent to using the opposite sign convention (as stated in the preceding tool). Multiplying one equation by  $-1$  on both sides yields the other equation.
- TOOL:** We skip voltage loops where we would be forced to define a voltage for a current source. Writing an equation for such a loop adds a new variable and a new equation. Thus, we merely create more equations in more unknowns rather than moving closer to the goal of writing  $n$  equations in  $n$  unknowns.
- TOOL:** We write voltage loops for all inner loops, if appropriate. If we are able to write an equation for each inner loop, we have all the voltage loop equations we need. When we must skip an inner loop equation (because we would have to define a voltage for a current source), we write a voltage-loop equation for the next larger voltage loop containing some portion of that inner loop. If we must skip that next larger loop, we proceed to the next larger loop, and so on recursively. If we must skip even the largest voltage loop, then that voltage loop is unnecessary. (In that case, the other voltage-loop and current-summation equations will be sufficient to solve the circuit.)
- TOOL:** When necessary, we supplement voltage-loop equations with equations that equate voltages across circuit elements that are in parallel.

- 
- DEF:** A circuit node is any point to which three or more circuit elements are attached.
- NOTE:** The goal of writing current summation and voltage loop equations is to obtain  $n$  equations in  $n$  unknowns that we can solve to find all the currents and voltages in a circuit.
- TOOL:** Nodes connected by wires are considered to be a single node.
- TOOL:** We set the sum of currents measured as flowing out of a node to zero. If the current measurement arrow points toward a node, that current appears with a minus sign in the current-sum equation.
- TOOL:** We skip current sums for nodes where we would be forced to define a current for a voltage source. Writing an equation for such a node adds a new variable and a new equation. Thus, we merely create more equations in more unknowns rather than moving closer to the goal of writing  $n$  equations in  $n$  unknowns.
- TOOL:** There is always at least one node we may skip when writing current-sum equations. The equation for that node would be redundant.
- TOOL:** When necessary, we supplement current sum equations with equations that equate currents flowing in circuit elements that are in series.
- NOTE:** Summing the currents flowing into a node yields an equation that is equivalent to the equation for summing the currents flowing out of that node. Multiplying one equation by  $-1$  on both sides yields the other equation.
- NOTE:** Summing currents measured as flowing into a node and setting that sum equal to currents measured as flowing out of that node yields an equation equivalent to setting the sum of all currents measured as flowing out of the node to zero.

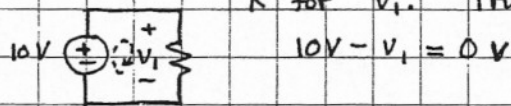


ex:

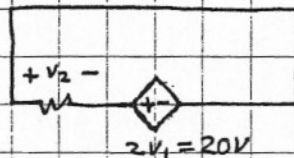


Find:  $V_1, V_2, i_1, i_2$ .

soln:  $V_1 = 10V$  from closed loop w/ 10V source and R for  $V_1$ . True even with intervening R.



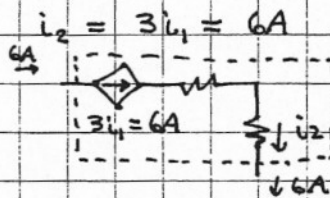
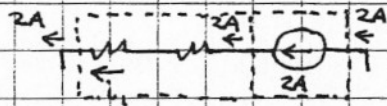
$V_2 = -2V_1 = -20V$  from closed <sup>loop</sup> involving only 2V1 source and R for  $V_2$



$2V_1 + V_2 = 0V$      $20 + V_2 = 0V$   
 $V_2 = -20V$

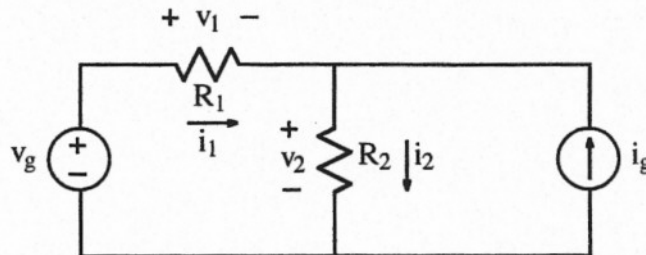
$i_1 = 2A$

$i_1 = 2A$  because resistor for  $i_1$  is in series with 2A source. 0A net current flows out of the dotted boxes.



$i_2 = 3i_1$  because resistor for  $i_2$  is in series with dependent source that creates  $i = 6A$ .  
 Current flow into dotted box = 6A.  
 $\therefore$  current flow out of dotted box = 6A.

**EX:** In the circuit below, use Kirchhoff's voltage and current laws to write equations relating voltages and currents.



**ANSWER:**  $-i_1 + i_2 - i_g = 0$

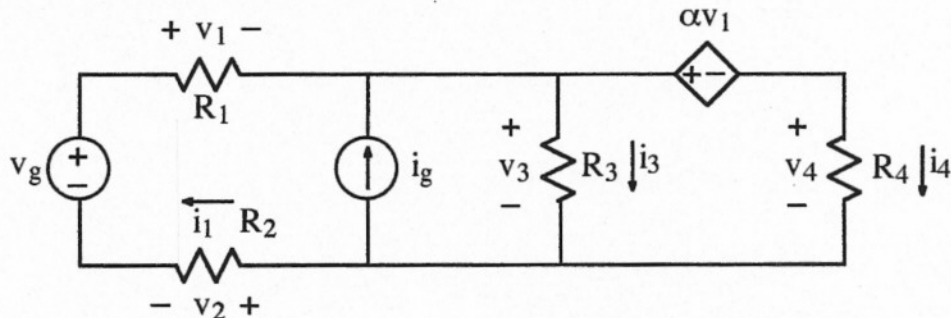
$$v_g - v_1 - v_2 = 0$$

**SOL'N:** We sum the currents flowing out of the top center node. Writing an equation for the bottom node would be redundant. Recall that we always have one extra node.

Because writing a v-loop equation for the right inner loop would require defining a voltage for a current source, we write a v-loop equation for only the left loop. Note that the only larger loop containing the right inner loop would also require defining a voltage for the current source. Thus, a voltage loop equation for the right side is unnecessary.

Our voltage loop on the left starts from the lower left and proceeds in a clockwise direction. We may start voltage loops wherever we desire, but being consistent tends to improve accuracy.

**EX:** In the circuit below, use Kirchoff's voltage and current laws to write equations relating voltages and currents.



**ANSWER:**

$$-i_1 - i_g + i_3 + i_4 = 0$$

$$+v_g - v_1 - v_3 - v_2 = 0$$

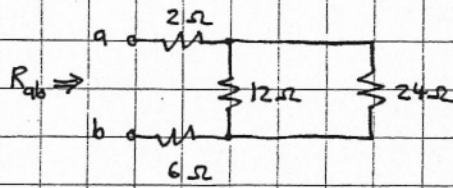
$$+v_3 - \alpha v_1 - v_4 = 0$$

**SOL'N:** We avoid labels defining the current for a voltage source or the voltage for a current source. Thus, we look for nodes where every branch has a labeled current, (i.e., contains at least one resistor or current source, as opposed to only v-sources), and v-loops where the loop where every element has a labeled voltage, (i.e., without current sources).

We sum the currents flowing out of the top center node. (Writing an equation for the bottom node would be redundant.) Note that, because a wire connects them, we consider the two top-center nodes as a single node. (We may redraw the circuit with the wire collapsed to a point.) Note also that current  $i_1$  continues around the loop to flow through  $R_1$  from left to right, and current  $i_4$  flows through the dependent source.

Because writing v-loop equations for the left or center inner loops would require defining a voltage for a current source, we write a v-loop equation for the next larger loop that goes around the current source. We also write a v-loop equation for the inner loop on the right.

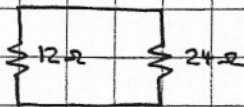
Our voltage loops start from the lower left and proceed in a clockwise direction. We may start voltage loops wherever we desire, but being consistent tends to improve accuracy.



series and parallel R's

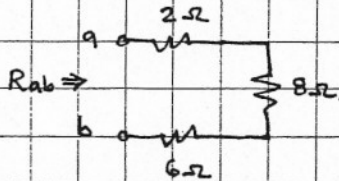
Find equivalent resistance  $R_{ab}$  looking into a b.

Sol'n:



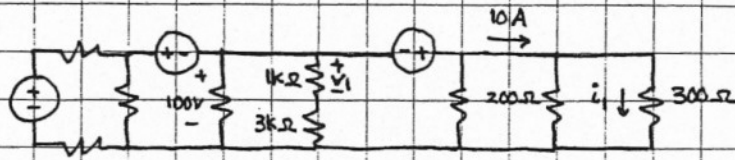
$$12\Omega \parallel 24\Omega = 12\Omega \cdot \frac{1}{2} = 12\Omega \cdot \frac{1 \cdot 2}{1+2}$$

$$= 12\Omega \cdot \frac{2}{3} = 8\Omega$$



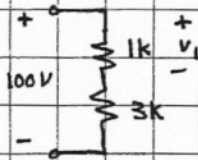
$$R_{ab} = 2\Omega + 8\Omega + 6\Omega = 16\Omega$$





a) Use V-divider to find  $v_1$ .

sol'n: We are given 100V drop across R to left of 1kΩ and 3kΩ. ∴ we have 100V drop across 1kΩ and 3kΩ in series.

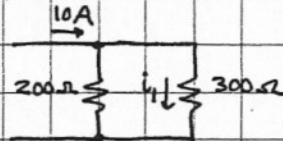


By V-divider formula,

$$v_1 = 100V \cdot \frac{1k\Omega}{1k\Omega + 3k\Omega} = 100V \cdot \frac{1}{4} = 25V$$

b) Use i-divider to find  $i_1$ .

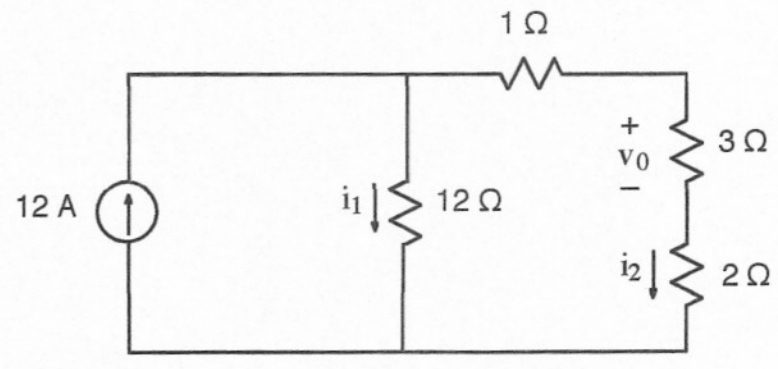
sol'n: We are given 10A thru 200Ω in parallel with 300Ω.



By i-divider formula,

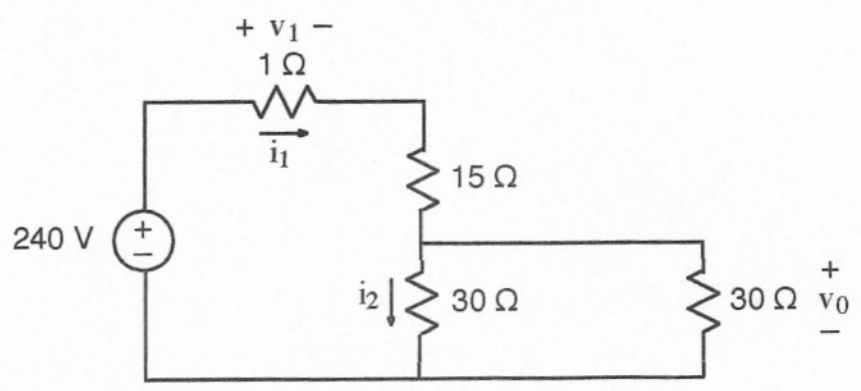
$$i_1 = 10A \cdot \frac{200\Omega}{200\Omega + 300\Omega} = 10A \cdot \frac{2}{5} = 4A$$

1.



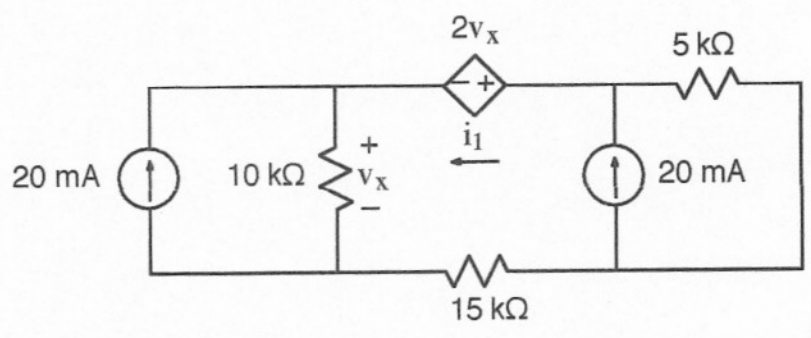
- a) Calculate  $i_1$ ,  $i_2$ , and  $v_0$ .
- b) Find the power dissipated for every component, including the current source.

2.



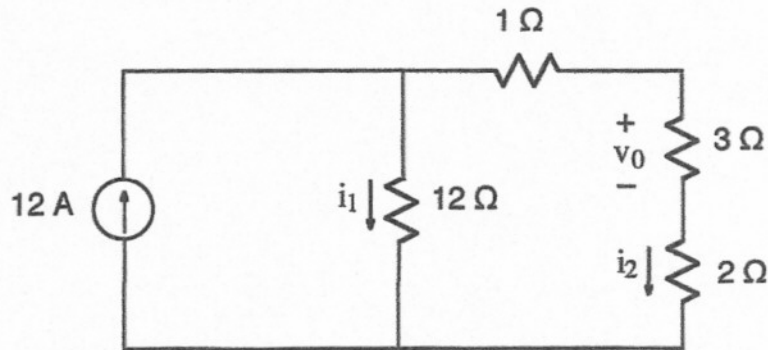
Calculate  $i_1$ ,  $i_2$ , and  $v_0$ .

3.



Find  $v_x$ ,  $i_1$ , and the power dissipated by the dependent source.

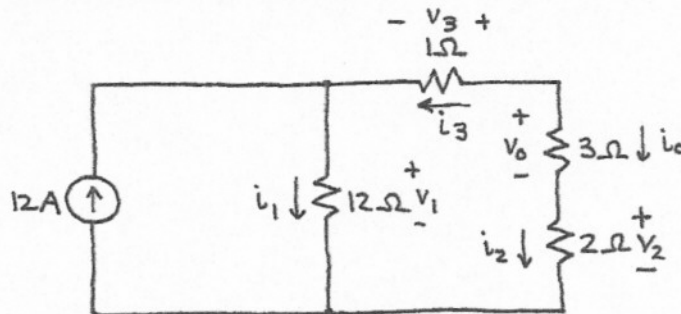
Ex:



- a) Calculate  $i_1, i_2,$  and  $v_0$ .
- b) Find the power dissipated for every component, including the current source.

sol'n: a) We first label current and voltage for each resistor. We follow the passive sign convention: the arrow for the direction of current measurement points toward the - sign of the voltage measurement.

For the  $1\Omega$  resistor, we may define the current measurement in either direction. For the sake of illustration, we define the direction of current measurement in a way that is somewhat awkward.



Now we write eq's for voltage loops. We try to write v-loop eq's for inner loops, but we avoid loops that include a current source. (The reason we do so is to avoid defining a new variable that requires another eq'n.) In this problem, there is only one v-loop without a current source. Going around the inner loop on the right side in a clockwise direction and using the sign where we exit a component, we have

$$+v_1 + v_3 - v_0 - v_2 = 0V$$

Next, we write eq's for current summations at nodes. We sum the currents measured flowing away from the top center node.

$$-12A + i_1 - i_3 = 0A$$

There is always one redundant node. So we only need this one eq'n.

Now we look for components in series that carry the same current.

$$i_3 = -i_0 \quad (\text{minus sign because currents measured in opposite directions})$$

$$i_0 = i_2$$

Our last set eq'ns comes from Ohm's Law for each resistor.

$$v_1 = i_1 \cdot 12\Omega$$

$$v_0 = i_0 \cdot 3\Omega$$

$$v_2 = i_2 \cdot 2\Omega$$

To solve the simultaneous eq'ns, we substitute  $i_2$  for  $i_0$  and  $-i_2$  for  $i_3$ . Then we substitute for  $v$ 's using the Ohm's law eq'ns.

Our  $v$ -loop eq'n becomes

$$i_1 \cdot 12\Omega - i_2 \cdot 1\Omega - i_2 \cdot 3\Omega - i_2 \cdot 2\Omega = 0V$$

Our  $i$ -sum eq'n becomes

$$-12A + i_1 + i_2 = 0A$$

Solving the second eq'n, we have, for  $i_2$ ,

$$i_2 = 12A - i_1$$

Substituting for  $i_2$  in the  $v$ -loop eq'n:

$$i_1 \cdot 12\Omega - (12A - i_1) \cdot \underbrace{(1\Omega + 3\Omega + 2\Omega)}_{6\Omega} = 0V$$

$$\text{or } i_1 (12\Omega + 6\Omega) = 12A \cdot 6\Omega$$

$$\text{or } i_1 = 12A \cdot 6\Omega / 18\Omega = 4A$$

Using an earlier eqn:

$$i_2 = 12A - i_1 = 12A - 4A = 8A$$

From earlier eqn:

$$V_0 = i_0 \cdot 3\Omega = i_2 \cdot 3\Omega = 8A \cdot 3\Omega = 24V$$

b) Power dissipated:  $p = i \cdot v = i^2 R$  for  $R$ 's

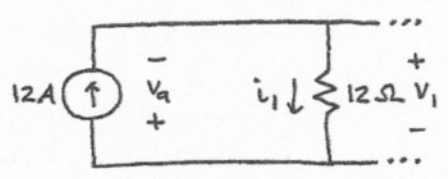
$$12\Omega: p = i_1^2 \cdot 12\Omega = 4A^2 \cdot 12\Omega = 192W$$

$$1\Omega: p = i_3^2 \cdot 1\Omega = (-8A)^2 \cdot 1\Omega = 64W$$

$$3\Omega: p = i_0^2 \cdot 3\Omega = (8A)^2 \cdot 3\Omega = 192W$$

$$2\Omega: p = i_2^2 \cdot 2\Omega = (8A)^2 \cdot 2\Omega = 128W$$

For the current source, we find the voltage drop from a v-loop on the left side.



$$\text{We have } -V_a - V_1 = -V_a - i_1 \cdot 12\Omega = 0V$$

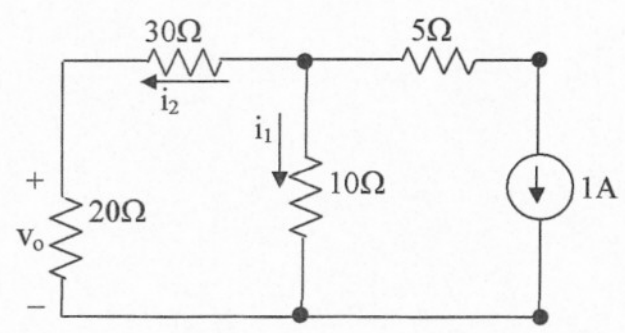
$$V_a = -i_1 \cdot 12\Omega = -4A \cdot 12\Omega = -48V$$

$$\text{Power for 12A src: } p = 12A \cdot V_a = 12A(-48V)$$

$$p = -576W$$

### Homework #1 Examples

1.

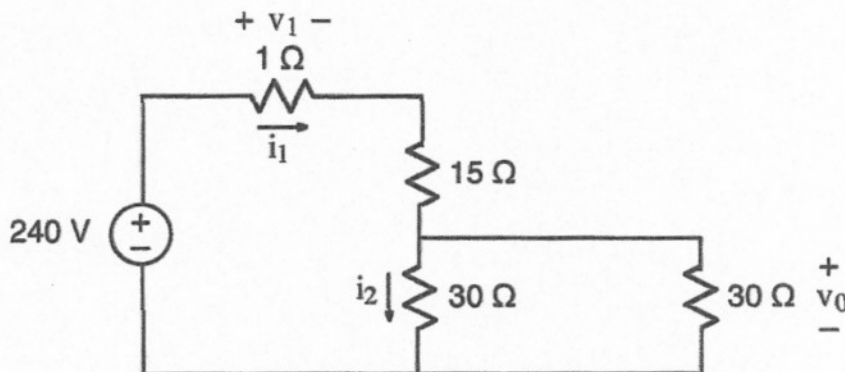


- (a) Calculate  $i_1$ ,  $i_2$ , and  $v_o$ .
- (b) Find the power dissipated for every component including the current source.

HOMEWORK #1 Solution Problem 2

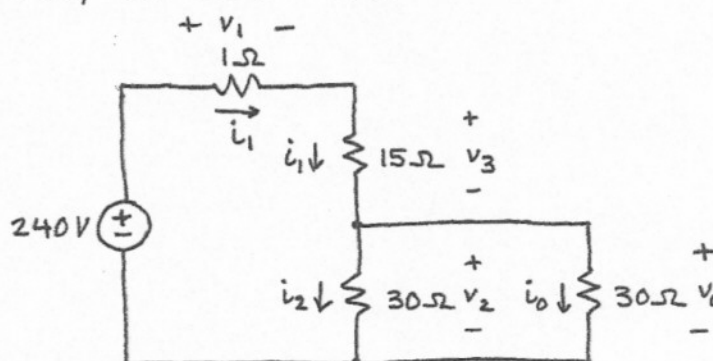


Ex:



Calculate  $i_1$ ,  $i_2$ , and  $v_0$ .

sol'n: First, we label R's.



Second, we write v-loop eq's. We write eq's for both inner loops.

$$+240V - v_1 - v_3 - v_2 = 0V$$

$$+v_2 - v_0 = 0V$$

Third, we write current-sum eq's for all but one node. For the node between R's on the right side we have

$$-i_1 + i_2 + i_0 = 0A$$



Fourth, we equate currents for components in series. We have already done this, however, by using  $i_1$  for both the  $1\Omega$  and  $15\Omega$  R's.

Fifth, we write Ohm's law eq'ns for every R:

$$v_1 = i_1 \cdot 1\Omega$$

$$v_3 = i_1 \cdot 15\Omega$$

$$v_2 = i_2 \cdot 30\Omega$$

$$v_0 = i_0 \cdot 30\Omega$$

Now we use the Ohm's law eq'ns to substitute for v's:

$$+240V - i_1 \cdot 1\Omega - i_1 \cdot 15\Omega - i_2 \cdot 30\Omega = 0V$$

$$+ i_2 \cdot 30\Omega - i_0 \cdot 30\Omega = 0V$$

Our current sum eq'n is unchanged:

$$-i_1 + i_2 + i_0 = 0A$$

From the 2nd of the above 3 eq'ns we have

$$i_0 = i_2$$

Using this in the 3rd eq'n gives

$$i_1 = 2i_2$$

Using this in the 1<sup>st</sup> eq'n gives

$$+240V - 2i_2(1\Omega + 15\Omega) - i_2 \cdot 30\Omega = 0V$$

$$\text{or } i_2(2 \cdot 16\Omega + 30\Omega) = 240V$$

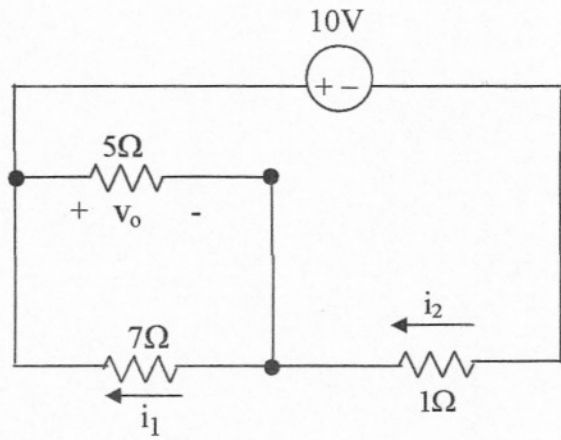
$$\text{or } i_2 = \frac{240V}{62\Omega} = \frac{120}{31} A \approx 3.87A$$

$$i_1 = 2i_2 = \frac{240}{31} A \approx 7.74A$$

$$v_o = i_o \cdot 30\Omega = i_2 \cdot 30\Omega = \frac{120(30)}{31} V$$

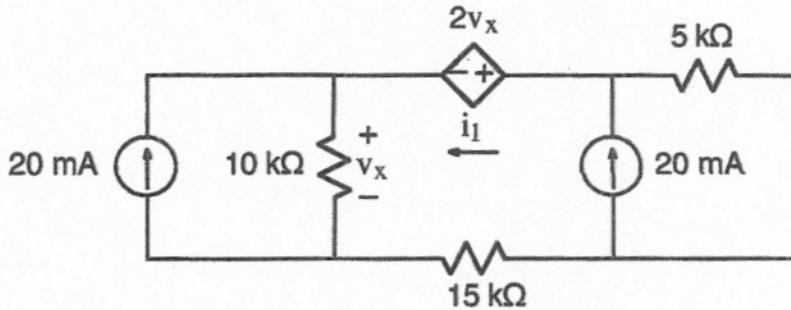
$$\text{or } v_o = \frac{3600}{31} V \approx 116V$$

2.



Calculate  $i_1$ ,  $i_2$ , and  $v_0$ .

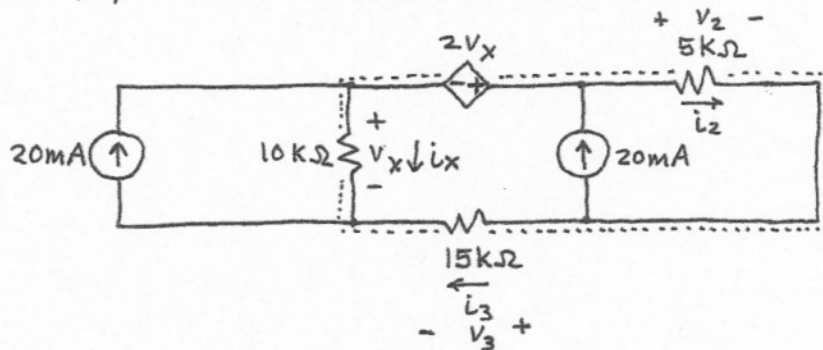
EX:



Find  $v_x$ ,  $i_1$ , and the power dissipated by the dependent source.

sol'n: We find  $i_1$ , the current for the dependent  $v$ -src after we solve the circuit.

First, label  $i$ 's and  $v$ 's for  $R$ 's:



Second, write  $v$ -loop eq's for loops not containing current src's. There is only one such loop, indicated by the dotted line:

$$+v_x + 2v_x - v_2 - v_3 = 0V$$

Third, write current-sum eq's for nodes (unless nodes are connected only by  $v$  src's). We don't use the nodes on top since they are connected by only the  $2v_x$  source.

For the node on the bottom, left of center, we have

$$+20mA - i_x - i_3 = 0A \tag{1}$$

For the node on the bottom, right of center, we have

$$i_3 + 20mA - i_2 = 0A \tag{2}$$

Fourth, we look components in series carrying the same current. Here, we lack any such components.

Fifth, we write Ohm's Law eq'ns for all the R's:

$$V_x = i_x \cdot 10k\Omega$$

$$V_2 = i_2 \cdot 5k\Omega$$

$$V_3 = i_3 \cdot 15k\Omega$$

Now substitute Ohm's Law for V's in V-loop eq'n:

$$i_x \cdot 10k\Omega + 2 i_x \cdot 10k\Omega - i_2 \cdot 5k\Omega - i_3 \cdot 15k\Omega = 0V \tag{3}$$

Solve one the three eq'ns (1-3) for a current:

$$i_3 = i_2 - 20mA$$

Substitute this in eq'ns (1) and (2):

$$20\text{ mA} - i_x - (i_2 - 20\text{ mA}) = 0\text{ A}$$

$$i_x(10\text{ k}\Omega + 20\text{ k}\Omega) - i_2(5\text{ k}\Omega) - (i_2 - 20\text{ mA})15\text{ k}\Omega = 0\text{ V}$$

Solving the first of these eq'ns for  $i_2$  gives

$$i_2 = 40\text{ mA} - i_x$$

Using this in the second of the two eq'ns gives:

$$i_x(30\text{ k}\Omega) - (40\text{ mA} - i_x)(5\text{ k}\Omega + 15\text{ k}\Omega) = -20\text{ mA} \cdot 15\text{ k}\Omega$$

$$\text{or } i_x(30\text{ k}\Omega + 20\text{ k}\Omega) = 40\text{ mA}(20\text{ k}\Omega) - 20\text{ mA} \cdot 15\text{ k}\Omega$$

$$\text{or } i_x(50\text{ k}\Omega) = 500\text{ V}$$

$$\text{or } i_x = \frac{500\text{ V}}{50\text{ k}\Omega} = 10\text{ mA}$$

Now we can find  $i_1$  from a current sum at the node on top to the left of center:

$$-20\text{ mA} + i_x - i_1 = 0\text{ mA}$$

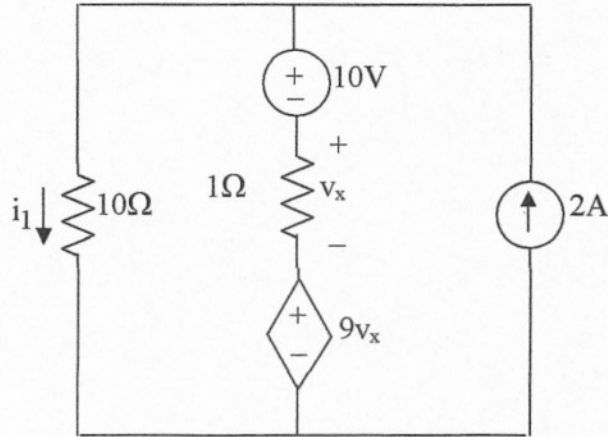
$$\text{or } i_1 = -20\text{ mA} + i_x = -20\text{ mA} + 10\text{ mA} = -10\text{ mA}$$

The power dissipated by the dependent source is

$$P = i_1 \cdot 2V_x = -10\text{ mA} \cdot 2 \cdot \underbrace{10\text{ mA} \cdot 10\text{ k}\Omega}_{V_x \text{ from Ohm's Law}}$$

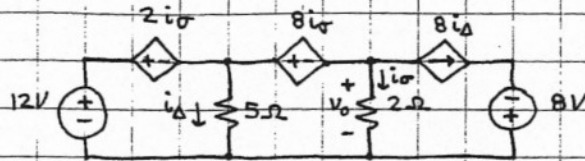
$$\text{or } P = -2\text{ W}$$

3.



Find  $v_x$ ,  $i_1$ , and the power dissipated by the dependent source.

ex:



a) Calculate  $i_Δ$  and  $v_σ$ . Use Ohm's law, Kirchoff's laws.

Left-side V loop (≠ Ohm's law):  $12V - 2i_σ - i_Δ 5Ω = 0V$

Center V loop (≠ Ohm's law, as always):  $v_σ + 8i_σ - i_Δ 5Ω = 0V$

but  $v_σ = i_σ 2Ω$  so center loop eqn is:  $i_σ 2Ω + 8i_σ - i_Δ 5Ω = 0V$

Now we have 2 eqns in two unknowns. Eliminate  $i_σ$ .

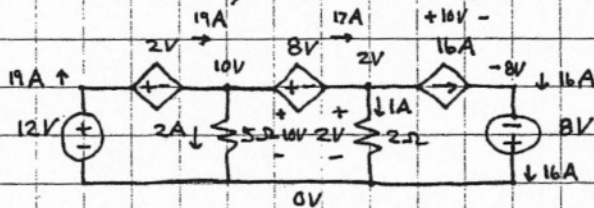
$10i_σ - i_Δ 5Ω = 0$  (2nd eqn)  $\therefore i_σ = i_Δ \frac{5}{10} = \frac{i_Δ}{2}$

Substitute into 1st eqn  $12V - \frac{2i_Δ}{2} - i_Δ 5Ω = 0V$

$i_Δ = \frac{12V}{6Ω} = 2A$  ( $i_σ = 1A$ )

and  $v_σ = i_σ \cdot 2Ω = \frac{i_Δ}{2} \cdot 2Ω = i_Δ \cdot 1Ω = 2V$

Consistency check: put all  $i$ 's and  $v$ 's on circuit diagram and verify that it obeys Kirchoff's laws and Ohm's law.



Left loop  $V = 0$  and 10V across 5Ω gives 2A ✓

Center loop  $V = 0$  and 2V across 2Ω gives 1A ✓



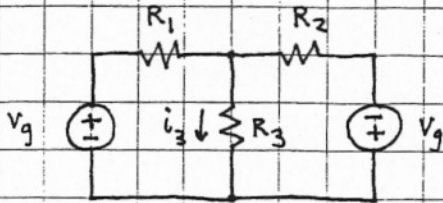
b) Show power delivered = power absorbed for sol'n to (a).

Consistency check diagram for (a) shows  $v$ 's and  $i$ 's we get from solution plus what we deduce from Kirchoff's laws and Ohm's law. (We make sure total current into nodes = 0 and  $V$  drops around loops sum to 0V.)

An element develops power if positive current flows out of terminal labeled + for voltage. An element absorbs power if positive current flows into terminal labeled + for voltage. These definitions are for  $V_{drop} > 0$ . Reverse them if  $V_{drop} < 0$  from terminal labeled + to terminal labeled -.

element	power developed = $-i \cdot V$	power absorbed = $i \cdot V$
12V src	$19A \cdot 12V = 228W$	
2 $i_r$ src		$19A \cdot 2V = 38W$
5 $\Omega$		$2A \cdot 10V = 20W$
8 $i_r$ src		$17A \cdot 8V = 136W$
2 $\Omega$		$1A \cdot 2V = 2W$
8 $i_d$ src		$16A \cdot 10V = 160W$
8V src	$16A \cdot 8V = 128W$	
tot	356W	tot 356W ✓

ex:

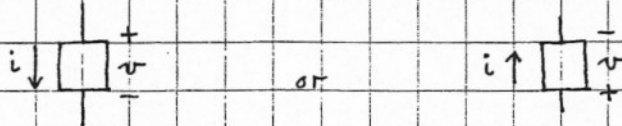


- i) Apply Kirchhoff's laws to find an expression for  $i_3$ .
- ii) Make at least 2 consistency checks on answer
- iii) Given  $V_g = 1V$ ,  $R_2 = 100\Omega$ , and  $R_3 = 10\Omega$ , find  $R_1$  so that  $i_3 = 0$ .
- iv) For  $V_g = 1V$ ,  $R_3 = 10\Omega$ , and  $R_1 =$  value found in part iii, find  $i_3$  when  $R_2 = 200\Omega$  and when  $R_2 = 50\Omega$ .

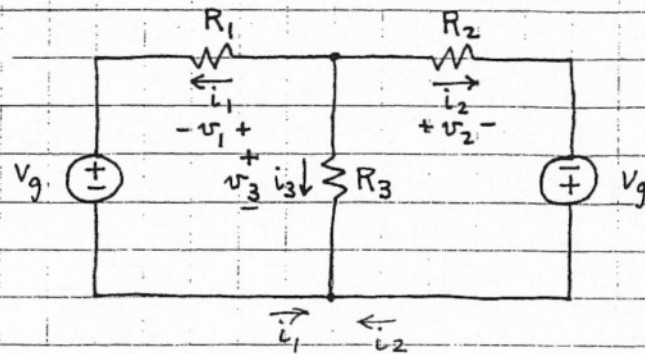
- ans:
- i)  $i_3 = V_g \frac{R_2 - R_1}{R_1 R_2 + R_2 R_3 + R_3 R_1}$
  - ii) see soln, various possibilities
  - iii)  $R_1 = 100\Omega$
  - iv)  $i_3 = 4.35 \text{ mA}$  for  $R_2 = 200\Omega$        $i_3 = -7.69 \text{ mA}$  for  $R_2 = 50\Omega$

soln: i) We first label voltages and currents. By the passive sign convention, the current measurement arrow must point from the + sign of the voltage measurement toward the - sign of the voltage measurement.

Note: If we follow the passive sign convention, we still have two choices for the direction of the current arrow and the corresponding voltage measurement:



In both cases the current measurement arrow points from + to -. The direction of the physical current flow is, of course, unchanged. If, for example, the physical current is flowing down, then we would get a negative value when we solve for  $i$  if we use the measurement scheme on the right.



Now we apply Kirchhoff's Laws:

- 1) Sum of currents flowing out of node = 0.

top node at the  
For the junction of  $R_1$ ,  $R_2$ , and  $R_3$  we have

$$i_1 + i_2 + i_3 = 0 \text{ A}$$

All the terms appear with plus signs because all three current measurement arrows point out from the node. If a current measurement arrow points toward a node, we subtract the current.

Thus, for the bottom node we have

$$-i_1 - i_2 - i_3 = 0 \text{ A}$$

We observe that is just  $i_1 + i_2 + i_3 = 0 \text{ A}$  multiplied by  $-1$  on both sides. Consequently, this 2<sup>nd</sup> eqn doesn't give us any new constraints that help us solve the problem.

moral: At most, we have one less current eqns than the number of circuit nodes, (where three or more elements connect).

- 2) Sum of voltage drops around any loop = 0.

We have a loop on the left and a loop on the right. There is also an outer loop.

The outer loop, it turns out, is redundant. If we add the eq'ns for the smaller loops together, we obtain the eq'n for the outer loop.

This observation is typical. We normally just need the smallest loops. Current sources complicate the picture somewhat, however.

If current sources are present, we avoid using loops that include them. The reason we do so is that the voltage drop across a current source creates a new unknown that we must solve for. Thus, we fail to gain ground in our effort to find as many eq'ns as we have unknowns.

Here, we have no current sources and we can write two loop eq'ns.

$$\text{Left loop: } v_g + v_1 - v_3 = 0V$$

$$\text{Right loop: } v_3 - v_2 + v_g = 0V$$

Note: I usually travel clockwise around the loop, but the other direction may be used. I treat a voltage drop as positive if I exit the circuit element at the + sign. In other words, I add that  $v$ -drop to the total. I subtract that  $v$ -drop from the total if I exit the circuit element at the - sign. The Text does the opposite.

Now we apply Ohm's Law to obtain 3 more eq'ns:

$$v_1 = i_1 R_1 \quad v_2 = i_2 R_2 \quad v_3 = i_3 R_3$$

Substitute for voltages and solve for  $i_3$ .

Our three eqns become

$$i_1 + i_2 + i_3 = 0A \quad (1)$$

$$v_g + i_1 R_1 - i_3 R_3 = 0V \quad (2)$$

$$i_3 R_3 - i_2 R_2 + v_g = 0V \quad (3)$$

Using the first eqn (which has the fewest terms)  
we solve for  $i_1$ :

$$i_1 = -(i_2 + i_3)$$

Substituting into the 2<sup>nd</sup> and 3<sup>rd</sup> eqns, we have:

$$v_g + -(i_2 + i_3)R_1 - i_3 R_3 = 0V \quad (2)$$

$$i_3 R_3 - i_2 R_2 + v_g = 0V \quad (3)$$

We now solve for  $i_2$  using the <sup>new</sup> third eqn (which has the fewest terms):

$$i_2 = \frac{i_3 R_3 + v_g}{R_2}$$

Substituting into the 2<sup>nd</sup> eqn, we have:

$$v_g - \left( \frac{i_3 R_3 + v_g}{R_2} + i_3 \right) R_1 - i_3 R_3 = 0V$$

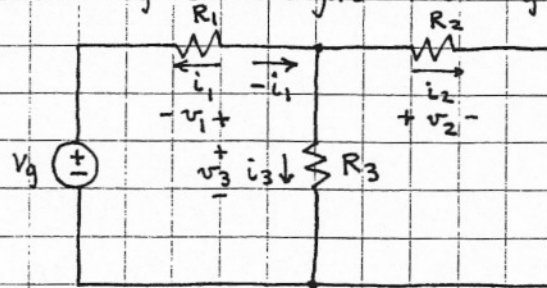
We solve this eqn for  $i_3$ :

$$i_3 \left( \frac{R_3 R_1 + R_1 + R_3}{R_2} \right) = v_g \left( 1 - \frac{R_1}{R_2} \right)$$

$$i_3 = v_g \frac{(R_2 - R_1)}{R_1 R_3 + R_1 R_2 + R_2 R_3} \quad \text{units consistent} \checkmark$$

(cont) ii) • Consider  $V_g = 0$ : No energy sources  $\Rightarrow i_3 = 0$   
Agrees with our formula. ✓

• Consider  $V_g = 0V$  on right side:  $V_g = 0V \Rightarrow$  short circuit



Total current,  $-i_1 = \frac{V_g}{R_1 + R_2 \parallel R_3}$

By current-divider formula,  $i_3 = -i_1 \frac{R_2}{R_2 + R_3} = \frac{R_2 \parallel R_3}{R_3}$

$$\begin{aligned} \therefore i_3 &= \frac{V_g}{R_1 + R_2 \parallel R_3} \frac{R_2 \parallel R_3}{R_3} = \frac{V_g}{R_1 + \frac{R_2 R_3}{R_2 + R_3}} \frac{\frac{R_2 R_3}{R_2 + R_3}}{R_3} \\ &= V_g \frac{R_2}{R_1(R_2 + R_3) + R_2 R_3} \end{aligned}$$

Now a trick: By symmetry, if we set  $V_g = 0V$  on the right side (instead of the left) and turn it upside down (as in the original circuit), we must have

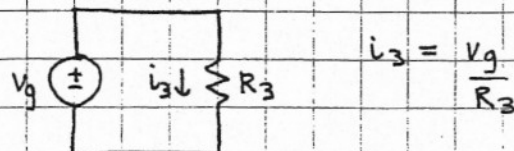
$$i_3 = -V_g \frac{R_1}{R_2(R_1 + R_3) + R_1 R_3}$$

If we add these two  $i_3$  values together, we get our formula for  $i_3$  from part (i). ✓

This is a preview of the idea of superposition. More on this later on in the course.

• Consider  $R_3 = \infty$ :  $i_3 = 0A$  since open circuit for  $R_3$   
Agrees with our formula. ✓

ii (cont) • Consider  $R_1 = 0 \Omega$  and  $R_2 = \infty \Omega$ : Just  $V_g$  and  $R_3$



Our formula with  $R_1 = 0$ :  $i_3 = \frac{V_g R_2}{R_2 R_3} = \frac{V_g}{R_3}$

$$\lim_{R_2 \rightarrow \infty} \frac{V_g}{R_3} = \frac{V_g}{R_3} \quad \checkmark$$

Note: It actually doesn't matter what  $R_2$  is if  $R_1 = 0 \Omega$ . The voltage across  $R_3$  will be  $V_g$ , and the current thru  $R_3$  must be  $V_g/R_3$ .

iii) If  $R_2 = 100 \Omega$  then  $i_3 = 0$  when  $R_1 = R_2 = 100 \Omega$

iv) Given:  $R_1 = 100 \Omega$ ,  $R_3 = 10 \Omega$ ,  $V_g = 1V$

$$\text{If } R_2 = 200 \Omega, \text{ then } i_3 = 1V \frac{(200\Omega - 100\Omega)}{100\Omega \cdot 10\Omega + 100\Omega \cdot 200\Omega + 200\Omega \cdot 10\Omega}$$

$$\text{or } i_3 = 1V \cdot \frac{100\Omega}{100\Omega (10\Omega + 200\Omega + 20\Omega)} = \frac{1V}{230\Omega}$$

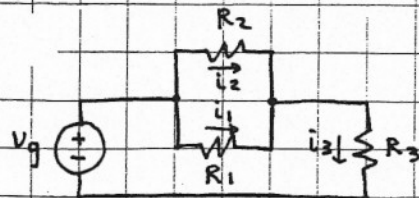
$$\text{or } i_3 = 4.35 \text{ mA.}$$

$$\text{If } R_2 = 50 \Omega, \text{ then } i_3 = 1V \frac{(50\Omega - 100\Omega)}{100\Omega \cdot 10\Omega + 100\Omega \cdot 50\Omega + 50\Omega \cdot 10\Omega}$$

$$\text{or } i_3 = 1V \cdot \frac{(-50\Omega)}{50\Omega (20\Omega + 100\Omega + 10\Omega)} = \frac{-1V}{130\Omega}$$

$$\text{or } i_3 = 7.69 \text{ mA.}$$

ex:



$V_g, R_2, R_3$  fixed nonzero vals

Explain how  $i_3$  would change as  $R_1$  ranged from 0 to  $\infty$ .  
 " "  $i_1, i_2$  " " " " " " " "

Sol'n:  $R_1$  and  $R_2$  form current divider for current  $i_3$ .

$$\therefore i_1 = i_3 \cdot \frac{R_2}{R_1 + R_2} \quad i_2 = i_3 \cdot \frac{R_1}{R_1 + R_2}$$

$$\text{Also, } i_3 = \frac{V_g}{\text{tot } R} = \frac{V_g}{R_1 \parallel R_2 + R_3}$$

$$\begin{aligned} \text{Now, } R_1 \parallel R_2 &= 0 \quad (R_1 = 0) & R_1 \parallel R_2 + R_3 &= R_3 \quad (R_1 = 0) \\ &= R_2/2 \quad (R_1 = R_2) & &= R_2/2 + R_3 \quad (R_1 = R_2) \\ &= R_2 \quad (R_1 = \infty) & &= R_2 + R_3 \quad (R_1 = \infty) \end{aligned}$$

$$\begin{aligned} \text{Thus, } i_3 &= \frac{V_g}{R_3} \quad (R_1 = 0) \\ &= \frac{V_g}{\frac{R_2 + R_3}{2}} \quad (R_1 = R_2) \\ &= \frac{V_g}{R_2 + R_3} \quad (R_1 = \infty) \end{aligned}$$

$$i_1 = \frac{i_3 R_2}{0 + R_2} = 1 \cdot i_3 \quad (R_1 = 0) \quad i_2 = \frac{i_3 \cdot 0}{0 + R_2} = 0$$

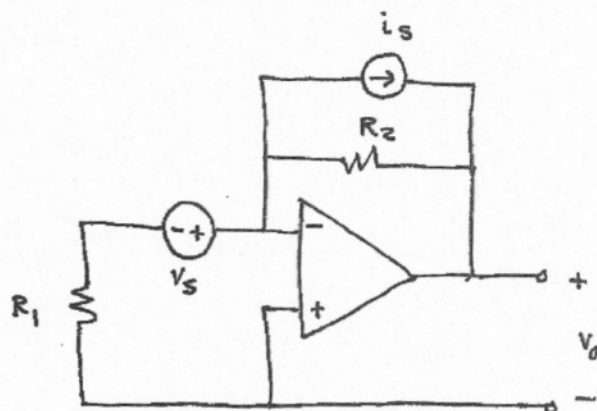
$$i_1 = i_3 \frac{R_2}{R_2 + R_2} = \frac{i_3}{2} \quad (R_1 = R_2) \quad i_2 = i_3 \frac{R_2}{R_2 + R_2} = \frac{i_3}{2}$$

$$i_1 = i_3 \frac{R_2}{\infty + R_2} = 0 \quad (R_1 = \infty) \quad i_2 = i_3 \frac{\infty}{\infty + R_2} = i_3$$

use L'Hopital's rule

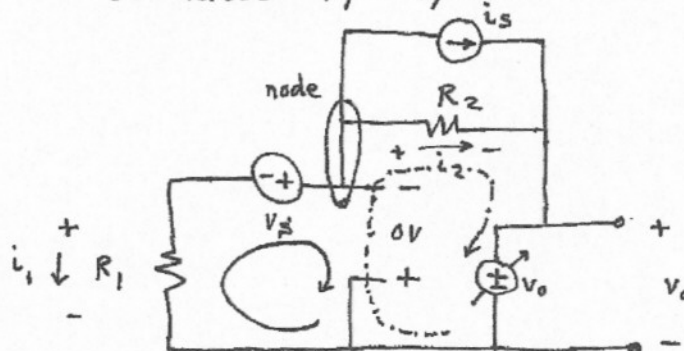


## Op Amp Example

sp05  
Dr. Cotter

Derive expression for  $v_o$  in terms of not more than  $v_s$ ,  $i_s$ ,  $R_1$ , and  $R_2$ .

sol'n: Replace op-amp with  $v_o$  source and assume 0V across  $+$ ,  $-$  inputs.



Use v-loop on left that includes 0V drop across  $+$ ,  $-$  inputs.

$$+i_1 R_1 + v_s + 0V = 0V$$

$$\text{or } i_1 = -\frac{v_s}{R_1}$$

Use v-loop on right that includes 0V drop across  $+$ ,  $-$  inputs.

$$-0V - i_2 R_2 - v_o = 0V$$

$$\text{or } v_o = -i_2 R_2$$

Op Amp sol'n: cont.

Use current sum at node next to - input.

$$i_1 + i_2 + i_s = 0A$$

$$\text{or } i_2 = -(i_1 + i_s) = -\left(-\frac{v_s}{R_1} + i_s\right) = \frac{v_s}{R_1} - i_s$$

Use this in 2nd v-loop eq'n.

$$v_o = -i_2 R_2 = -\left(\frac{v_s}{R_1} - i_s\right) R_2$$

$$v_o = \left(i_s - \frac{v_s}{R_1}\right) R_2$$

Consistency check:

If  $R_2 = 0$  then 2nd v-loop implies

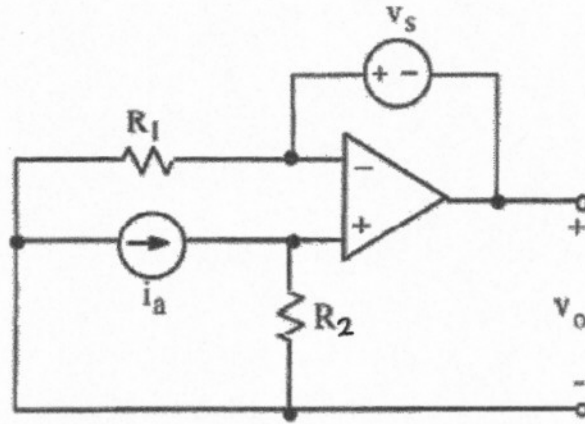
$$v_o = 0V. \quad v_o = \left(i_s - \frac{v_s}{R_1}\right) \cdot 0 = 0V \quad \checkmark$$

If  $i_s = 0A$  and  $v_s = 0V$ , we should

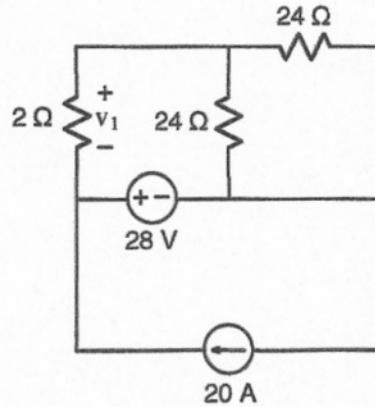
$$\text{get } v_o = 0V. \quad v_o = \left(0 - \frac{0}{R_1}\right) R_2 = 0V \quad \checkmark$$

### Op Amp Examples

5. The op-amp operates in the linear mode. Using an appropriate model of the op amp, derive an expression for  $v_o$  in terms of not more than  $v_s$ ,  $i_a$ ,  $R_1$ , and  $R_2$ .



Ex:



Calculate  $v_1$ .

sol'n: We can use a voltage divider consisting of the 28V src, the 2Ω Resistor, and the two 24Ω resistors in parallel:

$$v_1 = -28V \cdot \frac{2\Omega}{2\Omega + 24\Omega \parallel 24\Omega}$$

$$= -28V \cdot \frac{2\Omega}{2\Omega + 24\Omega \cdot \frac{1}{1+1}}$$

$$\frac{1}{1+1} = \frac{1 \cdot 1}{1+1} = \frac{1}{2}$$

$$= -28V \cdot \frac{2\Omega}{2\Omega + 12\Omega}$$

$$v_1 = -4V$$

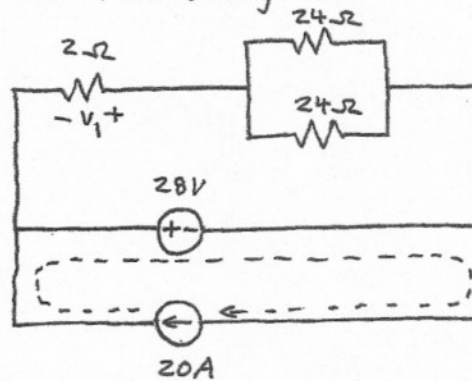
Note: We have a minus sign whenever the + sign of the resistor voltage measurement is on the side away from the + sign of the v src.

**HOMEWORK #2 Solution Prob 1 (cont.)**



Note: We have a voltage divider when the following conditions are met:  
i) The voltage across two or more  $R$ 's in series is known.  
ii) The current thru the  $R$ 's in series is the same.

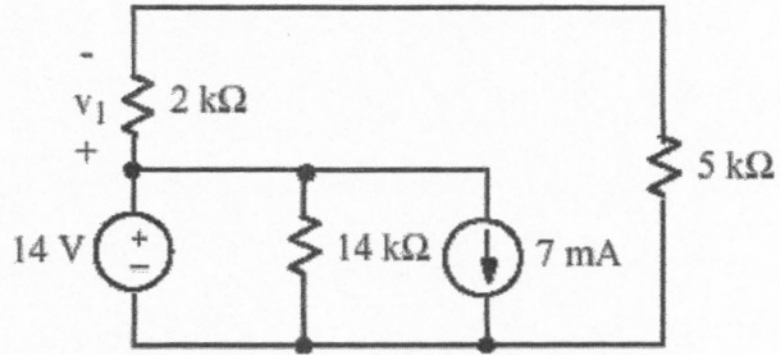
Note: We can verify that we have a  $v$ -divider in this circuit by redrawing it.



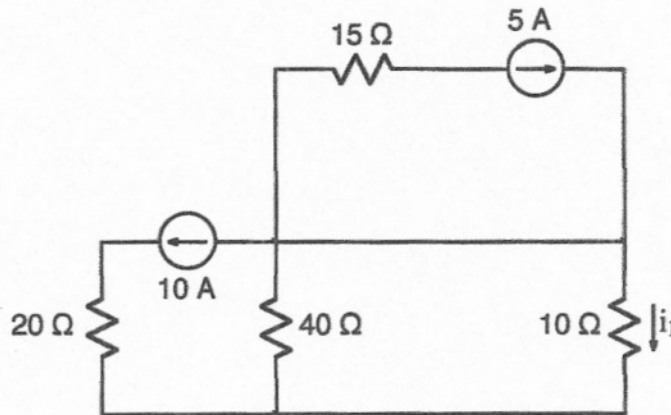
The 20A affects the current in the 28V source, but we still have 28V across the  $R$ 's. The 20A just circulates in the bottom half of the circuit.

### HW #2 Examples

1. Calculate  $v_1$ .



Ex:



Calculate  $i_1$ .

sol'n: We have a current divider consisting of the 10A src and the 40Ω and 10Ω R's in parallel.

To verify that we have a current divider, we observe that the circuit satisfies the following conditions:

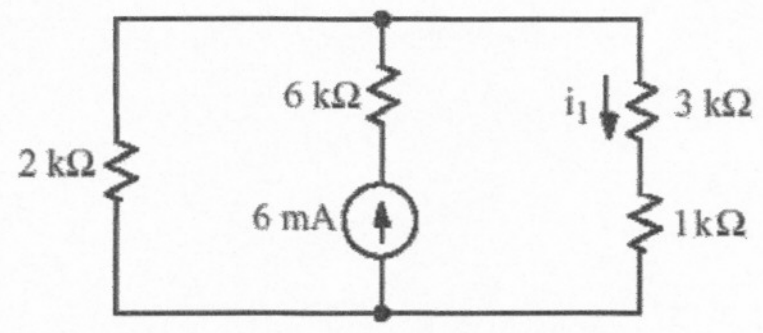
- i) The 10A is the total current flowing into one end of the 40Ω and 10Ω resistors, and
- ii) The opposite ends of the 40Ω and 10Ω R's are connected so that the v-drop across the 10Ω and 40Ω resistors is the same.

Using the current-divider formula (with a minus sign because  $i_1$  is measured in a direction opposite to the 10A src:

$$i_1 = -10A \cdot \frac{40\Omega}{40\Omega + 10\Omega} = -8A$$

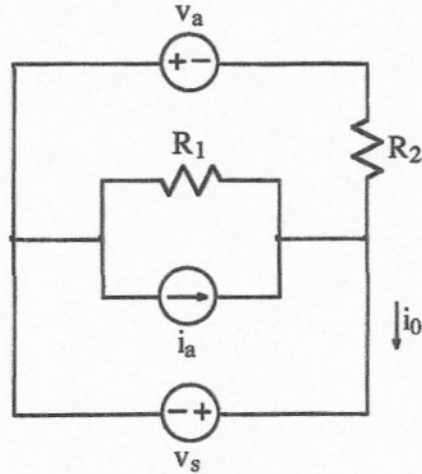
### HW #2 Examples

2. Calculate  $i_1$ .



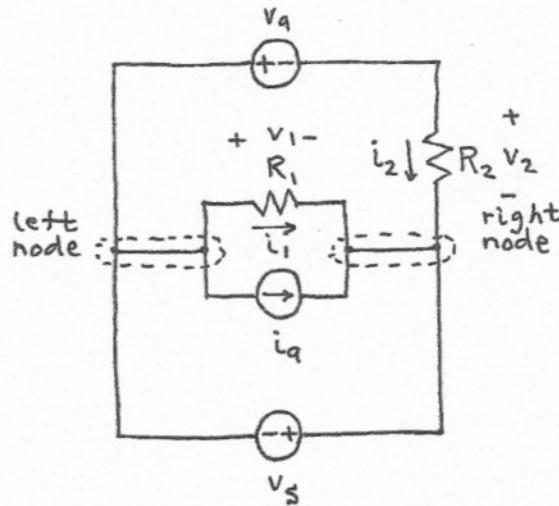


EX:



Derive an expression for  $i_0$ . The expression must not contain more than the circuit parameters  $v_a, v_s, i_a, R_1$ , and  $R_2$ .

sol'n: Label R's first.



v-loops: ( $v_a$  and  $R_1, R_2$ ;  $v_s$  and  $R_1$ )

$$-v_a - v_2 + v_1 = 0V$$

$$-v_1 - v_s = 0V$$

We observe that we can solve these 2 eqns in 2 unknowns without proceeding further.

$$v_1 = -v_s \quad \text{from 2nd eqn}$$

$$v_2 = -v_a + v_1 = -v_a - v_s \quad \text{from 1st eqn}$$

Note: If we try to write current-sum eqns, we find that the left node and right node are connected by only v-src  $v_s$ . Thus, we should not write i-sum eqns. (And we don't need them!)

Note: We also have no components in series that carry the same current, (except v-src  $v_a$  and  $R_2$ ).

We now use Ohm's law:

$$v_1 = i_1 R_1$$

$$v_2 = i_2 R_2$$

Using v-egns:

$$i_1 = \frac{v_1}{R_1} = \frac{-v_s}{R_1}$$

$$i_2 = \frac{v_2}{R_2} = \frac{-v_a - v_s}{R_2}$$

Now that we have solved the circuit,  
we can find  $i_0$  from an  $i$ -sum eq'n  
for the node on the right.

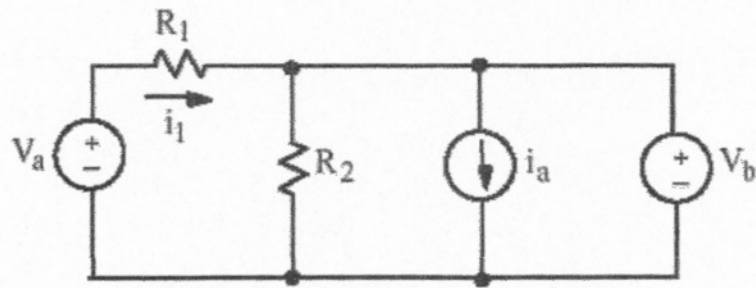
$$-i_2 - i_1 - i_a + i_0 = 0A$$

$$\text{or } i_0 = i_1 + i_2 + i_a$$

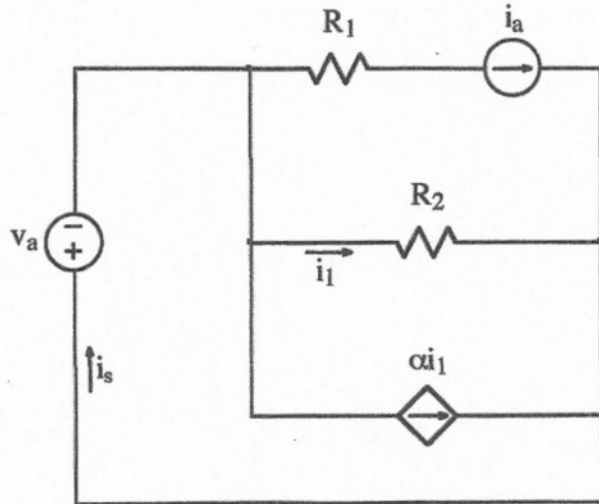
$$\text{or } i_0 = -\frac{v_s}{R_1} - \frac{v_a + v_s}{R_2} + i_a$$

### HW #2 Examples

3. Derive an expression for  $i_1$ . The expression must not contain more than the circuit parameters  $V_a$ ,  $V_b$ ,  $i_a$ ,  $R_1$ , and  $R_2$ .

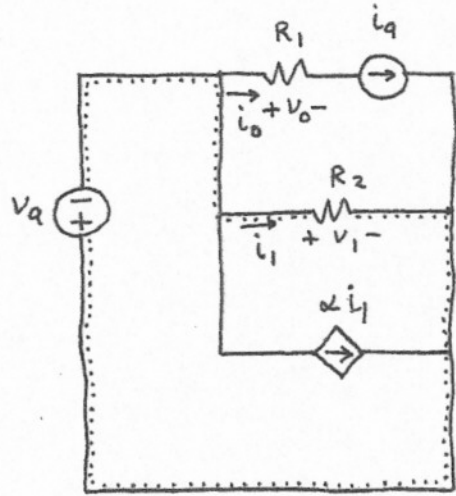


EX:



- a) Derive an expression for  $i_s$ . The expression must not contain more than the circuit parameters  $\alpha$ ,  $i_a$ ,  $v_a$ ,  $R_1$ , and  $R_2$ . (Make sure to eliminate  $i_1$  from the answer.)
- b) Make at least one consistency check (other than a units check) on your expression. Explain the consistency check clearly.

sol'n: a) Label R's



Only v-loop without current source is thru  $v_a$  and  $R_2$ , (dotted line).

$$-v_q - v_1 = 0V$$

We look for nodes where we can write  $i$ -sum eq'ns. Here, however, we really only have two nodes, and they are connected by only  $v$ -src  $v_q$ .

Thus, we have no  $i$ -sum eq'ns.

We look for components in series carrying the same current.

$$i_o = i_q$$

From Ohm's law:

$$v_o = i_o R_1 = i_q R_1$$

$$v_1 = i_1 R_2$$

Substituting for  $v_1$  in our  $v$ -loop eq'n:

$$-v_q - i_1 R_2 = 0V$$

$$\text{or } i_1 = -\frac{v_q}{R_2}$$

It follows that  $i_1 = -\frac{v_q}{R_2}$ .

Now we write  $i$ -sum eq'n (for node consisting of wire on right side) to find  $i_s$ .

$$i_s - \alpha i_1 - i_1 - i_q = 0A$$

$$\text{or } i_s = \alpha i_1 + i_1 + i_q$$

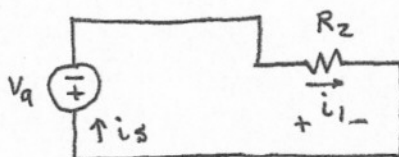
$$\text{or } i_s = (\alpha + 1) \left( \frac{-V_q}{R_2} \right) + i_q$$

$$\text{or } i_s = i_q - (\alpha + 1) \frac{V_q}{R_2}$$

b) Many consistency checks are possible. The idea is to pick component values that make the circuit so simple that we can solve it by inspection.

One example is to eliminate current sources:

Let  $i_q = 0A$  and  $\alpha = 0$ :



$$\text{We have } i_s = i_1 = - \frac{V_q}{R_2}$$

Now we verify that our eqn from (a) agrees:

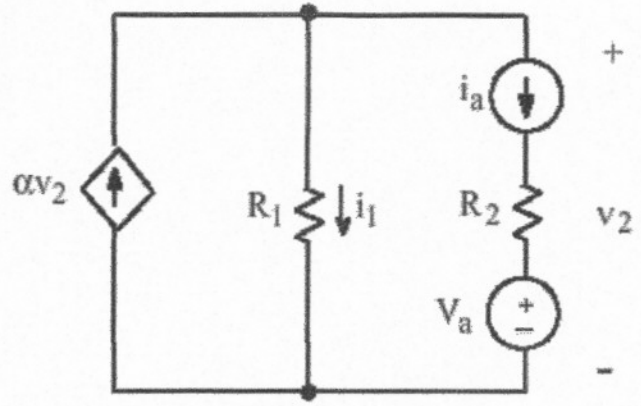
$$i_s = 0 - (0 + 1) \frac{V_q}{R_2} = - \frac{V_q}{R_2} \checkmark$$

Another example is to set  $V_q = 0V$ ,  $\alpha = 0$ .

Then  $i_s = i_q$ . Our eqn gives  $i_s = i_q - (0 + 1) \cdot \frac{0}{R_2} = i_q \checkmark$

### HW #2 Examples

4. Derive an expression for  $i_1$ . The expression must not contain more than the circuit parameters  $\alpha$ ,  $V_a$ ,  $i_a$ ,  $R_1$ , and  $R_2$ .

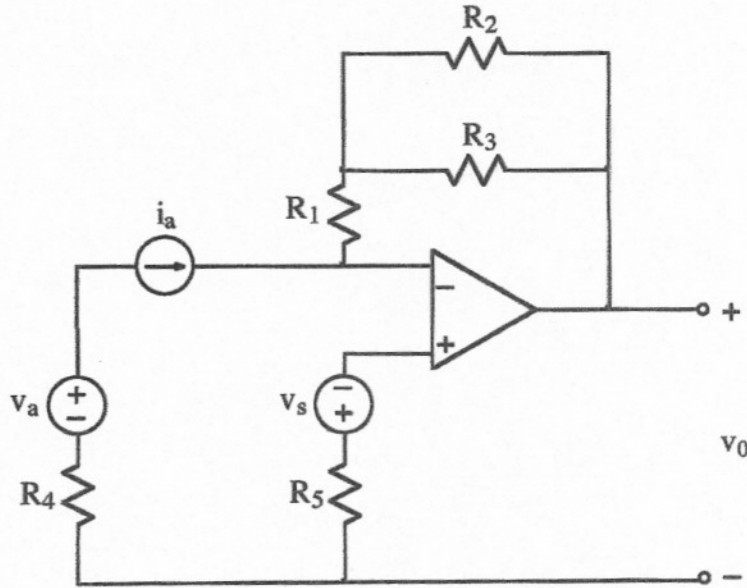


Make at least one consistency check (other than a units check) on your expression for problem 4. Explain the consistency check clearly.



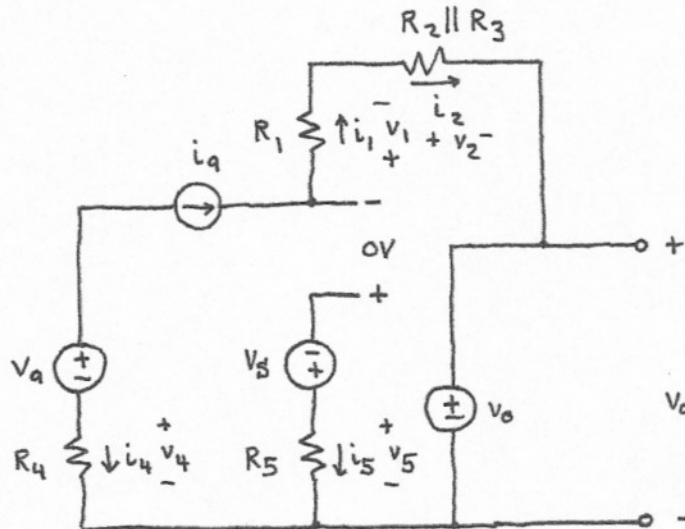
Dr. Neil Cotter

EX:



The op-amp operates in the linear mode. Using an appropriate model of the op amp, derive an expression for  $v_o$  in terms of not more than  $v_a$ ,  $v_s$ ,  $i_a$ ,  $R_1$ ,  $R_2$ ,  $R_3$ ,  $R_4$  and  $R_5$ .

sol'n: Replace op-amp with src called  $v_o$  and assume  $v$ -drop across + and - terminals is 0V. We also combine  $R_2$  and  $R_3$ .



**HOMEWORK #2 Solution Prob 5 (cont.)**



Write v-loops passing thru 0V across + and - terminals:

$+v_4 + v_9 + ?$  Don't use left-side v-loop because of current src.

$+v_5 - v_3 - 0V - v_1 - v_2 - v_6 = 0V$

Write current sums at nodes.  
The only true node is on the bottom.

$-i_4 - i_5 - i_2 = 0A$

Look for components in series carrying the same current:

$i_4 = -i_9$

$i_1 = i_9$

$i_2 = i_1 = i_9$

$i_5 = 0A$  (since it is in series with an open circuit)

We see that  $i_9$  flows all the way around the outer loop.

We need only substitute for v's in v-loop using  $i_9$  and Ohm's law for each resistor:

$V_1 = i_1 R_1 = i_9 R_1$

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$$v_2 = i_2 \cdot R_2 \parallel R_3 = i_a \cdot R_2 \parallel R_3$$

$$v_5 = i_5 R_5 = 0 \cdot R_5 = 0V$$

Our v-loop becomes:

$$0V - v_3 - 0V = i_a R_1 - i_a \cdot R_2 \parallel R_3 - v_o = 0V$$

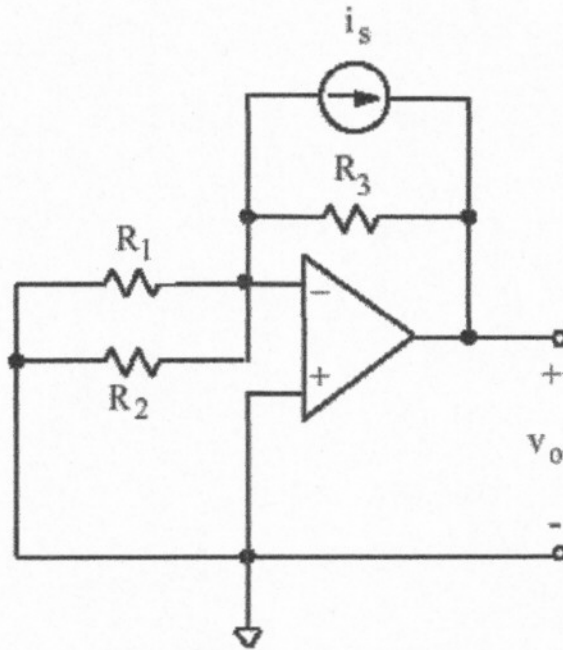
Solving  $v_o$  gives the expression we seek:

$$v_o = -v_3 - i_a (R_1 + R_2 \parallel R_3)$$

### HW #2 Examples

5.

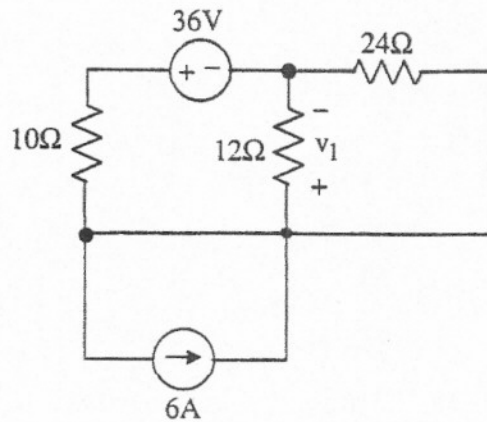
The op amp operates in the linear mode. Using an appropriate model of the op amp, derive an expression for  $v_o$  in terms of not more than  $i_s$ ,  $R_1$ ,  $R_2$ , and  $R_3$ .



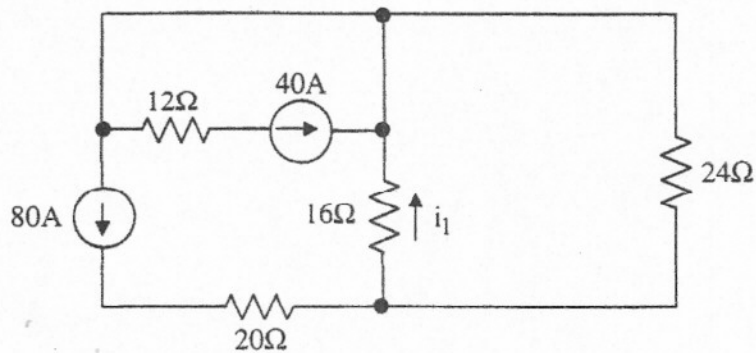
# REVIEW

1. a. (

Calculate  $v_1$ .

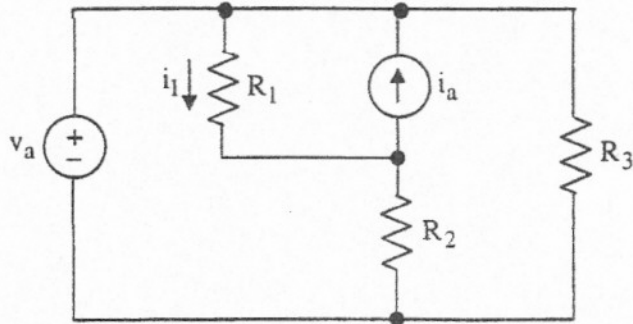


Calculate  $i_1$ .



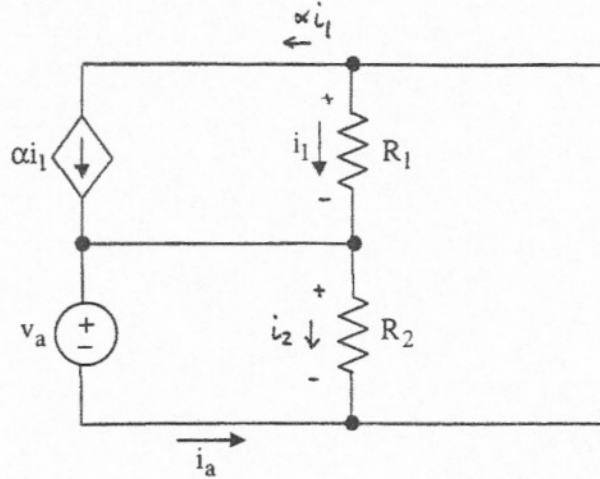
2.

Derive an expression for  $i_1$ . The expression must not contain more than the circuit parameters  $v_a$ ,  $i_a$ ,  $R_1$ ,  $R_2$ , and  $R_3$ .



3.

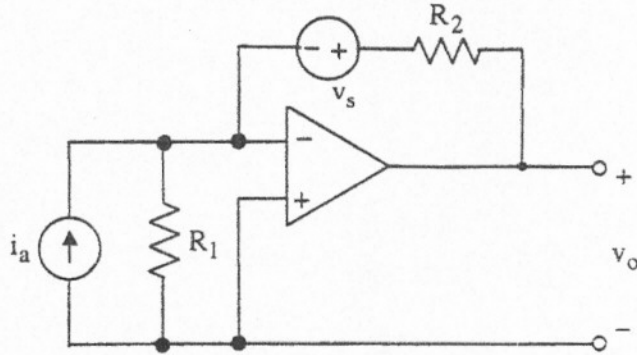
- a. Derive an expression for  $i_a$ . The expression must not contain more than the circuit parameters  $\alpha$ ,  $v_a$ ,  $R_1$ , and  $R_2$ .



- b. Make at least one consistency check (other than a units check) on your expression. Explain the consistency check clearly.

4.

The op-amp operates in the linear mode. Using an appropriate model of the op amp, derive an expression for  $v_o$  in terms of not more than  $v_s$ ,  $i_a$ ,  $R_1$ , and  $R_2$ .







a) Solve following eqns for  $V_1$  and  $V_2$  in terms of  $R_4$ :

$$V_1 \left( \frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_3} \right) - V_2 \frac{1}{R_3} = 10$$

$$\frac{V_1 - V_2}{R_3} = \frac{V_2}{R_4}$$

sol'n: Rewrite 2nd eqn as  $\frac{V_1}{R_3} = V_2 \left( \frac{1}{R_3} + \frac{1}{R_4} \right)$

Use parallel R notation  $R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$

Our two eqns become:

$$\frac{V_1}{R_1 \parallel R_2 \parallel R_3} - \frac{V_2}{R_3} = 10$$

$$\frac{V_1}{R_3} - \frac{V_2}{R_3 \parallel R_4} = 0$$

Now use 2nd eqn to write  $V_2$  in terms of  $V_1$ :

$$V_2 = V_1 \frac{R_3 \parallel R_4}{R_3} = V_1 \frac{R_3 R_4}{(R_3 + R_4) \cdot R_3} = V_1 \frac{R_4}{R_3 + R_4}$$

Now substitute this into first eqn:

$$\frac{V_1}{R_1 \parallel R_2 \parallel R_3} - \frac{V_1}{R_3} \frac{R_4}{R_3 + R_4} = 10 \text{ A}$$

$$V_1 = 10 / \left( \frac{1}{R_1 \parallel R_2 \parallel R_3} - \frac{R_4}{R_3} \frac{1}{R_3 + R_4} \right) = 10 / \left( \frac{1}{R_1 \parallel R_2 \parallel R_3} - \frac{R_3 \parallel R_4}{R_3^2} \right)$$

$$V_2 = V_1 \frac{R_3 \parallel R_4}{R_3} = 10 / \frac{R_3}{R_3 \parallel R_4} \left( \frac{1}{R_1 \parallel R_2 \parallel R_3} - \frac{R_4}{R_3} \frac{1}{R_3 + R_4} \right)$$

$$V_2 = 10 / \left( \frac{R_3 + R_4}{R_4} \frac{1}{R_1 \parallel R_2 \parallel R_3} - \frac{R_3 \parallel R_4}{R_3 (R_3 + R_4)} \right)$$

$$V_2 = 10 / \left( \frac{R_3 / R_4 \parallel R_3}{R_1 \parallel R_2 \parallel R_3} - \frac{1}{R_3} \right) = 10 R_3 / \left( \frac{R_3^2 \parallel R_4}{R_1 \parallel R_2 \parallel R_3} - 1 \right)$$

b) Make at least two consistency checks on answers to (a).

soln: Units:  $V_1 = A / \left( \frac{1}{\Omega} - \frac{\Omega}{\Omega} \frac{1}{\Omega} \right) = A \cdot \Omega = V \checkmark$

$$V_2 = A \Omega / \left( \frac{\Omega^2/\Omega}{\Omega} - 1 \right) = A \cdot \Omega = V \checkmark$$

When  $R_3 \rightarrow \infty$  2nd eqn gives  $V_2 = 0$ , 1st eqn gives

$$V_1 = 10A / \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$

In our  $V_1$  eqn with  $R_3 \rightarrow \infty$  we get  $R_1 \parallel R_2 \parallel R_3 = R_1 \parallel R_2$ ,

$$\frac{R_4}{R_3} \frac{1}{R_3 + R_4} = \frac{R_4}{\infty (\infty + R_4)} = \frac{R_4}{\infty} = 0$$

$$\therefore V_1 = \frac{10}{\frac{1}{R_1 \parallel R_2}} = 10 \cdot R_1 \parallel R_2 = 10 \cdot \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \checkmark$$

$$V_2 = 10 / \left( \frac{\infty R_4}{R_1 \parallel R_2} - \frac{0}{\infty} \right) = 10 \cdot \frac{R_1 \parallel R_2}{\infty} = 0 \checkmark$$

Make another consistency check.

$R_4 \rightarrow \infty$  gives  $V_1 = V_2$  from 2nd eqn. and  $R_3 \parallel R_4 = R_3$ .

Our eqns:  $V_1 = 10 / \left( \frac{1}{R_1 \parallel R_2 \parallel R_3} - \frac{R_3}{R_3^2} \right) = 10 / \left( \frac{1}{R_1 \parallel R_2 \parallel R_3} - \frac{1}{R_3} \right)$

$$V_2 = 10 / \left( \frac{R_3/R_3}{R_1 \parallel R_2 \parallel R_3} - \frac{1}{R_3} \right) = 10 / \left( \frac{1}{R_1 \parallel R_2 \parallel R_3} - \frac{1}{R_3} \right) \checkmark$$

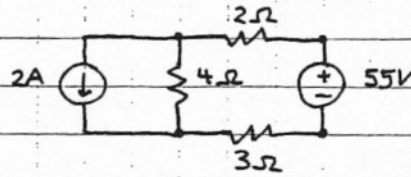
One more check.  $R_1 = \infty$   $R_2 = \infty$   $R_3 = R_4$  Then  $V_1 = 2V_2$ ,

$$V_2 = \frac{V_1}{2}, \quad V_1 - V_2 = 10 R_3, \quad V_1 - \frac{V_1}{2} = 10 R_3, \quad V_1 = 2 \cdot 10 R_3.$$

Our eqns:  $V_1 = 10 / \left( \frac{1}{R_3} - \frac{R_3/2}{R_3^2} \right) = 10 / \frac{1}{2R_3} = 2 \cdot 10 R_3 \checkmark$

$$V_2 = 10 / \left( \frac{R_3/(R_3/2)}{R_3} - \frac{1}{R_3} \right) = 10 / \frac{1}{R_3} = 10 R_3 = \frac{V_1}{2} \checkmark$$

ex:



Use Node-Voltage method to find how much power the 2A source extracts from circuit.

First we use terminology:

# nodes = 4 (two or more circuit elements join)

# essential nodes = 2 (three or more circuit elements join; they are nodes for 4Ω resistor, top & bottom)

paths:  $2A \rightarrow 4\Omega$ ,  $4\Omega \rightarrow 2\Omega \rightarrow 55V \rightarrow 3\Omega$ ,  $2\Omega \rightarrow 55V$ ,  $2A \rightarrow 2\Omega \rightarrow 55V$  are a few examples (trace of connected circuit elements without passing thru any element twice)

branch: (path that connect 2 nodes)  $2A$ ,  $4\Omega$ ,  $2\Omega$ ,  $3\Omega$ ,  $2\Omega \rightarrow 55V$ ,  $55V \rightarrow 3\Omega$ ,  $3\Omega \rightarrow 55V$  (either direction OK),  $55V$ ,  $2\Omega \rightarrow 55V \rightarrow 3\Omega$

essential branch: (path connecting essential node w/o passing thru essential node)  $2A$ ,  $4\Omega$ ,  $2\Omega \rightarrow 55V \rightarrow 3\Omega$ , or  $3\Omega \rightarrow 55V \rightarrow 2\Omega$

loops: (path with last node = start node)  $2A \rightarrow 4\Omega$ ,  $2A \rightarrow 2\Omega \rightarrow 55V \rightarrow 3\Omega$ ,  $4\Omega \rightarrow 2\Omega \rightarrow 55V \rightarrow 3\Omega$

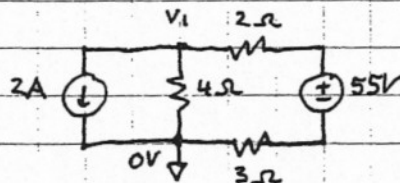
mesh: (loop not enclosing any other loop)  $2A \rightarrow 4\Omega$ ,  $4\Omega \rightarrow 2\Omega \rightarrow 55V \rightarrow 3\Omega$  but not  $2A \rightarrow 2\Omega \rightarrow 55V \rightarrow 3\Omega$

planar circuit: (can draw circuit w/o crossover branches) is planar

For Node-V method, we use all but 1 essential nodes after we define a ref node.

Choose node at bottom of 4Ω as ref node (i.e. 0V.)

↙ symbol  $\equiv 0V$



Node at top of 4Ω is the other essential node. Label it  $v_1$

Although we call it the Node-V method, (because we get an equation that we solve for voltage), we are writing an equation for sum of currents out of node = 0.

$$2A + \frac{V_1 - 0V}{4\Omega} + \frac{V_1 - 55V}{2\Omega + 3\Omega} = 0A$$

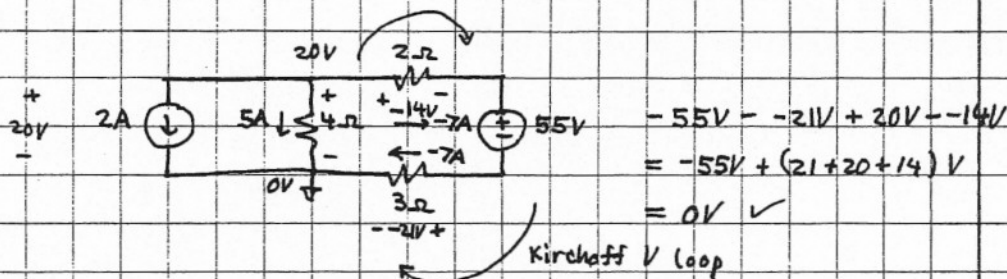
Note that current thru  $2\Omega$  is equal to total V-drop (i.e.  $V_1 - 55V$ ) across the  $2\Omega$  and  $3\Omega$  R's.

$$\text{or } V_1 \left( \frac{1}{4\Omega} + \frac{1}{5\Omega} \right) = -2A + \frac{55V}{5\Omega}$$

$$\text{or } \frac{V_1}{4\Omega \parallel 5\Omega} = 11A - 2A = 9A$$

$$\text{or } V_1 = 9A \cdot 4\Omega \parallel 5\Omega = 9A \cdot \frac{4\Omega \cdot 5\Omega}{4\Omega + 5\Omega} = 20V$$

$$\text{check: } \frac{V_1}{4\Omega} = \frac{20V}{4\Omega} = 5A \quad \frac{V_1 - 55V}{2\Omega + 3\Omega} = \frac{-35V}{5\Omega} = -7A$$

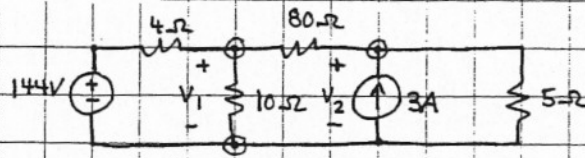


check: current out of top node for  $4\Omega$  is  $2A + 5A - 7A = 0$  ✓

Calculate power for  $2A$  source:  $p = i \cdot V = 2A \cdot 2V = 40W$   
 $p > 0 \Rightarrow$  power absorbed.

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ex:

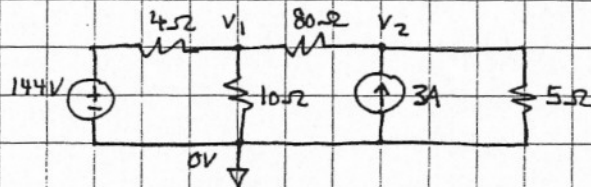


Use Node-V method to find  $V_1$  &  $V_2$ .

sol'n: nodes are marked by dots. Essential nodes are marked by  $\odot$ . Note that node under 3A source is considered to be part of essential node under 10Ω Resistor, (because they are connected by a wire).

3 essential nodes  $\Rightarrow 3-1=2$  equations needed.

Put ref V of 0V, (i.e.  $\downarrow$ ), at bottom so top nodes are  $V_1$  and  $V_2$ .



Node-V eqns give sum of currents out of node = 0:

$$\text{node 1: } \frac{V_1 - 144V}{4\Omega} + \frac{V_1 - 0V}{10\Omega} + \frac{V_1 - V_2}{80\Omega} = 0A$$

$(V_1)$

$$\text{node 2 } \frac{V_2 - V_1}{80\Omega} + -3A + \frac{V_2 - 0V}{5\Omega} = 0A$$

$(V_2)$

$$\text{or node 1: } V_1 \left( \frac{1}{4\Omega} + \frac{1}{10\Omega} + \frac{1}{80\Omega} \right) - \frac{V_2}{80\Omega} = \frac{144V}{4\Omega}$$

$$\text{node 2: } -V_1 \frac{1}{80\Omega} + V_2 \left( \frac{1}{80\Omega} + \frac{1}{5\Omega} \right) = 3A$$

$$\text{or node 1: } \frac{V_1}{4\Omega \parallel 10\Omega \parallel 80\Omega} - \frac{V_2}{80\Omega} = \frac{144V}{4\Omega}$$

$$\text{node 2: } -\frac{V_1}{80\Omega} + \frac{V_2}{80\Omega \parallel 5\Omega} = 3A$$

$$4\Omega \parallel 10\Omega \parallel 80\Omega = (4\Omega \parallel 10\Omega) \parallel 80\Omega$$

$$= \frac{40}{14}\Omega \parallel 80\Omega$$

$$= 40\Omega \cdot \frac{1}{14} \parallel 2$$

$$= 40\Omega \cdot \frac{2/14}{2 + 1/14}$$

$$= 40\Omega \cdot \frac{2}{29} \quad (\text{mult top \& bottom by 14})$$

$$= \frac{80}{29}\Omega$$

$$80\Omega \parallel 5\Omega = 5\Omega \cdot \frac{16}{17} \parallel 1 = 5\Omega \cdot \frac{16}{17} = \frac{80}{17}\Omega$$

Thus, we have: node 1  $v_1 \frac{29}{80\Omega} - \frac{v_2}{80\Omega} = 36\text{ A}$

node 2  $-\frac{v_1}{80\Omega} + \frac{v_2}{17} = 3\text{ A}$

or node 1  $29v_1 - v_2 = 2880\text{ V}$

node 2  $-v_1 + 17v_2 = 240\text{ V}$

From node 1:  $-v_2 = 2880\text{ V} - 29v_1$ ; substitute this into Node 2:

$$\text{node 2: } -v_1 - 17(2880\text{ V} - 29v_1) = 240\text{ V}$$

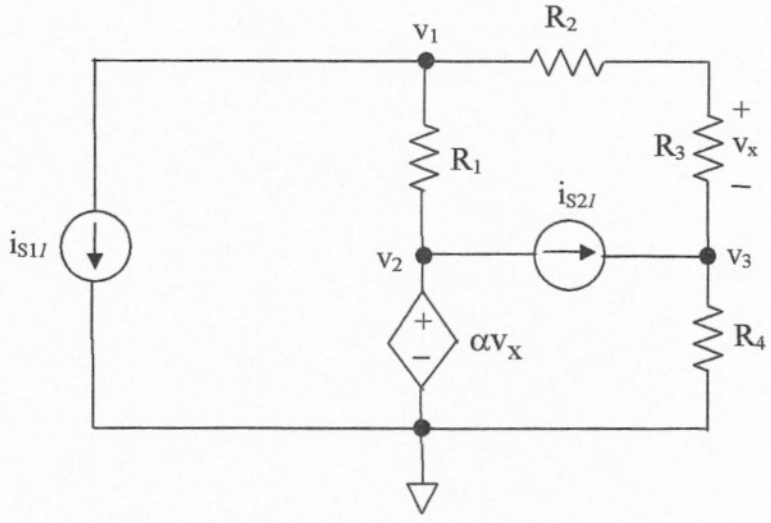
$$492v_1 = 240\text{ V} + 48960$$

$$v_1 = 100\text{ V}$$

$$v_2 = \frac{240\text{ V} + v_1}{17} = \frac{340\text{ V}}{17} = 20\text{ V}$$

### Node Voltage Example

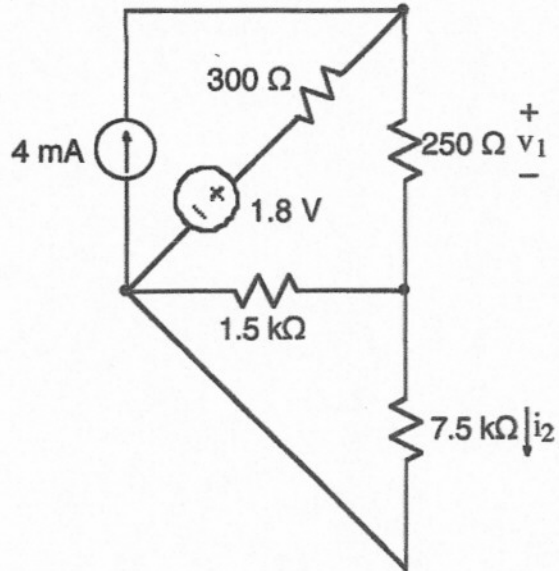
1. a.



For the circuit shown, write three independent equations for the node voltages  $v_1$ ,  $v_2$ , and  $v_3$ . The quantity  $v_x$  must not appear in the equations.

b. Make a consistency check on your equations for problem 1 by setting resistors and sources to values for which the values of  $v_1$ ,  $v_2$ , and  $v_3$  are obvious. State the values of resistors, sources, and  $v_1$ ,  $v_2$ ,  $v_3$  for your consistency check, and show that your equations for problem 1(a) are satisfied for these values. (In other words, plug the values into your equations for problem 1(a).)

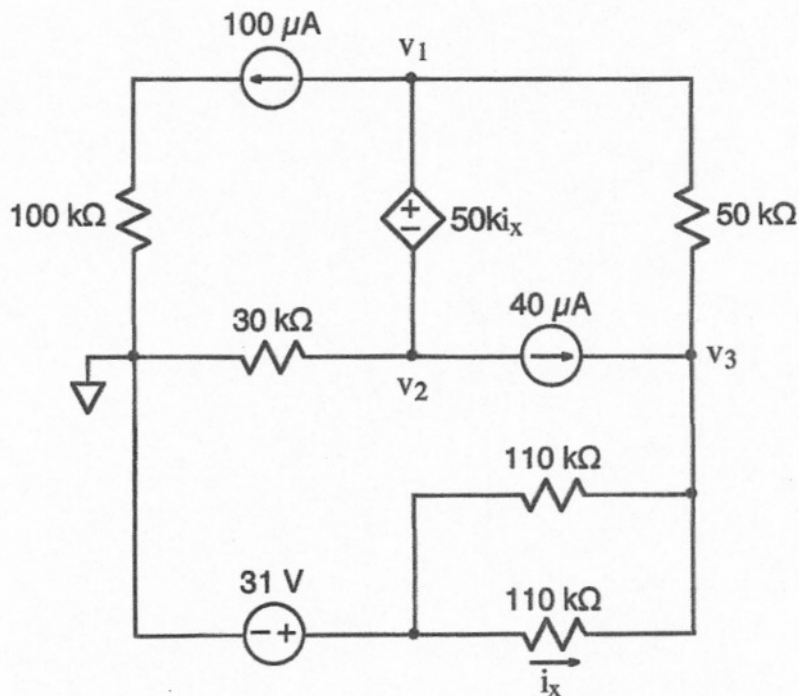
EX:



- a) Use the node-voltage method to calculate  $v_1$  and  $i_2$ .
- b) Calculate the power in the  $300 \Omega$  resistor.



Ex:



Use the node-voltage method to find  $v_1, v_2,$  and  $v_3$ .

sol'n: We first write the variable for the dependent source,  $i_x$ , in terms of node  $V$ 's.

$$i_x = \frac{31V - v_3}{110k\Omega}$$

We have a super node for  $v_1$  and  $v_2$  since these nodes are connected only by a  $V$  src. Thus, we sum all the currents flowing out of a bubble drawn around  $v_1, v_2,$  and the  $V$ -src between them.

$$v_1, v_2 \text{ node: } 100\mu A + \frac{v_1 - v_3}{50k\Omega} + \frac{v_2}{30k\Omega} + 40\mu A = 0A$$

We also write a voltage eq'n for  $v_1$  &  $v_2$ :

$$v_1 = v_2 + 50k\Omega i_x = v_2 + 50k\Omega \left( \frac{31V - v_3}{110k\Omega} \right)$$

(Remember to use only node  $v$ 's in eq'ns.)

For the  $v_3$  node, we only have a current sum.

$$v_3 \text{ node: } \frac{v_3 - v_1}{50k\Omega} + -40\mu A + \frac{v_3 - 31V}{\underbrace{110k\Omega \parallel 110k\Omega}_{55k\Omega}} = 0A$$

Now we solve the 3 eq'ns. We put terms multiplying  $v_1$ ,  $v_2$ , and  $v_3$  on the left side and constant terms on the right side.

$$v_1 \frac{1}{50k\Omega} + v_2 \frac{1}{30k\Omega} + v_3 \left( \frac{-1}{50k\Omega} \right) = -100\mu A - 40\mu A$$

$$v_1 - v_2 + v_3 \frac{50k\Omega}{110k\Omega} = 31V \cdot \frac{50k\Omega}{110k\Omega}$$

$$v_1 \left( \frac{-1}{50k\Omega} \right) + v_3 \left( \frac{1}{50k\Omega} + \frac{1}{55k\Omega} \right) = 40\mu A + \frac{31V}{55k\Omega}$$

We multiply both sides of the 1<sup>st</sup> eq'n by  $150k\Omega$  to clear the denominators.

$$v_1 \cdot 3 + v_2 \cdot 5 + v_3 (-3) = -140\mu A \cdot 150k\Omega = -21V$$

We multiply both sides of the 2<sup>nd</sup> eq'n by  $11\Omega$  to clear the denominators.

$$v_1 (11) + v_2 (-11) + v_3 (5) = 31V (5) = 155V$$

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We multiply the 3<sup>rd</sup> eq'n by  $1100\text{k}\Omega = 1.1\text{M}\Omega$

$$V_1(-22) + V_3(22 + 20) = 40\mu\text{A} \cdot 1.1\text{M}\Omega + 31\text{V}(20)$$

$$\text{or } V_1(-22) + V_3(42) = 44\text{V} + 620\text{V} = 664\text{V}$$

Now we start eliminating variables. From the 1<sup>st</sup> eq'n, we have

$$V_2 = \frac{-21\text{V} - 3V_1 + 3V_3}{5}$$

Our 2<sup>nd</sup> and 3<sup>rd</sup> eq'ns with this  $V_2$  become

$$11V_1 - 11\left(\frac{-21\text{V} - 3V_1 + 3V_3}{5}\right) + 5V_3 = 155\text{V}$$

and

$$-22V_1 + 42V_3 = 664\text{V}$$

Solving the last eq'n for  $V_3$  gives

$$V_3 = \frac{664\text{V} + 22V_1}{42}$$

Substituting into the 1<sup>st</sup> eq'n (and collecting terms multiplying  $V_3$ ) yields the following eq'n:

$$\left(11 + \frac{33}{5}\right)V_1 + \left(-\frac{33}{5} + 5\right)\left(\frac{664\text{V} + 22V_1}{42}\right) = 155\text{V} - \frac{21(11)\text{V}}{5}$$

or, after multiplying both sides by 5,

$$88V_1 - 8\left(\frac{664\text{V} + 22V_1}{42}\right) = 775\text{V} - 231\text{V} = 544\text{V}$$

Dividing both sides by 4 and moving constant term:

HOMEWORK #3 Solution Prob 3 (cont.)

$$22v_1 - \frac{2(22)}{42}v_1 = 136V + \frac{2(664)}{42}V.$$

Multiplying both sides by 42 gives

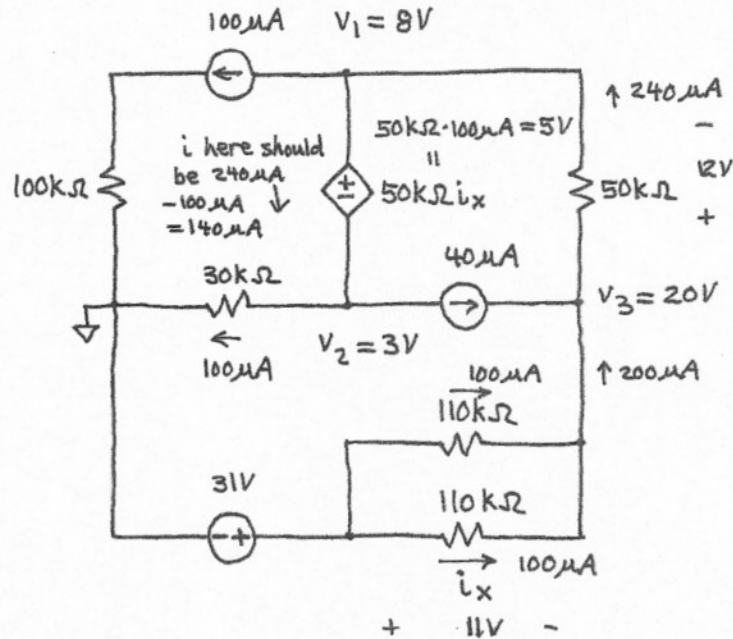
$$[42(22) - 44]v_1 = 136V(42) + 2(664)V$$

$$\text{or } v_1 = \frac{5712 + 1328V}{924 - 44} = \frac{7040V}{880} = 8V$$

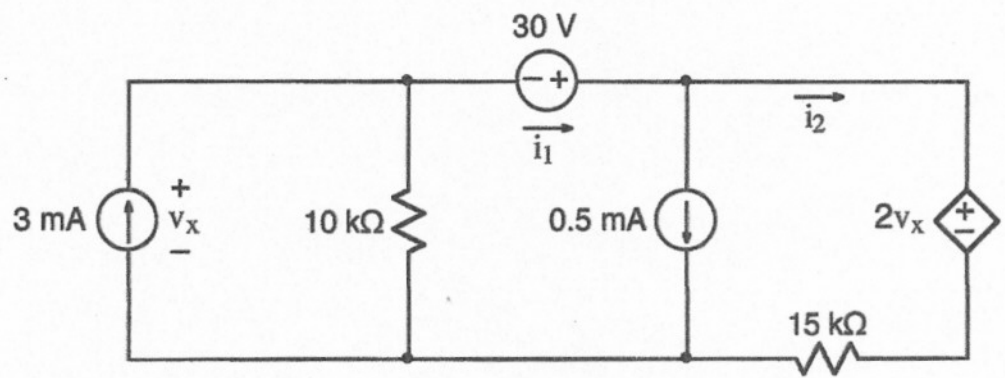
$$\text{Then } v_3 = \frac{664V + 22(8V)}{42} = 20V$$

$$\text{and } v_2 = \frac{-21V - 3(8V) + 3(20V)}{5} = 3V.$$

Consistency check: Calculate currents from these voltages and verify that currents sum to zero at nodes... They do!



EX:



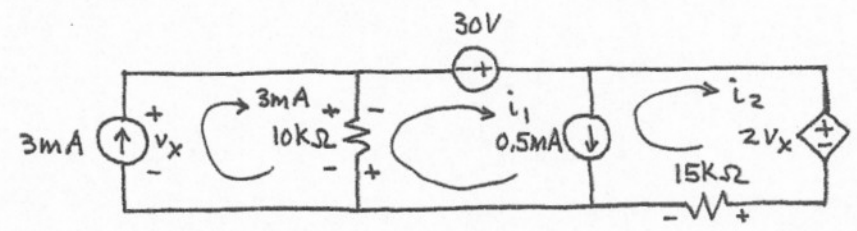
- a) Use the mesh-current method to find  $i_1$  and  $i_2$ .
- b) Find the power dissipated by the dependent source.

sol'n: a) We follow a step-by-step procedure:

1) We define mesh currents. If, however, we have any current sources on outside edges of the circuit, the mesh currents for those loops will be the same as the current source.

In this circuit, we have a current source on the left edge. Thus, the mesh current for the left loop is 3mA.

Since  $i_1$  and  $i_2$ , as defined, are on the outside edge of the circuit, we may use them as our mesh currents.



- 2) We define the voltage from the dependent src,  $v_x$ , in terms of mesh currents. Here, we observe that  $v_x$  is across the  $10k\Omega$  resistor, too. For the  $10k\Omega$  resistor, we have

$$v_x = 3\text{mA} \cdot 10k\Omega - i_1 \cdot 10k\Omega$$

- 3) We look for loops with a current source in between, meaning we have a super mesh. This is the case for the  $i_1, i_2$  loops. For the  $i_1, i_2$  supermesh, we take a  $v$ -loop around the outside edge of the  $i_1$  and  $i_2$  loops, (bypassing the  $0.5\text{mA}$  src).

$$i_1, i_2 \text{ } v\text{-loop: } -i_1 \cdot 10k\Omega + 30V - \overbrace{2(3\text{mA} - i_1)}^{v_x} 10k\Omega + 3\text{mA} \cdot 10k\Omega$$

$$-i_2 \cdot 15k\Omega = 0V$$

Add a current eq'n for the  $0.5\text{mA}$  src between the loops:

$$i_1 - i_2 = 0.5\text{mA} = \frac{1}{2}\text{mA}$$

Note: we have  $-i_2$  for current measured opposite the arrow in the current src.

- 4) We solve our eqns for  $i_1$  and  $i_2$ .

We group  $i_1$  and  $i_2$  terms on the left and move constant to the right side.

$$i_1 \underbrace{(-10k\Omega + 2 \cdot 10k\Omega)}_{= 10k\Omega} + i_2(-15k\Omega) = -60V + 60V$$

$$i_1 - i_2 = \frac{1}{2} \text{ mA}$$

Solving the 2<sup>nd</sup> eq'n for  $i_1$ , we have

$$i_1 = i_2 + \frac{1}{2} \text{ mA}$$

Substituting into 1<sup>st</sup> eq'n, we have

$$(i_2 + \frac{1}{2} \text{ mA}) 10k\Omega + i_2(-15k\Omega) = 30V$$

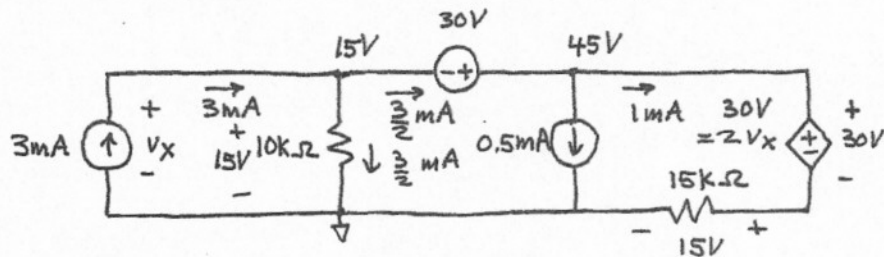
$$\text{or } i_2(10k\Omega - 15k\Omega) = 30V - \frac{1}{2} \text{ mA} \cdot 10k\Omega$$

$$\text{or } -i_2(5k\Omega) = -5V$$

$$\text{or } i_2 = 1 \text{ mA}$$

$$\text{Then } i_1 = 1 \text{ mA} + \frac{1}{2} \text{ mA} = \frac{3}{2} \text{ mA}$$

Consistency check: calculate v-drops for  $i_1, i_2$  and verify v-loops.



$$v_x = \frac{3}{2} \text{ mA} \cdot 10k\Omega = 15V$$

All v-loops sum to 0V, and all current sums at nodes = 0A. ✓

b) We know  $v_x = (3\text{mA} - i_1) 10\text{k}\Omega$

$$= \frac{3}{2} \text{mA} \cdot 10\text{k}\Omega$$

$$v_x = 15\text{V}$$

The current for the dependent src is  $i_2$ .

$$i_2 = 1\text{mA}$$

Thus, power for the dependent src is

$$p = v \cdot i = 2v_x i_2 = 2(15\text{V}) \cdot 1\text{mA}$$

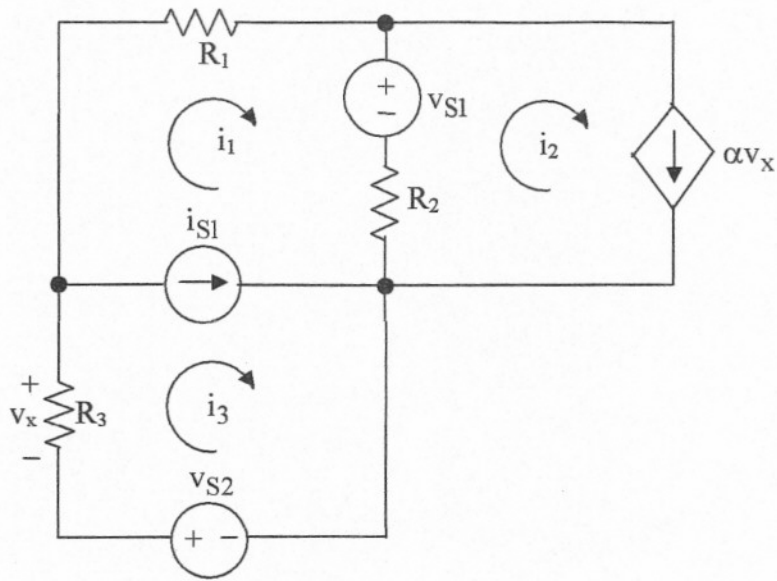
or  $p = 30\text{mW}$ .



# Mesh Current Example

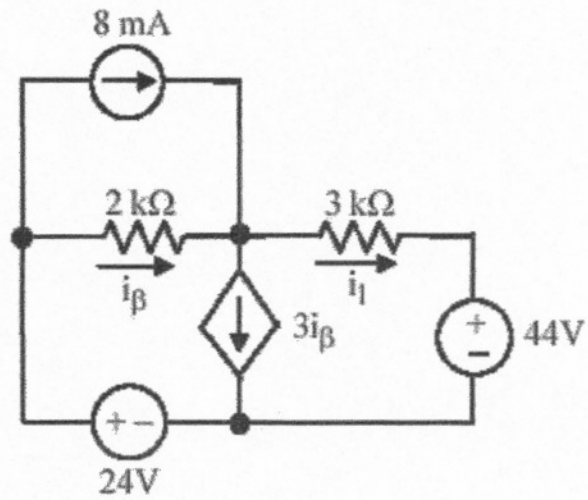
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For the circuit shown, write three independent equations for the three mesh currents  $i_1$ ,  $i_2$ , and  $i_3$ . The quantity  $v_x$  must not appear in the equations.



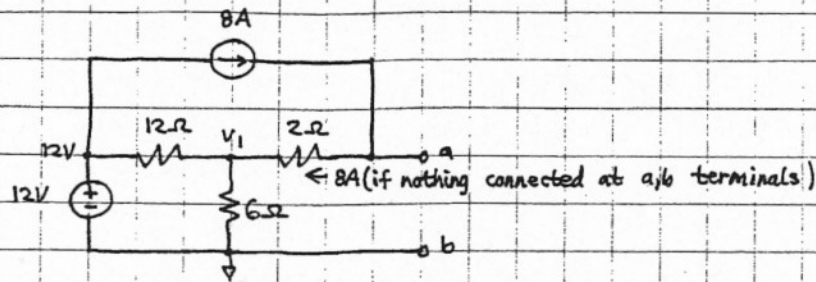
# Mesh Current Example

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- Use the mesh-current method to find  $i_1$ .
- Find the power dissipated by the dependent current source.

ex:



Find the Thevenin equivalent with respect to terminals a,b.

(In other words, create a circuit with a  $V$ -source and internal  $R$  that has same  $i$  and  $v$  at its terminals as above circuit has at a,b terminals.)

Use Node-V method to find  $V_1$ , then use  $V_a = V_1 + 8A \cdot 2\Omega = V_{Th}$ .

(We are finding  $V_{Th}$  by calculating voltage at a,b terminals with nothing connected to a,b terminals.)

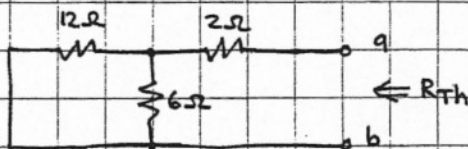
Node  $V_1$  eq'n: 
$$\frac{V_1 - 12V}{12\Omega} + \frac{V_1}{6\Omega} - 8A = 0A$$

or 
$$\frac{V_1}{12\Omega \parallel 6\Omega} = 8A + \frac{12V}{12\Omega} = 9A$$

or 
$$V_1 = 9A \cdot 12\Omega \parallel 6\Omega = 9A \cdot 6\Omega \left(\frac{2}{3}\right) = 54V \cdot \frac{2}{3} = 36V$$

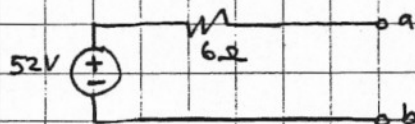
$$V_{Th} = V_a = V_1 + 8A \cdot 2\Omega = 36V + 16V = 52V$$

To find  $R_{Th}$  we turn independent sources to zero and find  $R$  looking into a,b terminals:



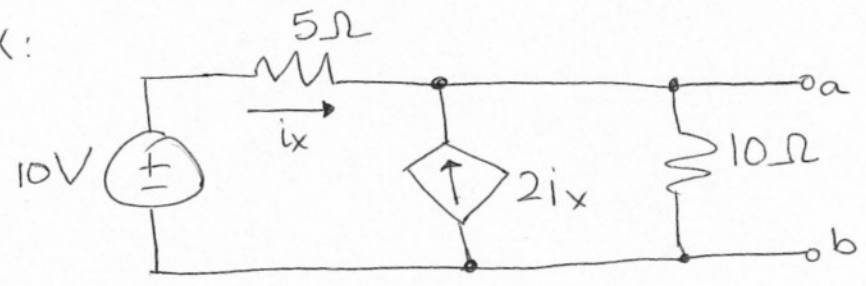
$$\begin{aligned} R_{Th} &= (6\Omega \parallel 12\Omega) + 2\Omega \\ &= 6\Omega \cdot \left(\frac{1}{2}\right) + 2\Omega \\ &= 6\Omega \cdot \frac{2}{3} + 2\Omega \\ &= 6\Omega \end{aligned}$$

Thevenin equivalent:



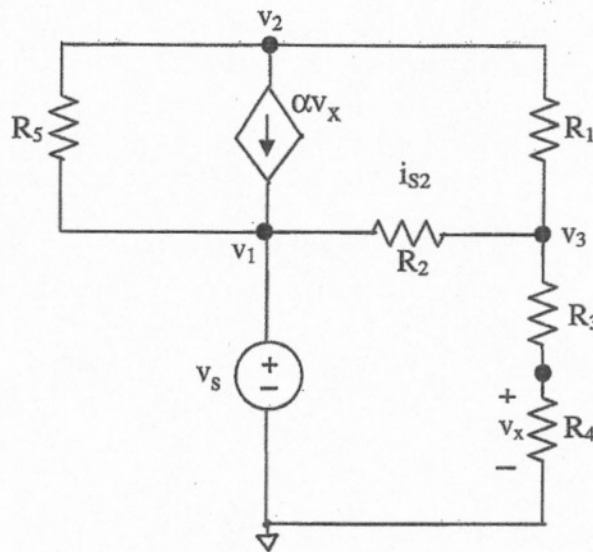
# Thévenin Equivalent

EX:



• Find Thévenin equivalent circuit between nodes a and b.

1.



For the circuit shown, write three independent equations for the node voltages  $v_1$ ,  $v_2$ , and  $v_3$ . The quantity  $v_x$  must not appear in the equations.

2. Make a consistency check on your equations for problem 1 by setting resistors and sources to values for which the values of  $v_1$ ,  $v_2$ , and  $v_3$  are obvious. State the values of resistors, sources, and  $v_1$ ,  $v_2$ ,  $v_3$  for your consistency check, and show that your equations for problem 1(a) are satisfied for these values. (In other words, plug the values into your equations for problem 1(a).)

sol'n: 1. 1)  $v_1 = v_3$  (Connected to ref node by v-src)

$$2) \frac{v_2 - v_1}{R_5} + \alpha \frac{v_3 R_4}{R_3 + R_4} + \frac{v_2 - v_3}{R_1} = 0A$$

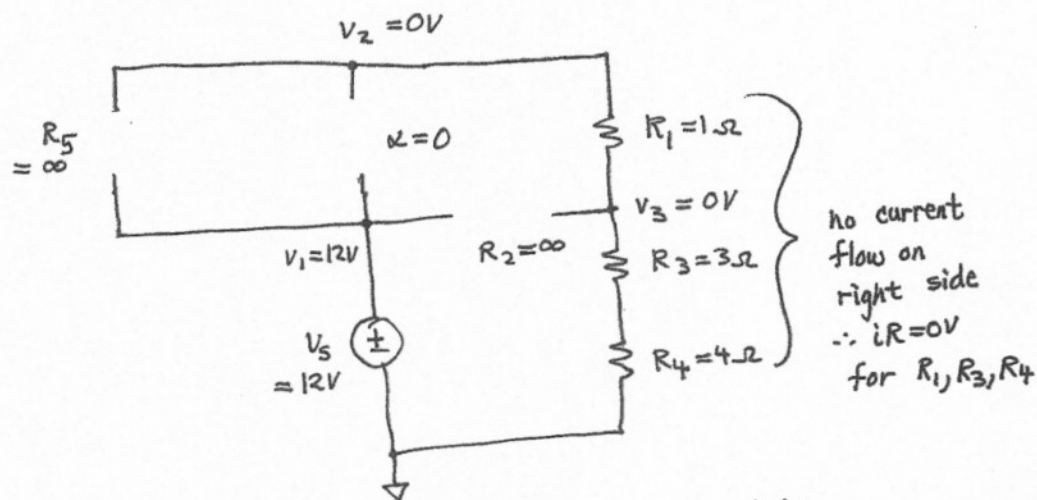
This is  $v_x$  in terms of node voltage. We are using a v-divider.

$$3) \frac{v_3 - v_2}{R_1} + \frac{v_3 - v_1}{R_2} + \frac{v_3 - 0V}{R_3 + R_4} = 0A$$

Note: we only have to write the eq'ns, not solve them.

2. Many consistency checks are possible.  
Choose values of  $R$ 's and  $s$ 's such that  
values of  $v_1, v_2, v_3$  are obvious from inspection.

My choices:



Plug all the component values and  $v$ 's into  
eqns in prob 1 to verify that eqns are satisfied.

$$1) \quad 12V = 12V \quad \checkmark$$

$$v_1 = v_3$$

$$2) \quad \frac{0 - 12V}{\infty} + 0 \cdot \frac{0V \cdot 4\Omega}{3 + 4\Omega} + \frac{0 - 0V}{1\Omega} = 0A \quad \checkmark$$

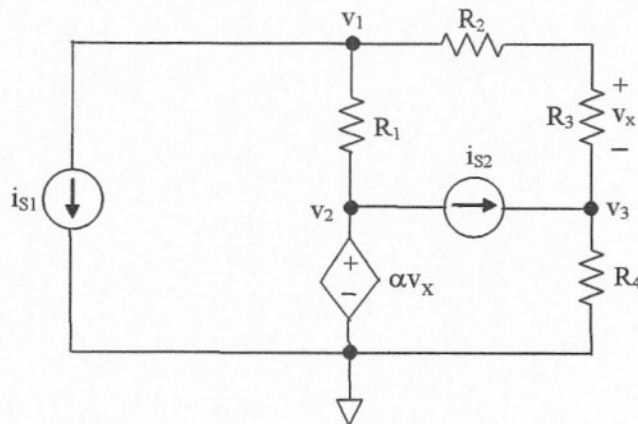
$$\frac{v_2 - v_1}{R_5} + \alpha \frac{v_3 R_4}{R_3 + R_4} + \frac{v_2 - v_3}{R_1} = 0A$$

$$3) \quad \frac{0 - 0V}{1\Omega} + \frac{0 - 12V}{\infty} + \frac{0V - 0V}{3 + 4\Omega} = 0A \quad \checkmark$$

$$\frac{v_3 - v_2}{R_1} + \frac{v_3 - v_1}{R_2} + \frac{v_3 - 0V}{R_3 + R_4} = 0A$$

## Homework #4 Example

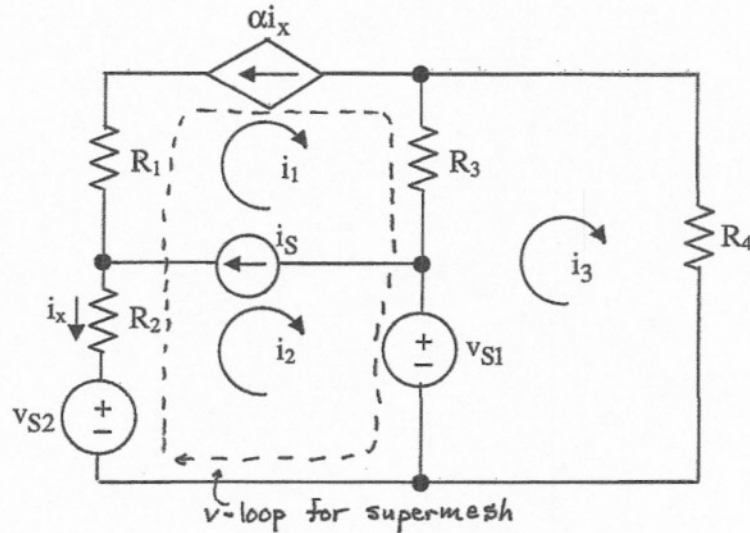
1.



(a) For the circuit shown, write three independent equations for the node voltages  $v_1$ ,  $v_2$ , and  $v_3$ . The quantity  $v_x$  must not appear in the equations.

(b) Make a consistency check on your equations for (a) by setting resistors and sources to values for which the values of  $v_1$ ,  $v_2$ , and  $v_3$  are obvious. State the values of resistors, sources, and  $v_1$ ,  $v_2$ ,  $v_3$  for your consistency check, and show that your equations for problem (a) are satisfied for these values. (In other words, plug the values into your equations for problem (a).)

3.



For the circuit shown, write three independent equations for the three mesh currents  $i_1$ ,  $i_2$ , and  $i_3$ . The quantity  $i_x$  must not appear in the equations.

sol'n: First, write  $i_x$  in terms of mesh currents.

$$i_x = -i_2 \quad (\text{use mesh currents only})$$

Now do v-loops for mesh currents.

We have current source between loops for  $i_1$  and  $i_2$ .

$\therefore$  we have super-mesh loop around outside of  $i_1$  and  $i_2$  loops (that avoids having to define  $v$  for  $i_3$ ).

supermesh v-loop:  $+v_{S2} - i_2 R_2 - i_1 R_1 -$  stop! ↓ don't use  $i_x$  here! current src not allowed

We must not need this v-loop. Indeed, we have a current src on the outside edge, giving us  $i_1 = -i_x = -(-i_2)$

We must also have a current eq'n for  $i_3$  source to complete super mesh:

$$i_3 = i_1 - i_2$$

↑ flows against  $i_3$  so  $i_3$  minus

Normal v-loop for  $i_3$ :

$$+v_{S1} - i_3 R_3 - i_3 R_4 = 0V$$

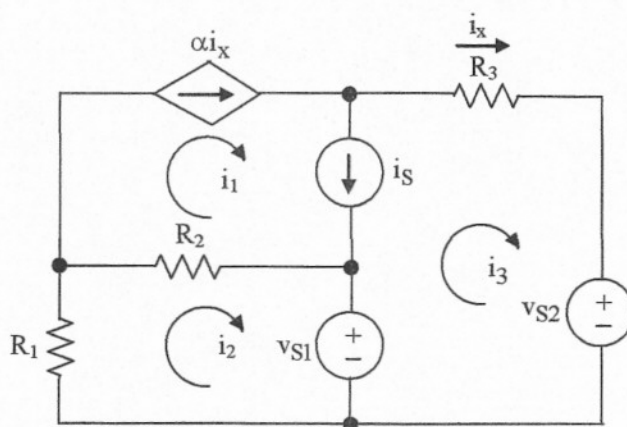
$$+ i_1 R_3$$

Question only requires 3 eq'ns we could solve, but we don't have to solve them.



### Homework #4 Example

2.

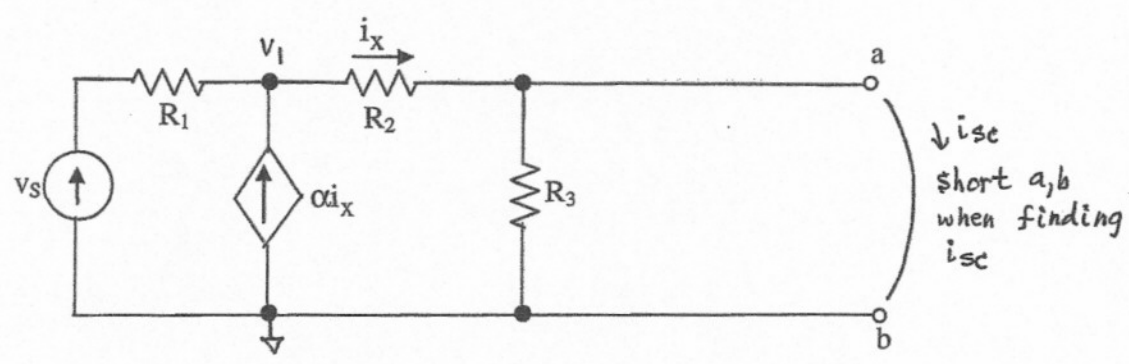


Write 3 independent equations for  $i_1$ ,  $i_2$ , and  $i_3$ .  $i_x$  must not appear in the equations.

Homework #4 Examples

Dr. Neil Cotter

4.



Find the Thevenin's equivalent circuit at terminals a-b.  $i_x$  must not appear in your solution. **Hint:** Use the node voltage method. **Note:**  $\alpha > 0$ .

sol'n: Assuming  $v_s$  is current src:

$$V_{Thev} = V_{a,b} \text{ open circuit}$$

Use node-voltage method to find  $v_1$ :

$$-v_s - \alpha \left[ \frac{v_1}{R_2 + R_3} \right] + \frac{v_1}{R_2 + R_3} = 0A$$

$= i_x$

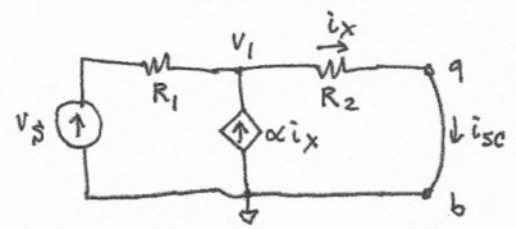
$$\text{or } v_1 \frac{1 - \alpha}{R_2 + R_3} = v_s$$

$$v_1 = v_s \frac{R_2 + R_3}{1 - \alpha}$$

$$V_{Thev} = v_1 \cdot \frac{R_3}{R_2 + R_3} = v_s \frac{R_3}{1 - \alpha} \quad \text{v-divider}$$

Now short a,b terminals and find  $i_{sc}$  flowing from a to b.

Note that  $R_3$  will carry no current; it all flows thru the short.  $\therefore$  we may ignore  $R_3$ .



$$\text{Now, } i_{sc} = i_x = \frac{v_1}{R_2} = \frac{v_s}{1 - \alpha}$$

$$R_{Thev} = V_{Thev} / i_{sc} = R_3$$

Use node voltage to find  $v_1$ .

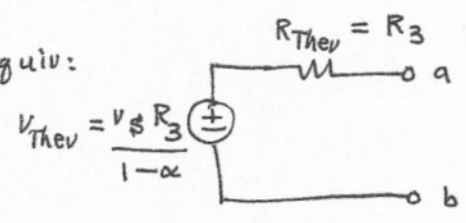
$$-v_s - \alpha \left[ \frac{v_1}{R_2} \right] + \frac{v_1}{R_2} = 0A$$

$= i_x$

$$v_1 = v_s \frac{R_2}{1 - \alpha}$$

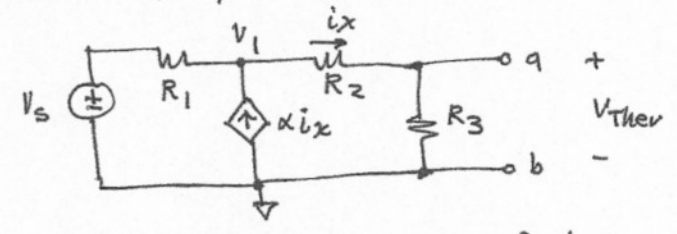
4. cont.

Thevenin equiv:



Assuming  $v_s$  is voltage src  $v_s$ :

$V_{Thev} = V_{a,b}$  open circuit



Use node-voltage method to find  $v_1$ :

$$\frac{v_1 - v_s}{R_1} - \alpha \frac{v_1}{R_2 + R_3} + \frac{v_1}{R_2 + R_3} = 0A$$

$= i_x$

$$v_1 \left( \frac{1}{R_1} + \frac{1 - \alpha}{R_2 + R_3} \right) = \frac{v_s}{R_1}$$

mult both sides by  $R_1$  and re-arrange:

$$v_1 = \frac{v_s}{1 + \frac{(1 - \alpha) R_1}{R_2 + R_3}}$$

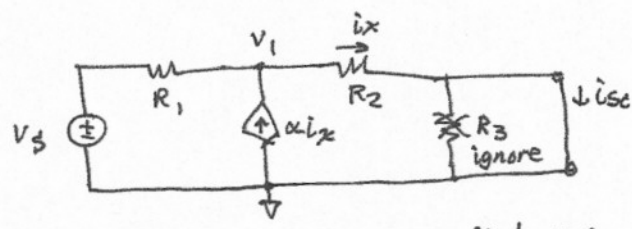
$$V_{Thev} = v_1 \cdot \frac{R_3}{R_2 + R_3} = \frac{v_s R_3}{\left( 1 + \frac{(1 - \alpha) R_1}{R_2 + R_3} \right) (R_2 + R_3)}$$

$$V_{Thev} = \frac{v_s R_3}{R_2 + R_3 + (1 - \alpha) R_1}$$

Now short a,b terminals and find  $i_s$  flowing from a to b. Note that  $R_3$  will be bypassed. All current will flow thru short.

$\therefore$  we may ignore  $R_3$ .

4. cont.



use node-voltage to find  $v_1$ :

$$\frac{v_1 - v_s}{R_1} - \alpha \frac{v_1}{R_2} + \frac{v_1}{R_2} = 0A$$

$\frac{v_1}{R_2} = i_x$

$$v_1 \left( \frac{1}{R_1} + \frac{1-\alpha}{R_2} \right) = \frac{v_s}{R_1}$$

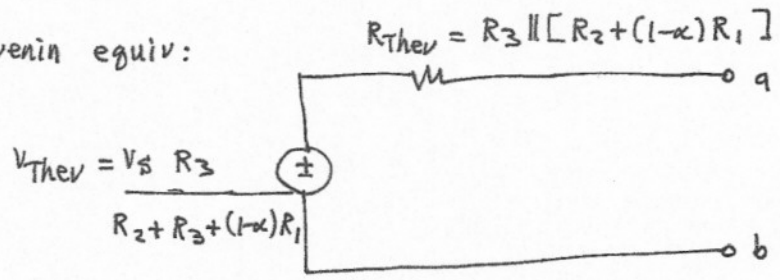
$$\text{or } v_1 = \frac{v_s}{1 + \frac{(1-\alpha)R_1}{R_2}}$$

$$i_{sc} = i_x = \frac{v_1}{R_2} = \frac{v_s}{R_2 + (1-\alpha)R_1}$$

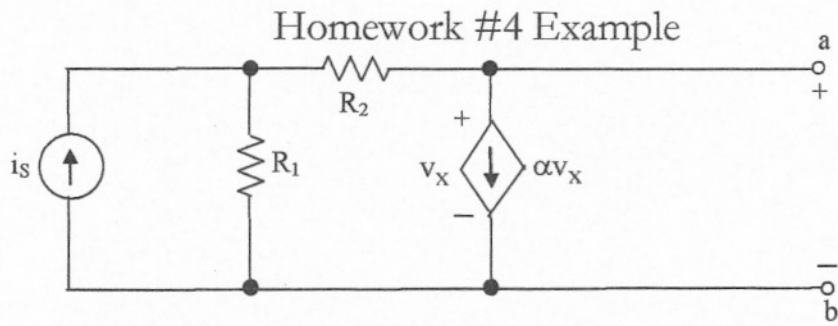
$$R_{Thev} = \frac{v_{Thev}}{i_{sc}} = \frac{R_3}{R_2 + R_3 + (1-\alpha)R_1} \cdot R_2 + (1-\alpha)R_1$$

$$R_{Thev} = R_3 \parallel [R_2 + (1-\alpha)R_1]$$

Thevenin equiv:



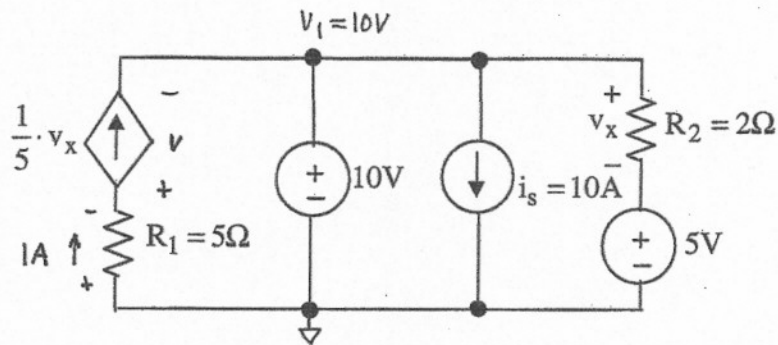
3.



Find the Thevenin's equivalent circuit at terminals a-b.  $v_x$  must not appear in your solution.

**Hint:** Use node voltage method to find  $v$  above  $R_1$ . **Note:**  $\alpha > 0$ .

5.



Calculate the power consumed by the dependent current source, (labeled  $\frac{1}{5}v_x$ ). **Note:** If a source supplies power, the power it consumes is negative.

sol'n:  $v$  on top rail = 10V from 10V src (if ref on bottom).

$$\text{Then } v_x = 5V = 10V - 5V$$

$$\therefore \frac{1}{5}v_x = \frac{1}{5}5V = 1A \text{ for dependent src}$$

$$\text{Using } v\text{-loop on left, } -1A \cdot R_1 - v - 10V = 0V$$

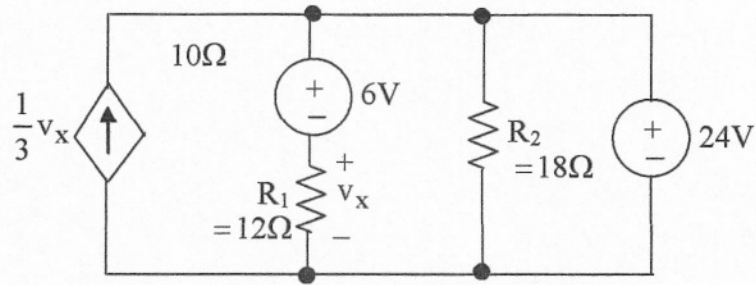
$$\text{or } v = -10V - 1A \cdot R_1 = -15V$$

$$\text{power } \equiv p = \frac{1}{5}v_x \cdot v = 1A (-15V) = -15W$$

$$p = -15W$$

### Homework #4 Example

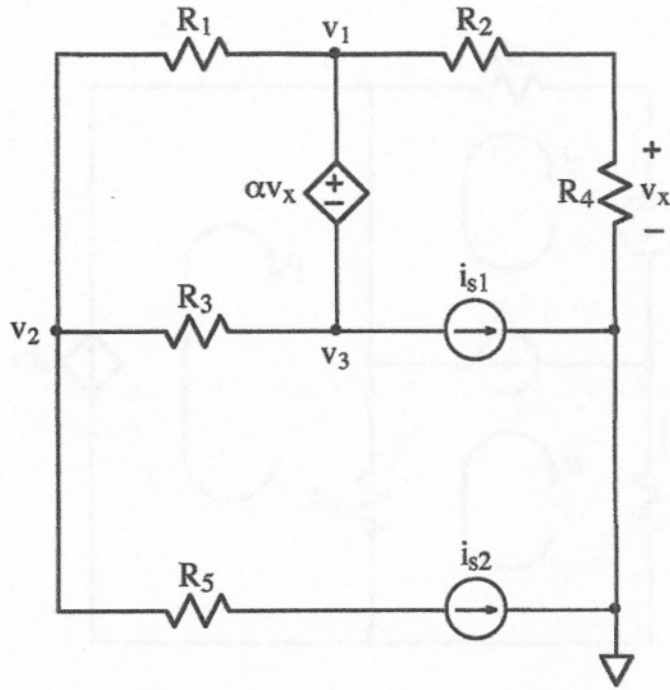
4.



Calculate the power consumed by the dependent current source, (labeled  $\frac{1}{3}v_x$ ).

**Note:** If a source supplies power, the power it consumes is negative.

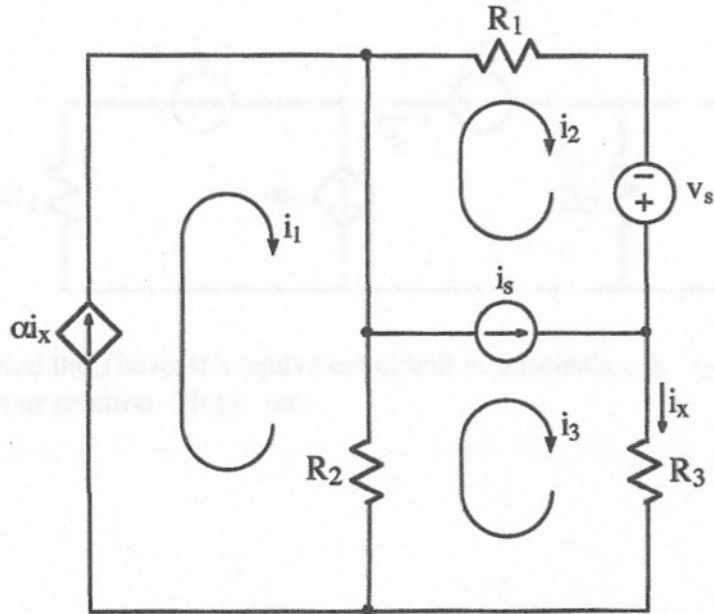
Ex:



For the circuit shown, write three independent equations for the node-voltages,  $v_1, v_2$ , and  $v_3$ . The quantity  $v_x$  must not appear in the equations.

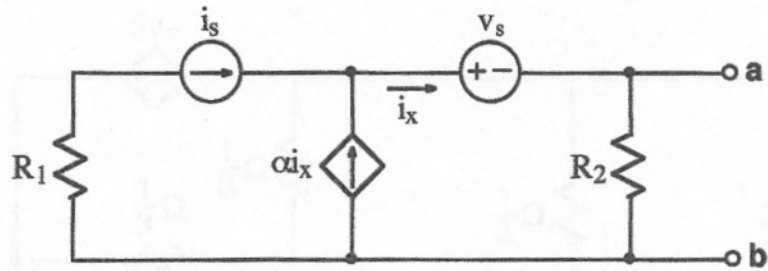


Ex:



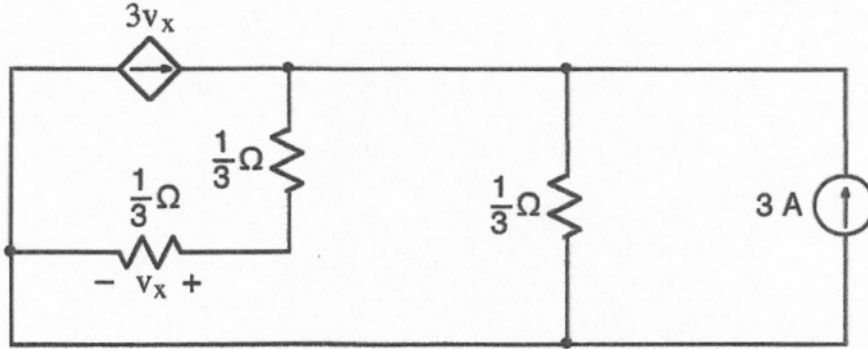
For the circuit shown, write three independent equations for the three mesh currents,  $i_1$ ,  $i_2$ , and  $i_3$ . The quantity  $i_x$  must not appear in the equations.

EX:



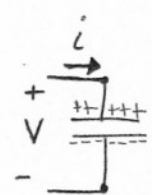
Find the Thevenin's equivalent circuit at terminals a-b.  $i_x$  must not appear in your solution. Note:  $\alpha \neq 1$ .

EX:



Calculate the power dissipated in the dependent current source, (labeled  $3v_x$ ).

Capacitor (C in Farads)



$$i(t) = C \frac{dV(t)}{dt}$$

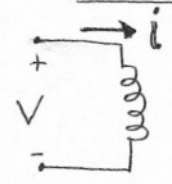
parallel plates dielectric between  
 V causes i to flow  
 • stores charge on plates like a small battery

V can not change instantly for C

V constant  $\Rightarrow i=0$  ( $\frac{dV}{dt}=0$ )

$$\text{Energy} = \frac{1}{2} CV^2$$

Inductor (L in Henry)



$$V(t) = L \frac{di(t)}{dt}$$

i cannot change instantly for L  
 i constant  $\Rightarrow V=0$  ( $\frac{di}{dt}=0$ )

$$\text{Energy} = \frac{1}{2} Li^2$$

<p>Cap</p> <p>Series: <math>C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}</math></p> <p>parallel: <math>C_{eq} = C_1 + C_2</math></p>	<p>Ind</p> <p>Series: <math>L_{eq} = L_1 + L_2</math></p> <p>parallel: <math>L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2}}</math></p>
--	--

General Equation:

$$x(t) = \text{Final} + [\text{Initial} - \text{Final}] e^{-\frac{(t-t_0)}{\tau}}$$

start time at switch (usually set to zero)

procedure:

1. Find Initial value on cap/inductor before switch ( $t=0^-$ )  
 $\{ i_L(t=0^-) \text{ OR } V_C(t=0^-) \}$

$\Rightarrow$  Note: voltage in C cannot change instantaneously

$$V_C(t=0^-) = V_C(t=0^+) \{ V \text{ src. } \}$$

$\Rightarrow$  Note: current in L cannot change instantaneously

$$i_L(t=0^-) = i_L(t=0^+) \{ I \text{ src. } \}$$

2. Find final value on C/L ( $t \rightarrow \infty$ )

$\Rightarrow$  Note: use  $V_C(t)=0$  (wire) ;  $i_C(t)=0$  (open)

3. Find  $\tau$  (use switch in FINAL position)

$\tau = R_{eq} C$  OR  $\tau = L/R_{eq}$ , where  $R_{eq}$  is R seen by element

4. Plug into general equation

Note: If finding other variables, relate to  $V_C(t)$  or  $i_L(t)$  eq.

# Derivation

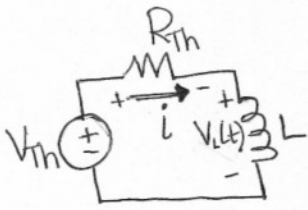
Recall #1

$$\int \frac{du}{au+b} = \frac{1}{a} \ln|au+b| + C$$

Recall #2

$$\ln a - \ln b = \ln\left(\frac{a}{b}\right)$$

(9/6)



V-loop  $\rightarrow$   
 $+V_{Th} - iR_{Th} - V_L(t) = 0$   
 $V_{Th} - iR_{Th} - L \frac{di(t)}{dt} = 0$   
 $L \frac{di(t)}{dt} = +V_{Th} - iR_{Th}$

$$\frac{di(t)}{dt} = \frac{-(iR_{Th} - V_{Th})}{L}$$

$$\int_{i(t_0)}^{i(t)} \frac{di(t)}{i(t)R_{Th} - V_{Th}} = \int_{t_0}^t -\frac{1}{L} dt$$

$$\frac{1}{R_{Th}} [\ln(R_{Th} i(t) - V_{Th}) - \ln(R_{Th} i(t_0) - V_{Th})] = \frac{-(t-t_0)}{L}$$

$$\ln \left[ \frac{R_{Th} i(t) - V_{Th}}{R_{Th} i(t_0) - V_{Th}} \right] = \frac{-(t-t_0) R_{Th}}{L}$$

$$R_{Th} i(t) - V_{Th} = (R_{Th} i(t_0) - V_{Th}) e^{-\frac{(t-t_0) R_{Th}}{L}}$$

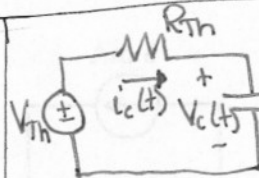
$$i(t) = \frac{V_{Th}}{R_{Th}} + \left( i(t_0) - \frac{V_{Th}}{R_{Th}} \right) e^{-\frac{(t-t_0) R_{Th}}{L}}$$

Note: after a long time ( $t \rightarrow \infty$ )  
 $V_{Th}$  = constant and so  $V_L(t \rightarrow \infty) = 0$  (wire)  
 so  $i(t \rightarrow \infty) = \frac{V_{Th}}{R_{Th}}$  (final value)

$i(t_0)$  = initial value

$$\tau = \frac{L}{R_{Th}}$$

$$i(t) = \text{Final} + (\text{initial} - \text{Final}) e^{-\frac{(t-t_0)}{\tau}}$$



$$+V_{Th} - i_c(t) \cdot R_{Th} - V_c(t) = 0$$

$$V_{Th} - C \frac{dV_c(t)}{dt} \cdot R_{Th} - V_c(t) = 0$$

$$\frac{dV_c(t)}{dt} = \frac{V_{Th} - V_c(t)}{R_{Th} C}$$

$$\int_{V_c(t_0)}^{V_c(t)} \frac{dV_c(t)}{V_c(t) - V_{Th}} = \int_{t_0}^t \frac{-1}{R_{Th} C} dt$$

$$\ln(V_c(t) - V_{Th}) - \ln(V_c(t_0) - V_{Th}) = \frac{-(t-t_0)}{R_{Th} C}$$

$$\ln \left[ \frac{V_c(t) - V_{Th}}{V_c(t_0) - V_{Th}} \right] = \frac{-(t-t_0)}{R_{Th} C}$$

$$V_c(t) - V_{Th} = (V_c(t_0) - V_{Th}) e^{-\frac{(t-t_0)}{R_{Th} C}}$$

$$V_c(t) = V_{Th} + (V_c(t_0) - V_{Th}) e^{-\frac{(t-t_0)}{R_{Th} C}}$$

Note: After a long time ( $t \rightarrow \infty$ )

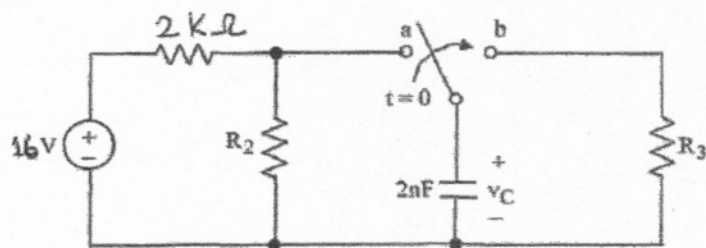
$V_{Th}$  = constant and so  $i_c(t) = 0$  (open)  
 so  $V_c(t \rightarrow \infty) = V_{Th}$  (final value)

$V_c(t_0)$  = initial value

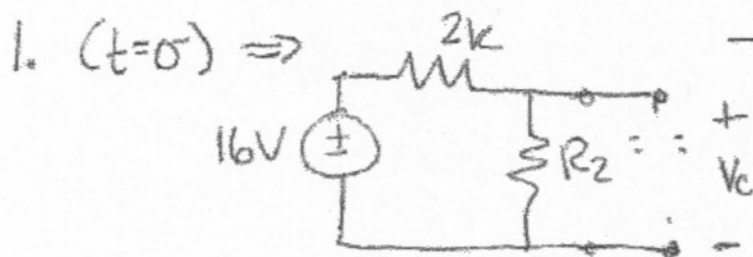
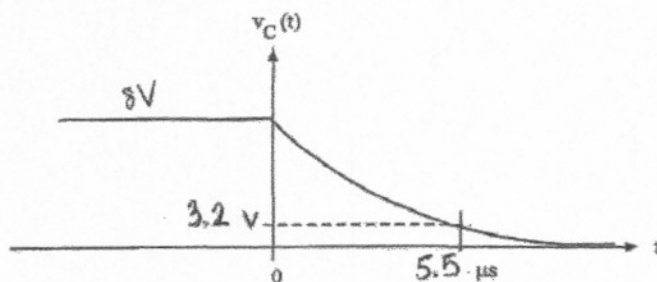
$$\tau = R_{Th} \cdot C$$

$$\therefore V_c(t) = \text{Final} + [\text{initial} - \text{Final}] e^{-\frac{(t-t_0)}{\tau}}$$

1.



After being in position a for a long time, the switch moves to position b at  $t = 0$ . Find  $R_2$  and  $R_3$  that give the following plot for  $v_C(t)$ :



$$v_C = \frac{16(R_2)}{R_2 + 2k}$$

( $t=0^+$ )  $\Rightarrow$  because cap v can not change instantaneously

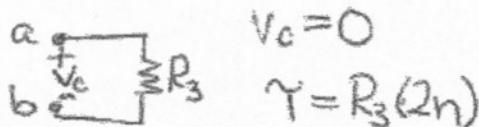
$$v_C(t=0^-) = v_C(t=0^+) = \frac{16(R_2)}{R_2 + 2k}$$

• from graph  $\rightarrow$   
 $v_C(t=0^+) = 8V$

$$8 = \frac{16(R_2)}{R_2 + 2k} \Rightarrow 8R_2 + 16k = 16R_2 - 8R_2$$

$$\boxed{R_2 = 2k\Omega}$$

( $t=\infty$ ):



$$v_C = 0$$

$$\tau = R_3(2n)$$

$$v_C(t) = 8 + (0 - 8)(1 - e^{-t/R_3(2n)}) = +8e^{-t/R_3(2n)}$$

from graph  $v_C(5.5\mu) = 3.2V$

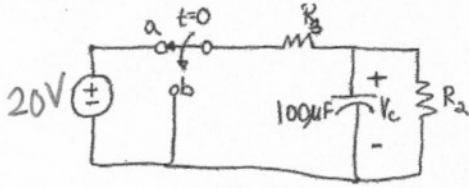
$$3.2 = 8e^{-5.5\mu/R_3(2n)}$$

$$\ln \frac{3.2}{8} = \frac{-5.5\mu}{R_3(2n)} \Rightarrow R_3 = \frac{-5.5\mu}{2n(\ln(\frac{3.2}{8}))} \approx \boxed{3k\Omega}$$

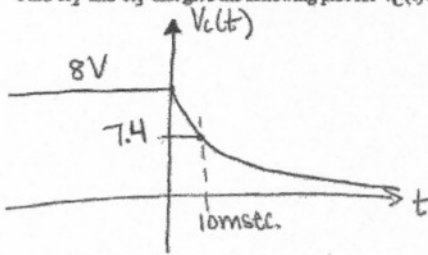
# Homework #5 Example

98

1.

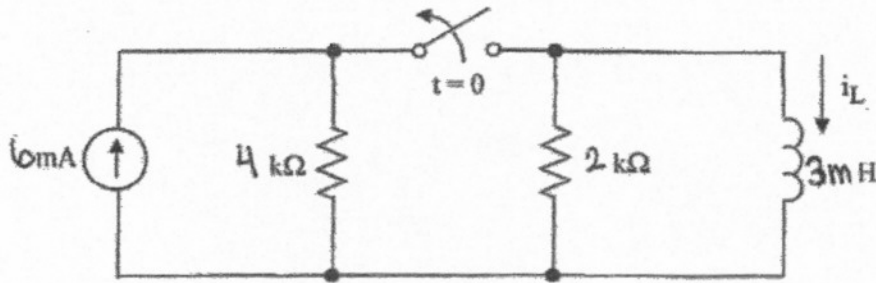


After being in position a for a long time, the switch moves to position b at  $t = 0$ . Find  $R_2$  and  $R_3$  that give the following plot for  $v_C(t)$ :

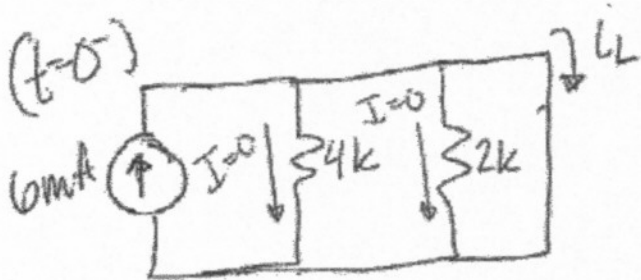


Homework #5 Examples

2.

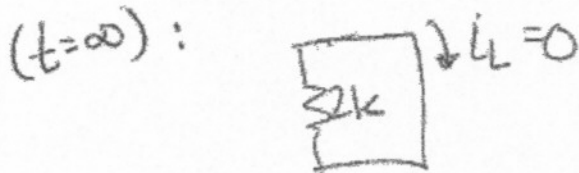


After being closed for a long time, the switch opens at  $t = 0$ . Find  $i_L(t)$  for  $t > 0$ .



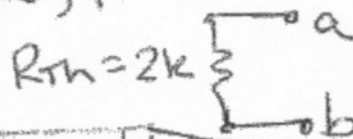
$i_L = 6\text{mA}$

$(t=0^+)$ : • inductor current remains the same  
 $i_L(t=0^-) = i_L(t=0^+) = 6\text{mA}$  ← (initial value)



$i_L(t) = 6\text{mA} + (0 - 6\text{mA})(1 - e^{-t/\tau})$

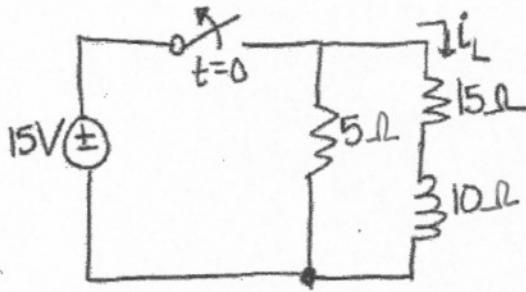
(switch at  $t=\infty$ ):



$i_L(t) = +6\text{mA} e^{-t/3\text{m}/2\text{k}} \text{ A}$

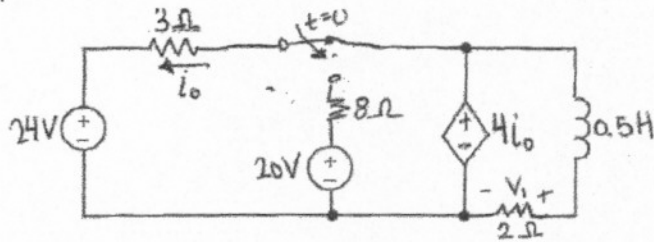


2.

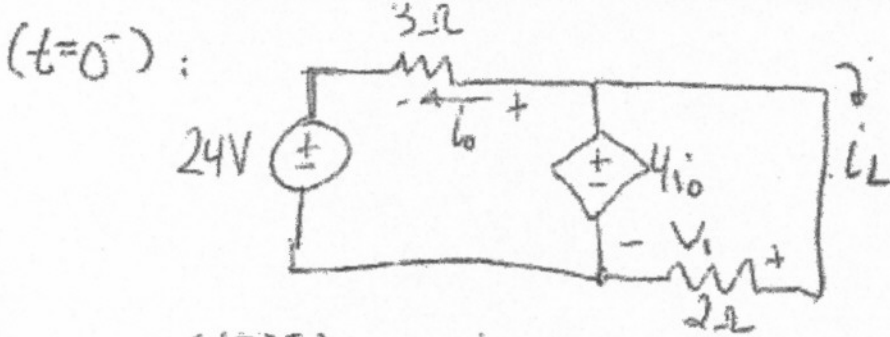


After being closed for a long time, the switch opens at  $t=0$ . Find  $i_L(t)$  for  $t > 0$ .

3.



After being in the above position for a long time, the switch changes at  $t=0$ . Find  $v_1(t)$  for  $t > 0$ .



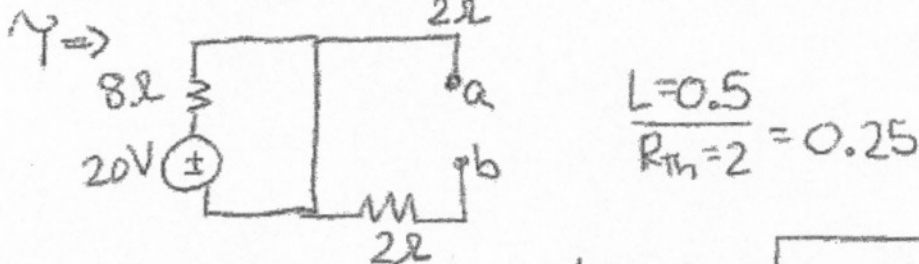
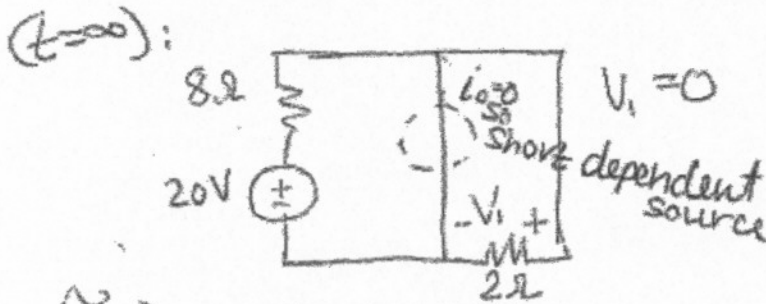
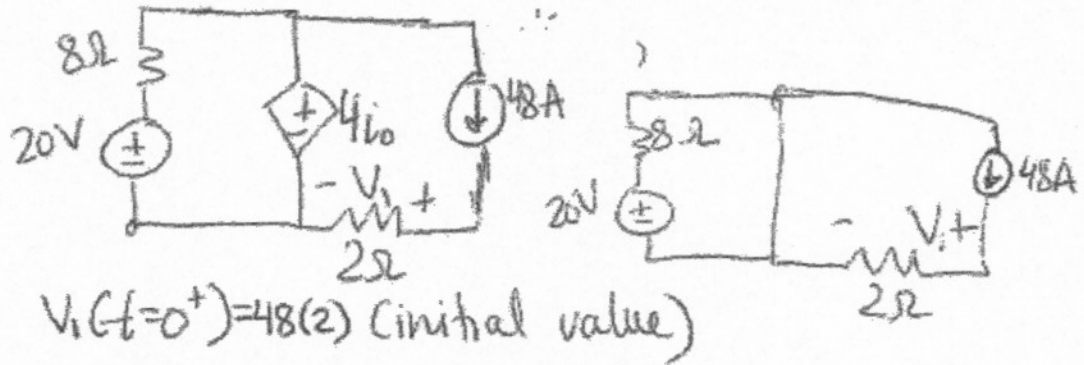
$$+4i_o - 3i_o - 24 = 0$$

$$i_o = +24V/\Omega$$

$$i_L = \frac{4i_o}{2} = \frac{4(24)}{2} = 48A$$

$(t=0^+)$ : Inductor current stays the same  $\Rightarrow$   
 $i_L(t=0^-) = i_L(t=0^+) = 48A$  ← initial value

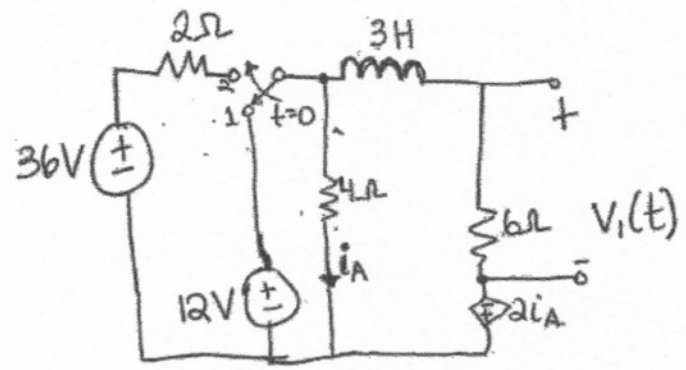
Note that  $v_1$  changes from  $t=0^-$  to  $t=0^+$ .



$$v_1(t) = 96 + (0 - 96)e^{-t/0.25} = \boxed{96e^{-4t} V}$$

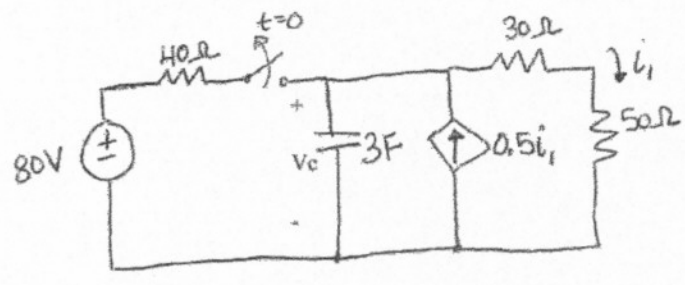
Homework #5 Example

3.



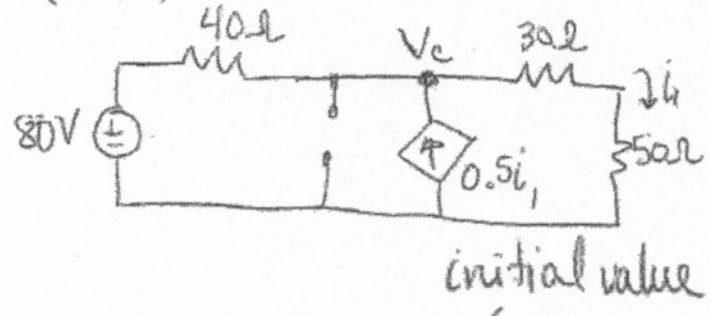
After being at position 1 for a long time, the switch moves to position 2 at  $t=0$ . Find  $v_1(t)$  for  $t > 0$ .

4.



After being closed for a long time, the switch opens at  $t=0$ . Find  $V_c(t)$  for  $t > 0$ .

$(t=0^-)$



$$\frac{V_c - 80}{40} - 0.5i_1 + \frac{V_c}{80} = 0$$

$$i_1 = \frac{V_c}{80}$$

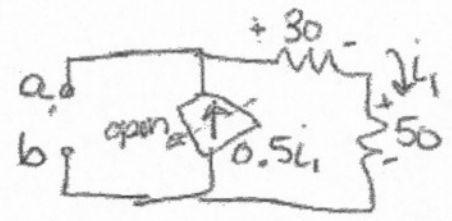
$$\frac{V_c}{40} - 0.5\left(\frac{V_c}{80}\right) + \frac{V_c}{80} = \frac{80}{40}$$

$64V = V_c(t=0^+)$

$$V_c \left( \frac{4}{160} - \frac{1}{160} + \frac{2}{160} \right) = \frac{80}{40}$$

$$V_c = 2 \frac{1}{\left(\frac{5}{160}\right)} = \frac{2(160)}{5}$$

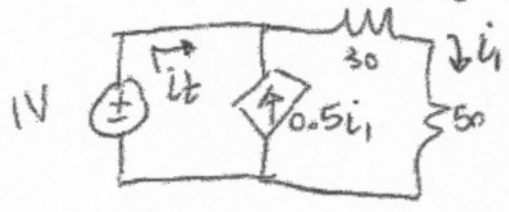
$(t \rightarrow \infty)$



$$0.5i_1 - i_1 = 0 \Rightarrow i_1 = 0$$

$$V_c = 0V \leftarrow \text{final}$$

Need to use a test supply since  $V_{Th} = 0$  and  $R_{Th} \neq 0$  or the capacitor will not discharge. It must though since  $V_c = 0$  as a final value and it started with 64V initially.



$$i_t + 0.5i_1 - i_1 = 0$$

$$i_t = 0.5i_1$$

$$i_1 = \frac{1}{80}$$

$$i_t = 0.5 \left( \frac{1}{80} \right) = \frac{1}{160}$$

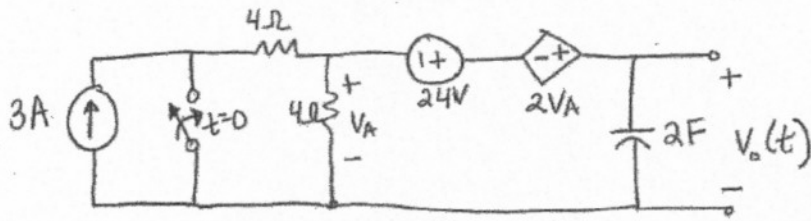
$$R_{Th} = \frac{1V}{\frac{1}{160}} = 160\Omega$$

$$V_c(t) = 64e^{-t/160} V$$

# Homework #5 Example

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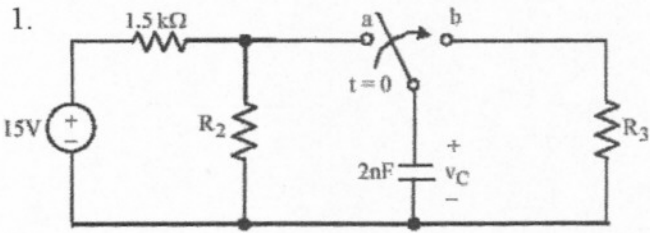
4.



After being open for a long time, the switch closes at  $t=0$ . Find  $V_o(t)$  for  $t > 0$ .

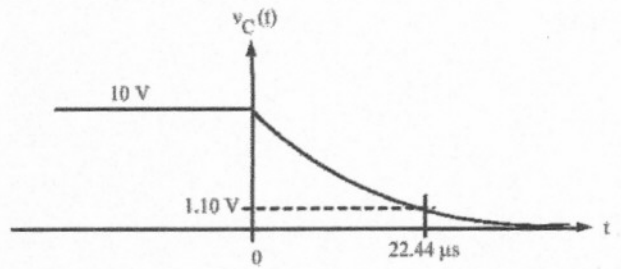
# RC/RL Examples

105

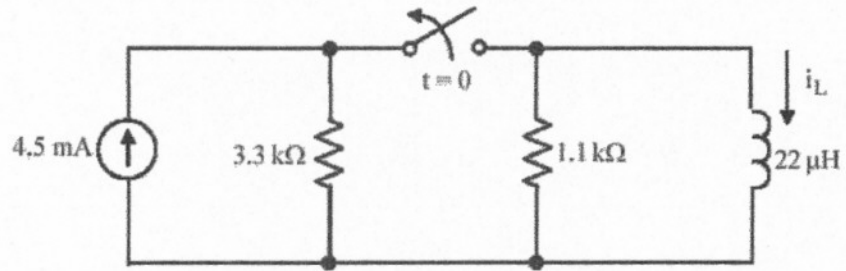


After being in position **a** for a long time, the switch moves to position **b** at  $t = 0$ .

Find  $R_2$  and  $R_3$  that give the following plot for  $v_C(t)$ :

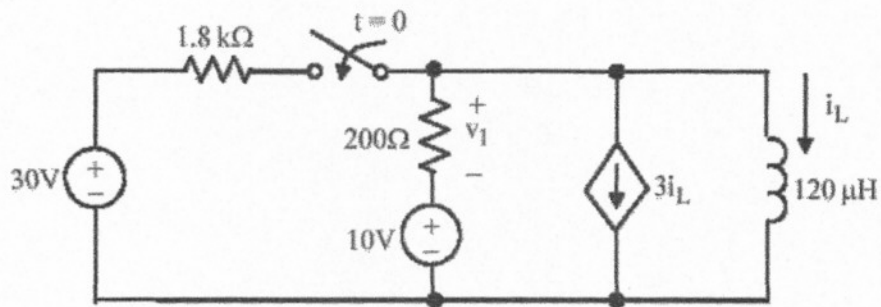


2.



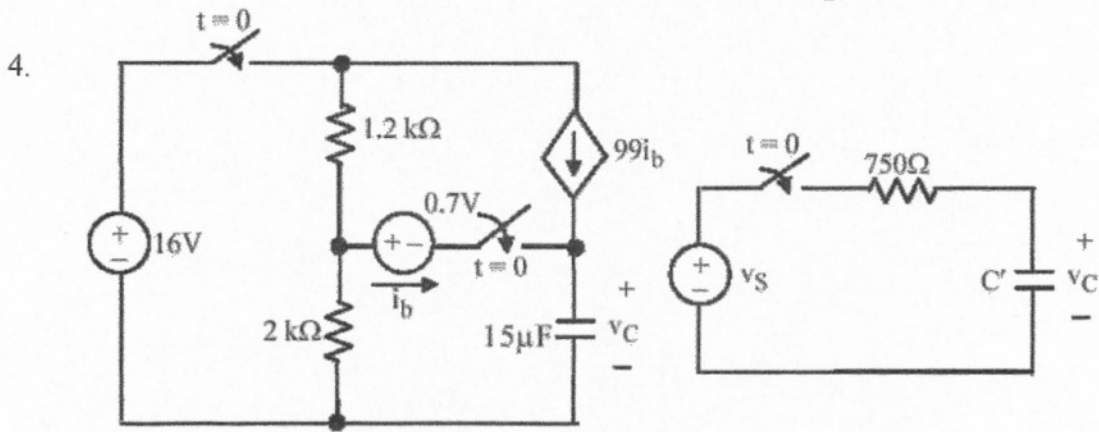
After being closed for a long time, the switch opens at  $t = 0$ . Find  $i_L(t)$  for  $t > 0$ .

3.



After being open for a long time, the switch closes at  $t = 0$ . Find  $v_1(t)$  for  $t > 0$ .



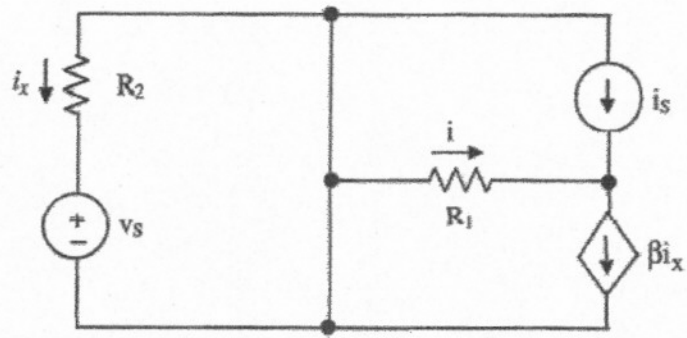


For both circuits, the switches close at  $t = 0$  after being open for a long time.

- Find  $v_C(t)$  for  $t > 0$  for the circuit on the left.
- Find  $v_s$  and  $C'$  values that make  $v_C$  for the circuit on the right equal  $v_C$  for the circuit on the left. Assume  $v_C(0^-) = 0\text{ V}$ .

# Superposition Example

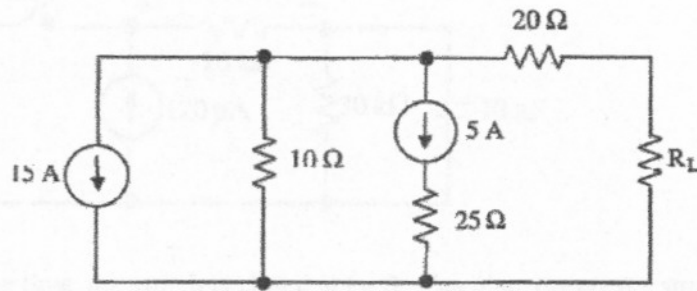
109



Using superposition, derive an expression for  $i$  that contains no circuit quantities other than  $i_s$ ,  $v_s$ ,  $R_1$ ,  $R_2$ , and  $\beta$ .

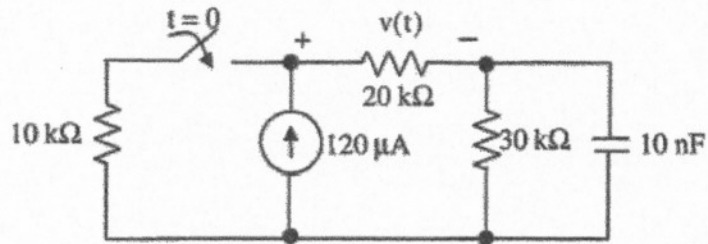
# Max Power Example

110



Calculate the value of  $R_L$  that would absorb maximum power, and calculate that value of maximum power  $R_L$  could absorb.

# Energy Example

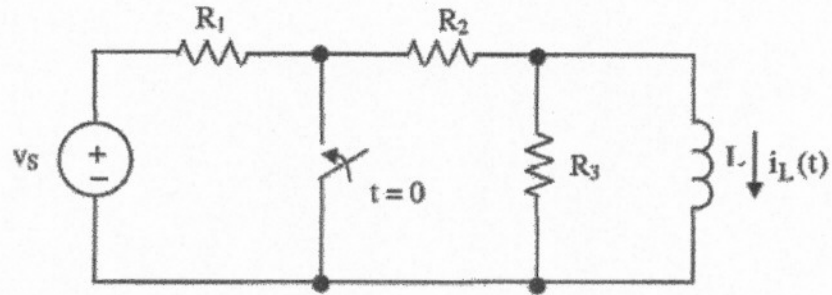


After being open for a long time, the switch is closed at  $t = 0$ . Calculate the energy stored on the capacitor at  $t \rightarrow \infty$

Write a numerical expression for  $v(t)$ ,  $t > 0$ .

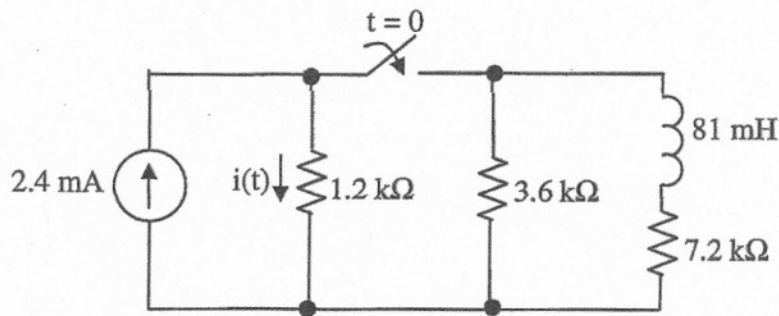
# Switch RL Example

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After being open for a long time, the switch is closed at  $t = 0$ . Write an expression for  $i_L(t)$ ,  $t > 0$ .

1.



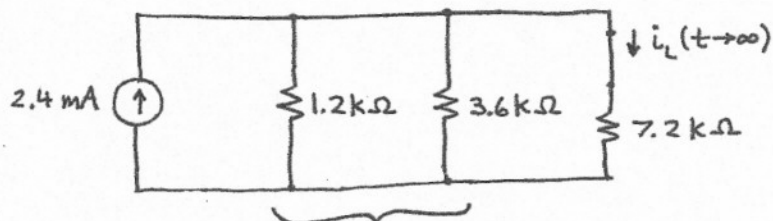
After being open for a long time, the switch is closed at  $t = 0$ .

Calculate the energy stored on the inductor at  $t \rightarrow \infty$ .

2. Write a numerical expression for  $i(t)$ ,  $t > 0$ .

Sol'n: 1. For  $t \rightarrow \infty$ , L acts like wire, switch is closed.

Find  $i_L(t \rightarrow \infty)$ . Energy,  $w = \frac{1}{2} L i_L^2$ .



$$R_{eq} = 1.2 \text{ k}\Omega \parallel 3.6 \text{ k}\Omega = 1.2 \text{ k}\Omega \cdot \frac{1}{1+3} = 1.2 \text{ k}\Omega \cdot \frac{3}{4}$$

$$R_{eq} = 0.9 \text{ k}\Omega$$

We have current divider.

$$i_L(t \rightarrow \infty) = 2.4 \text{ mA} \cdot \frac{R_{eq}}{R_{eq} + 7.2 \text{ k}\Omega} = 2.4 \text{ mA} \frac{0.9 \text{ k}\Omega}{8.1 \text{ k}\Omega}$$

$$i_L(t \rightarrow \infty) = \frac{2.4 \text{ mA}}{9} = \frac{24}{90} \text{ mA} = \frac{12}{45} \text{ mA} = \frac{4}{15} \text{ mA}$$

$$w = \frac{1}{2} 81 \text{ mH} \left( \frac{4}{15} \right)^2 \text{ mA}^2 = \frac{1}{2} 81 \cdot \frac{16}{25 \cdot 9} \text{ nJ}$$

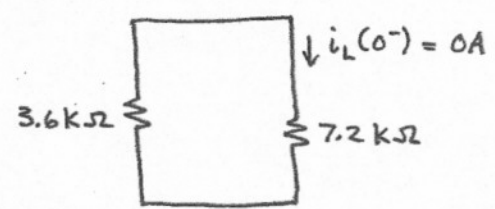
$$w = \frac{9(8)}{25} \text{ nJ} = \frac{72 \cdot 40}{25 \cdot 40} \text{ nJ} = 2.88 \text{ nJ}$$

$$w_L(t \rightarrow \infty) = 2.88 \text{ nJ}$$

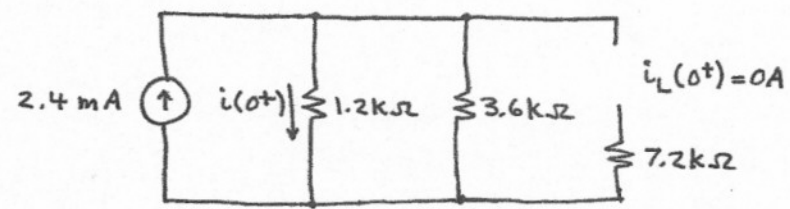
sol'n: 2.

We want to find  $i(t=0^+)$ . We start by finding  $i_L(t=0^-)$  so we will know  $i_L(t=0^+)$ , since  $i_L(t=0^+) = i_L(t=0^-)$ .

$t=0^-$ : L acts like wire, switch is open.  
No pwr src for L, so L discharges to give  $i_L(0^-) = 0A$ .



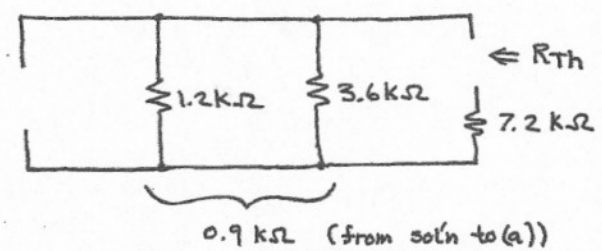
$t=0^+$ : L acts like current src of value  $i_L(0^+) = i_L(0^-) = 0A$  = open circuit  
Switch is closed.



This is a current divider.

$$i(0^+) = 2.4mA \cdot \frac{3.6k\Omega}{1.2k\Omega + 3.6k\Omega} = 1.8mA$$

For time constant  $\frac{L}{R_{Th}}$ , we look into the terminals where L is connected. We turn off independent 2.4mA src which becomes open circuit.



sol'n: 2. cont.

$$R_{Th} = 7.2k\Omega + 0.9k\Omega = 8.1k\Omega$$

Our time constant is  $\frac{L}{R_{Th}} = \frac{81mH}{8.1k\Omega} = 10\mu s$ .

Find  $i(t \rightarrow \infty)$  [see below],

Plug values into general solution.

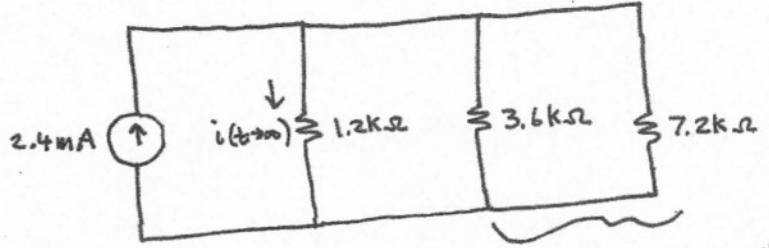
$$i(t > 0) = i(t \rightarrow \infty) + [i(0^+) - i(t \rightarrow \infty)] e^{-t/\frac{L}{R_{Th}}}$$

$$i(t > 0) = 1.6mA + [1.8mA - 1.6mA] e^{-t/10\mu s}$$

$$i(t > 0) = 1.6mA + 0.2mA e^{-t/10\mu s}$$

$t \rightarrow \infty$ : L acts like wire, switch closed

Find  $i(t \rightarrow \infty)$  (not  $i_L$ ).



$$3.6k \parallel 7.2k\Omega = 3.6k\Omega \cdot \frac{1}{2}$$

$$= 3.6k\Omega \cdot \frac{2}{3}$$

$$= 2.4k\Omega$$

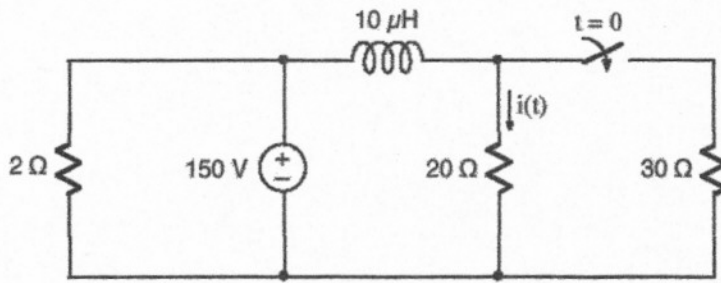
i-divider:  $i(t \rightarrow \infty) = 2.4mA \cdot \frac{2.4k\Omega}{2.4k + 1.2k\Omega} = 1.6mA$



# Homework #6/Review Examples

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1.

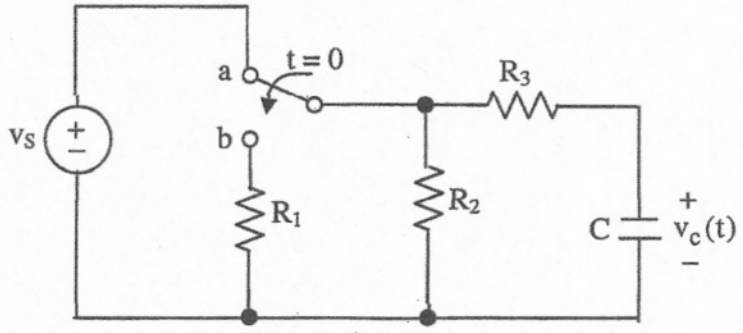


After being closed for a long time, the switch closes at  $t = 0$ .

Calculate the energy stored on the inductor as  $t \rightarrow \infty$ .

find  $i(t)$  for  $t > 0$ .

3.

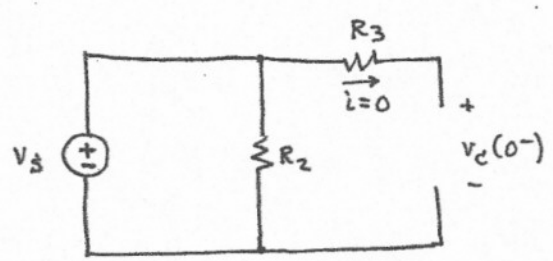


After being at position a for a long time, the switch moves to position b at time  $t=0$ .

- a. Write an expression for  $v_c(t=0^+)$ .
- b. Write an expression for  $v_c(t), t > 0$ .

sol'n: a)  $v_c(t=0^+) = v_c(t=0^-)$

At  $t=0^-$ , switch is in position a. C acts like open circuit.



No current in  $R_3$  means no v-drop for  $R_3$ .

$\therefore v_c(0^-) = v_s$

$v_c(0^+) = v_s$

- b) Find  $v_c(t \rightarrow \infty)$  and  $R_{TH}C =$  time constant.

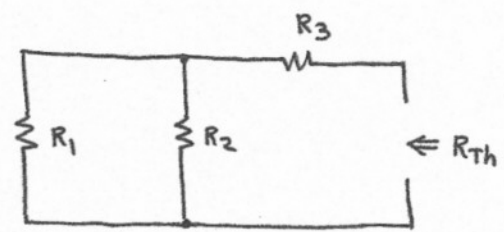
$t \rightarrow \infty$ : Switch is in position b. No pwr src.

C discharges thru R's.

$\therefore v_c(t \rightarrow \infty) = 0V$

$R_{TH}$  is resistance seen looking into terminals where C connected with switch in position b.

sol'n: 3.b) cont.



$$R_{Th} = (R_1 \parallel R_2) + R_3$$

Now plug values into general sol'n.

$$v_c(t > 0) = v_c(t \rightarrow \infty) + [v_c(0^+) - v_c(t \rightarrow \infty)] e^{-t/R_{Th}C}$$

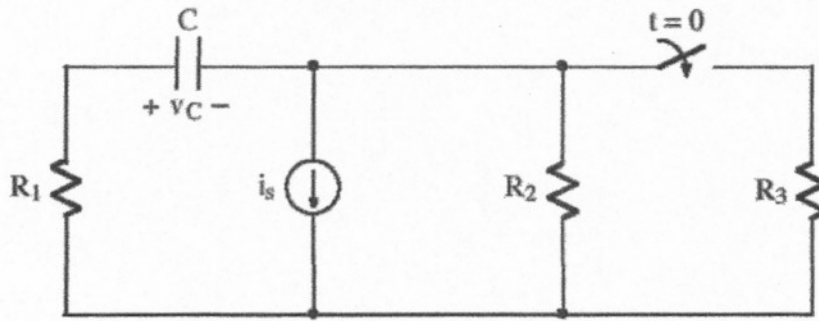
$$v_c(t > 0) = 0V + [V_s - 0V] e^{-t/(R_1 \parallel R_2 + R_3)C}$$

$$v_c(t > 0) = V_s e^{-t/(R_1 \parallel R_2 + R_3)C}$$

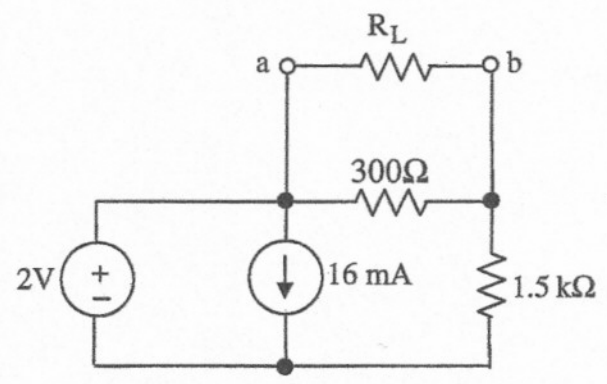
# Homework #6/Review Examples

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2.



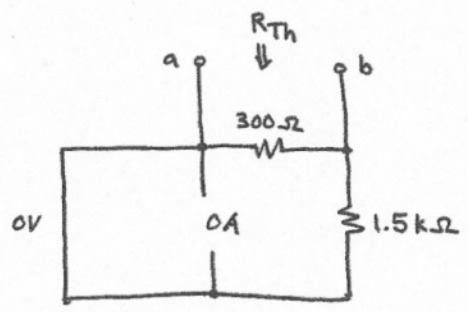
After being open for a long time, the switch closes at  $t = 0$ . Write an expression for  $v_C(t \geq 0)$  in terms of  $R_1, R_2, R_3, i_s$ , and  $C$ .



- a. Calculate the value of  $R_L$  that would absorb maximum power.
- b. Calculate that value of maximum power  $R_L$  could absorb.

sol'n: a)  $R_L = R_{Th}$  yields max pwr transfer

We find  $R_{Th}$  by removing  $R_L$ , turning independent src's off, and seeing what resistance we have looking into terminals a,b.



$$R_{Th} = 300\Omega \parallel 1.5k\Omega = 300\Omega \cdot \frac{1}{1 + 5} = 300\Omega \cdot \frac{5}{6}$$

$$R_{Th} = 250\Omega$$

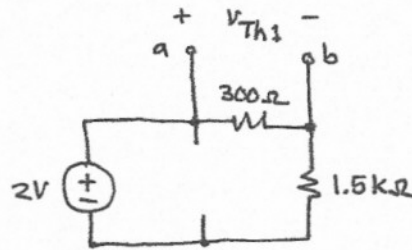
$$\therefore R_L = 250\Omega$$

sol'n: 4.b) Find Thevenin equivalent of circuit where  $R_L$  connected.

$$V_{Th} = V_{a,b} \text{ with } R_L \text{ removed.}$$

Use superposition (or other method such as node-voltage) to find  $V_{Th}$ .

case I: 2V src on, 16mA src off (open circuit)

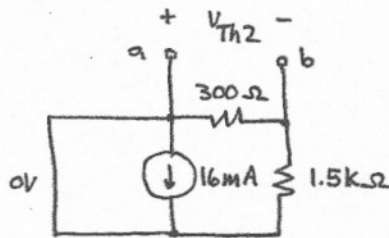


We have v-divider.

$$V_{Th1} = 2V \cdot \frac{300\Omega}{300\Omega + 1.5k\Omega}$$

$$V_{Th1} = \frac{2V}{6}$$

case II: 2V src off, 16mA src on  
= wire



We have i-divider.

All current flows thru wire.

$$\therefore V_{Th2} = 0V$$

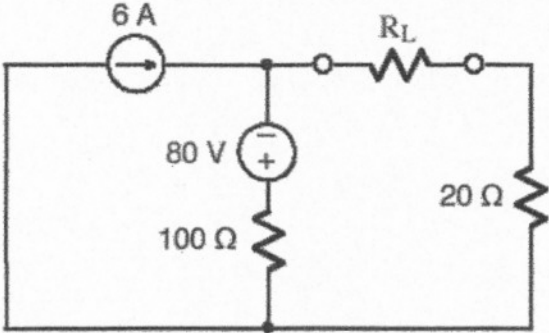
$$V_{Th} = V_{Th1} + V_{Th2} = \frac{2V}{6} + 0V = \frac{2V}{6} = \frac{1}{3}V$$

$$\max \text{ pwr} = \frac{V_{Th}^2}{4R_{Th}} = \frac{\left(\frac{1}{3}V\right)^2}{4 \cdot 250\Omega} = \frac{1}{9} \text{ mW}$$

$$\boxed{\max \text{ pwr} = \frac{1}{9} \text{ mW}}$$

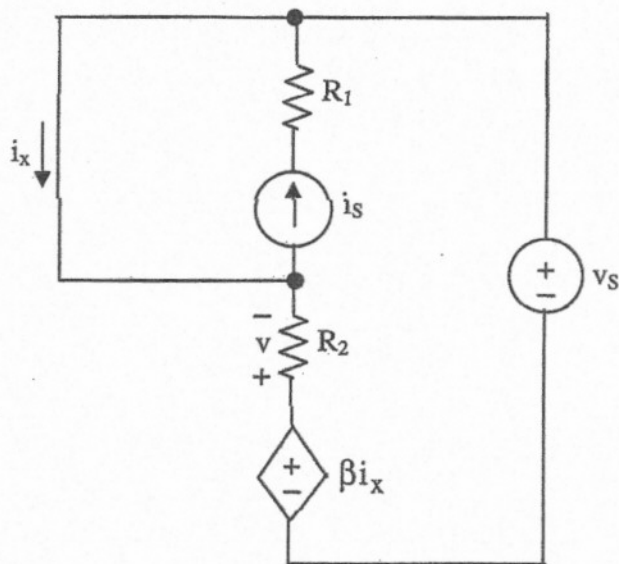
Homework #6/Review Examples

3.



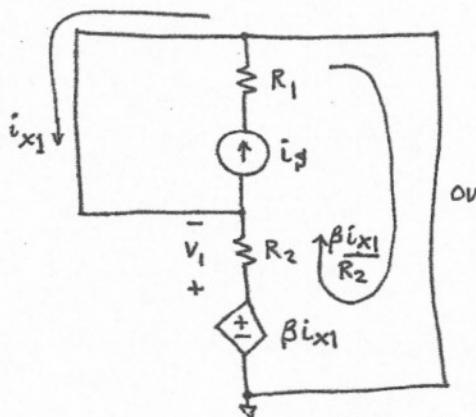
- a) Calculate the value of  $R_L$  that would absorb maximum power.
- b) Calculate that value of maximum power  $R_L$  could absorb.

5.



Using superposition, derive an expression for  $v$  that contains no circuit quantities other than  $i_s$ ,  $v_s$ ,  $R_1$ ,  $R_2$ , and  $\beta$ . Note:  $\beta > 0$ .

sol'n: case I:  $i_s$  on,  $v_s$  off = wire



$v_1 = \beta i_{x1}$  from outer  $v$  loop

Current summation at center node gives

$$-i_s + i_{x1} + \frac{\beta i_{x1}}{R_2} = 0A$$

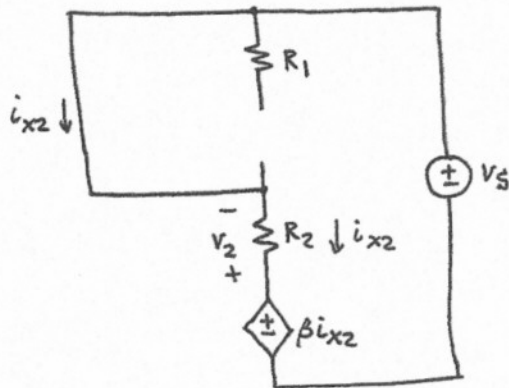
$$\text{or } i_{x1} \left(1 + \frac{\beta}{R_2}\right) = i_s$$

$$i_{x1} = \frac{i_s}{1 + \frac{\beta}{R_2}} = i_s \cdot \frac{R_2}{R_2 + \beta}$$

$$v_1 = \beta i_{x1} = \beta i_s \frac{R_2}{R_2 + \beta} = i_s \cdot R_2 \parallel \beta$$



sol'n: 5. cont. case II:  $i_s$  off,  $v_s$  on  
= open



$v$  loop around outside gives

$$\beta i_{x2} - i_{x2} \cdot R_2 - v_s = 0V$$

$$i_{x2} (\beta + R_2) = v_s$$

$$i_{x2} = \frac{v_s}{\beta + R_2}$$

$$v_2 = -i_{x2} R_2 = -v_s \frac{R_2}{\beta + R_2} = -v_s \frac{R_2}{R_2 + \beta}$$

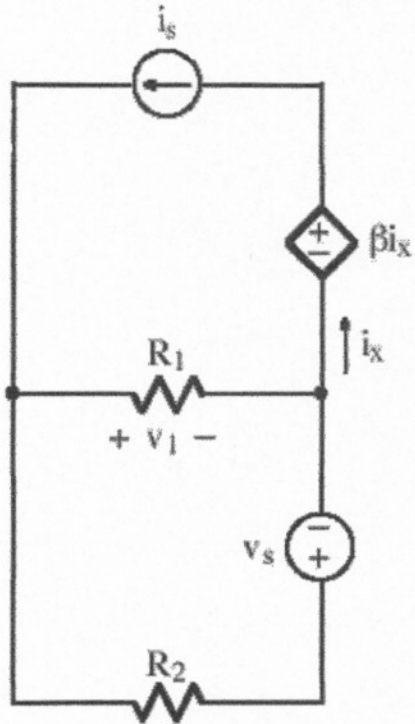
$$v = v_1 + v_2 = \beta i_s \frac{R_2}{R_2 + \beta} - v_s \frac{R_2}{R_2 + \beta} = (\beta i_s - v_s) \frac{R_2}{R_2 + \beta}$$

$$v = (\beta i_s - v_s) \frac{R_2}{R_2 + \beta}$$

Homework #6/Review Examples

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4.

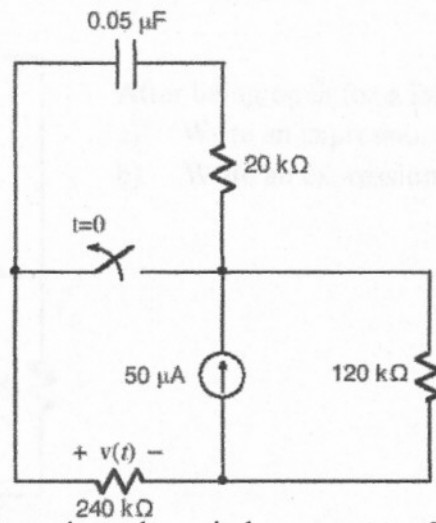


Using superposition, derive an expression for  $v_1$  that contains no circuit quantities other than  $i_s$ ,  $v_s$ ,  $R_1$ ,  $R_2$ , and  $\beta$ , where  $\beta > 0$ .

# Homework #6/Review Examples

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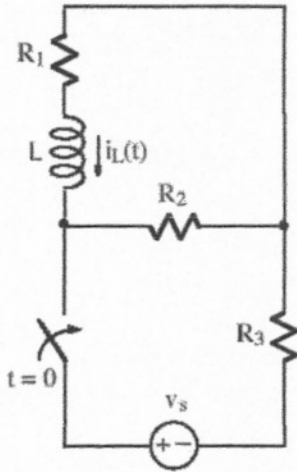
1.



After being closed for a long time, the switch opens at  $t = 0$ .

- a) Calculate the energy stored on the capacitor at  $t = 0^+$ .
- b) Write a numerical expression for  $v(t)$  for  $t > 0$ .

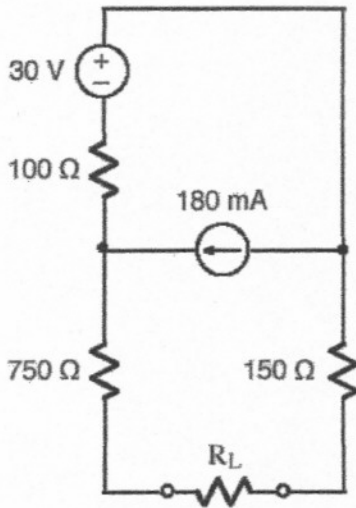
2.



After being open for a long time, the switch closes at  $t = 0$ .

- Write an expression for  $i_L(t = 0^+)$
- Write an expression for  $i_L(t > 0)$  in terms of  $R_1$ ,  $R_2$ ,  $R_3$ ,  $v_s$ , and  $L$ .

3.

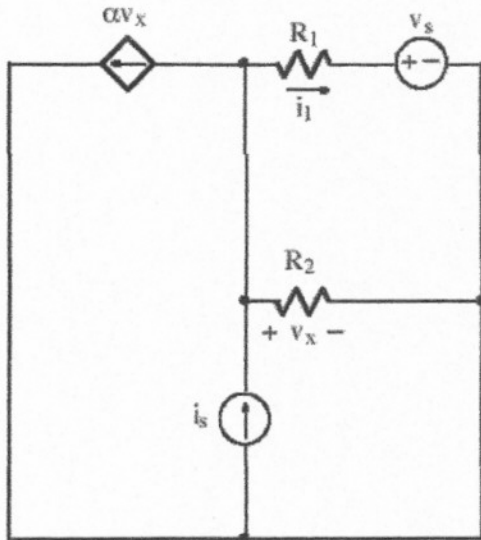


- a) Calculate the value of  $R_L$  that would absorb maximum power.
- b) Calculate that value of maximum power  $R_L$  could absorb.

# Homework #6/Review Examples

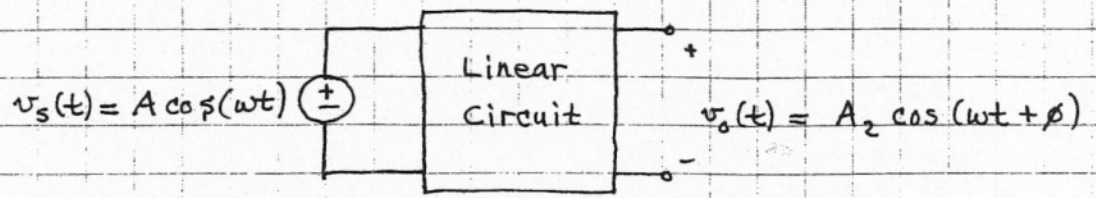
129

4.



Using superposition, derive an expression for  $i_1$  that contains no circuit quantities other than  $i_s$ ,  $v_s$ ,  $R_1$ ,  $R_2$ , and  $\alpha$ , where  $\alpha > 0$ .

### Sinusoidal Signals



- input sinusoid, output also sinusoid!  
freq unchanged!

amplitude & phase shift change  
calc. these

- Can write arbitrary input (periodic) as sum of sinusoids of different frequencies

### Imaginary #'s

What is j ?

9  
8  
7  
6  
5  
4  
3  
2  
1  
0  
4

0) j allows to find roots of all polynomials

1)  $j = \sqrt{-1}$

2) j is a unit vector  $\perp$  to real axis

3) j defines rule for multiplying vecs

4)  $j = 90^\circ$  rotation

5) j allows us to use phasors for  $A \cos(\omega t + \phi)$

6) j is a token placed in front of  $\sin()$  to make  $\cos()$

7) j allows us to write  $\cos(\omega t + \phi)$  as  $a + jb = \cos + j \sin$   
w/ no phase shift

8) j allows us to use complex exp for polar form  $A e^{j\phi}$  draw rect pict

9) j results from taking  $\frac{d}{dt} e^{j\phi(t)} = j e^{j\phi(t)} \frac{d\phi(t)}{dt}$  L's and C's

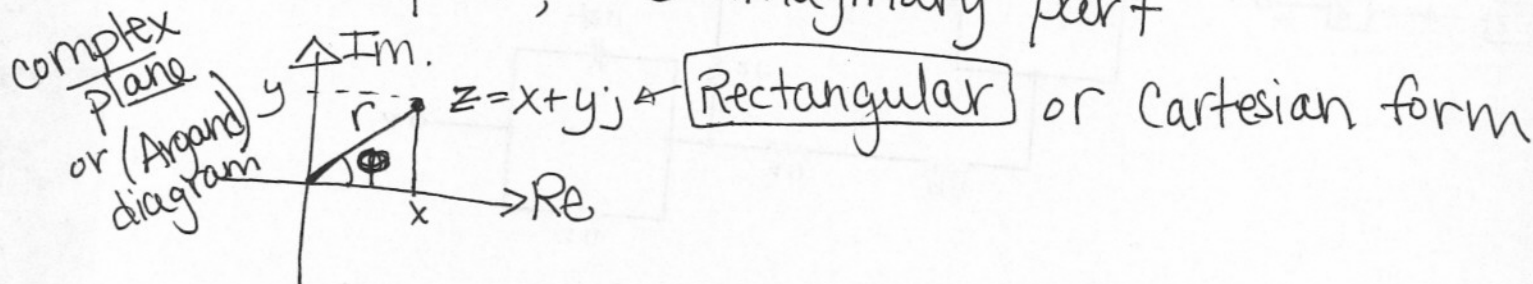
10) j allows us to write  $\cos = \frac{e^{j\phi} + e^{-j\phi}}{2}$

Complex #'s:

$a + bj$  ← used so that current ( $i$  symbol) not get confused

(131)

$a$  = real part,  $b$  = imaginary part



polar form exponential form

$$z = r e^{i\phi} \quad \text{where } \phi = \begin{cases} \tan^{-1}\left(\frac{y}{x}\right) & x > 0 \\ \tan^{-1}\left(\frac{y}{x}\right) + \pi & x < 0, y \geq 0 \\ \tan^{-1}\left(\frac{y}{x}\right) - \pi & x < 0, y < 0 \\ +\pi/2 & x = 0, y > 0 \\ -\pi/2 & x = 0, y < 0 \end{cases}$$

$$r = \sqrt{x^2 + y^2}$$

trigonometric form:

$$z = r(\cos \phi + j \sin \phi)$$

Suggestions  $\Rightarrow$

add/sub. using rect. form  $\Rightarrow a + bj + (c + dj) = (a+c) + (b+d)j$

mult./div. " exponential form

$$(r_1 e^{i\phi_1}) * (r_2 e^{i\phi_2}) = r_1 \cdot r_2 e^{i(\phi_1 + \phi_2)}$$



**EX:** Express  $3e^{j32^\circ} + 4e^{-j40^\circ}$  in rectangular and polar form.

**ANS:**  $5.608 - j0.981$  or  $5.693e^{-j9.922^\circ}$  (approx)

**SOL'N:** Use Euler's formula to write each complex number in rectangular form  $a + jb$ :

$$Ae^{j\phi} = A \cos \phi + jA \sin \phi$$

$$3e^{j32^\circ} = 3 \cos(32^\circ) + j3 \sin(32^\circ) = 2.544 + j1.590$$

$$4e^{-j40^\circ} = 4 \cos(-40^\circ) + j4 \sin(-40^\circ) = 3.064 - j2.571$$

Sum the real and imaginary parts:

$$3e^{j32^\circ} + 4e^{-j40^\circ} = 2.544 + 3.064 + j(1.590 - 2.571)$$

Our answer in rectangular form:

$$3e^{j32^\circ} + 4e^{-j40^\circ} = 5.608 - j0.981$$

Use the Pythagorean theorem to find the magnitude for polar form:

$$\left| 3e^{j32^\circ} + 4e^{-j40^\circ} \right| = \sqrt{5.608^2 + 0.981^2} = 5.693$$

Use tangent of phase angle = Im/Re to find angle for polar form:

$$\angle(3e^{j32^\circ} + 4e^{-j40^\circ}) = \tan^{-1}\left(\frac{-0.981}{5.608}\right) = -9.922^\circ$$

Our answer in polar form:

$$3e^{j32^\circ} + 4e^{-j40^\circ} = 5.693e^{-9.922^\circ}$$

EX: Rationalize  $\frac{3 + j1.5}{5 - j2.6}$

ANS:  $0.35 + j0.48$

SOL'N: Multiply the numerator and denominator by the denominator's complex conjugate. This turns the denominator into a real number equal to the magnitude squared of the original denominator.

$$\begin{aligned} \frac{3 + j1.5}{5 - j2.6} &= \frac{3 + j1.5}{5 - j2.6} \frac{5 + j2.6}{5 + j2.6} \\ &= \frac{(3 + j1.5)(5 + j2.6)}{5^2 + 2.6^2} \\ &= \frac{15 - 3.9 + j(7.8 + 7.5)}{31.76} \\ &= 0.35 + j0.48 \end{aligned}$$

OR

$$\begin{aligned} \frac{3 + j(1.5)}{5 - j2.6} &= \frac{\sqrt{9 + 1.5^2} e^{j \tan^{-1}\left(\frac{1.5}{3}\right)}}{\sqrt{25 + 2.6^2} e^{-j \tan^{-1}\left(\frac{2.6}{5}\right)}} \\ &= \frac{3.354}{5.636} e^{j(26.6^\circ + 27.5^\circ)} = 0.6 e^{j(54.1^\circ)} \\ &= 0.6 \cos(54.1^\circ) + 0.6 \sin(54.1^\circ)j \\ &= 0.35 + 0.49j \end{aligned}$$

EX: Evaluate  $\frac{4 + j3}{2 - j6}$

ANS:  $-\frac{1}{4} + j\frac{3}{4}$

SOL'N: Multiply the numerator and denominator by the denominator's complex conjugate. This turns the denominator into a real number equal to the magnitude squared of the original denominator. Divide the numerator's real and imaginary parts by this new denominator.

$$\begin{aligned} \frac{4 + j3}{2 - j6} &= \frac{4 + j3}{2 - j6} \frac{(2 + j6)^*}{(2 + j6)^*} \\ &= \frac{4 + j3}{2 - j6} \frac{2 + j6}{2 + j6} \\ &= \frac{8 - 18 + j(6 + 24)}{4 + 36} \\ &= \frac{-10 + j30}{40} \\ &= -\frac{1}{4} + j\frac{3}{4} \end{aligned}$$

OR

$$\begin{aligned} \frac{\sqrt{16+9} e^{j \tan^{-1}(\frac{3}{4})}}{\sqrt{4+36} e^{-j \tan^{-1}(\frac{6}{2})}} &= \frac{5}{6.32} e^{j(37^\circ + 72^\circ)} = 0.8 e^{j(109^\circ)} \\ &= +0.8 \cos(109^\circ) + 0.8 \sin(109^\circ)j = -26 + 0.76j \end{aligned}$$

**DEF:** Complex Conjugate of  $a + jb \equiv a - jb$   $\equiv$  complex number with imaginary part inverted

**NOT'N:**  $z^*$   $\equiv$  complex conjugate of  $z$

**TOOL:** To find the complex conjugate of an expression, change each  $j$  to  $-j$ .

**NOTE:** This is equivalent to (but easier than) converting the expression to form  $a + jb$  and changing it to  $a - jb$ .

**TOOL:**  $(Ae^{j\phi})^* = Ae^{-j\phi}$  when  $A$  is real

**TOOL:** To find  $z^*$ , reflect  $z$  around the real axis. In other words, preserve the magnitude but take the negative of the phase angle.

**EX:** Evaluate  $(2 + j3)^*$

**ANS:**  $2 - j3$

**SOL'N:** This is a direct application of the definition of complex conjugate.

**EX:** Evaluate  $(Re^{j\psi})^*$  where  $R$  is real

**ANS:**  $Re^{-j\psi}$

**SOL'N:** We retain the magnitude but invert the phase angle to find the conjugate of a complex number in polar form.

**EX:** Evaluate  $\left[ \frac{(6 - j2)e^{j35^\circ}}{(4 + j5)\sin(x)} \right]^*$  where  $x$  is real

**ANS:**  $\left[ \frac{(6 + j2)e^{-j35^\circ}}{(4 - j5)\sin(x)} \right]^*$

**SOL'N:** Change each  $j$  to  $-j$ . Thus,  $-j$  becomes  $-(-j) = j$ .

CONCEPTUAL TOOLS

By: Carl H. Durney and Neil E. Cotter

COMPLEX ANALYSIS  
BASIC MATH  
Re[]  
EXAMPLE 1

EX: Find  $\text{Re}[7e^{j25^\circ}]$ , (i.e., find the real part)

ANS:  $7\cos(25^\circ) \approx 6.34$

SOL'N: The answer follows directly from Euler's formula (see tools for rectangular and polar forms for complex numbers):

$$Ae^{j\phi} = A\cos\phi + jA\sin\phi$$

CONCEPTUAL TOOLS

By: Carl H. Durney and Neil E. Cotter

COMPLEX ANALYSIS  
BASIC MATH  
Magnitude  
EXAMPLE 1

EX: Find  $|2e^{j182^\circ}|$ , (i.e., find the magnitude)

ANS: 2

SOL'N: The magnitude of a product is the product of the magnitudes:

$$|2e^{j182^\circ}| = |2| \cdot |e^{j182^\circ}|$$

The magnitude of a real number is the absolute value of that real number:

$$|2| = 2$$

The magnitude of  $e^{jx}$  for any real x is 1:

$$|e^{jx}| = 1$$

Thus we have:

$$|e^{j182^\circ}| = 1$$

Putting our results together gives the answer:

$$|2e^{j182^\circ}| = 2$$

**EX:** Find  $\text{Re}\left[\frac{6+j3}{2-j4}e^{jx}\right]$ , (i.e., find the real part) where "x" is real

**ANS:**  $1.5 \cos(x + \pi/2)$

**SOL'N:** We may take one of several different approaches to convert the quantity inside the brackets into the form  $a + jb$  (where  $a$  is our final answer). We'll take the approach of rationalizing the fraction.

$$\begin{aligned}\text{Re}\left[\frac{6+j3}{2-j4}e^{jx}\right] &= \text{Re}\left[\frac{6+j3}{2-j4} \frac{2+j4}{2+j4}e^{jx}\right] \\ &= \text{Re}\left[\frac{12-12+j(24+6)}{2^2+4^2}e^{jx}\right] \\ &= \text{Re}\left[\frac{j30}{20}e^{jx}\right]\end{aligned}$$

We now use Euler's formula to expand the complex exponential:

$$\begin{aligned}&= \text{Re}\left[\frac{j30}{20}\{\cos(x) + j\sin(x)\}\right] \\ &= \text{Re}[-1.5\sin(x) + j1.5\cos(x)]\end{aligned}$$

Our final answer is the real part, which we may express in several ways.

$$\text{Re}\left[\frac{6+j3}{2-j4}e^{jx}\right] = -1.5\sin(x) \text{ or}$$

$$\text{Re}\left[\frac{6+j3}{2-j4}e^{jx}\right] = 1.5\cos(x + \pi/2) = 1.5\cos(x + 90^\circ)$$

**NOTE:** A curious feature of this problem is that the fraction consisting of complex numbers is purely imaginary. We now examine this symbolically.

$$k \cdot \frac{a+jb}{b-ja} = k \cdot \frac{j(b-ja)}{b-ja} = jk$$

Whenever the numerator and denominator of a fraction have the above pattern, we will find that the result is purely imaginary. Note the necessary minus sign.

## CONCEPTUAL TOOLS

By: Carl H. Durney and Neil E. Cotter

COMPLEX ANALYSIS  
 BASIC MATH  
 Roots and Powers  
 NTHROOTS  
 example

**EX:** Find the value (in polar form) of  $(6 + j5)^{1/5}$ .

**ANS:**  $1.51e^{j7.96^\circ}$  (approx)

**SOL'N:** First, we convert the number being raised to a power to polar form:

$$6 + j5 = \sqrt{6^2 + 5^2} e^{j \tan^{-1}\left(\frac{5}{6}\right)} \approx 7.81e^{j39.8^\circ}$$

Now take the power inside the parentheses and use the identity

$$(ab)^n = a^n b^n$$

giving the answer:

$$(6 + j5)^{1/5} \approx 7.81^{1/5} e^{j39.8^\circ/5} \approx 1.51e^{j7.96^\circ}$$

## CONCEPTUAL TOOLS

By: Carl H. Durney and Neil E. Cotter

COMPLEX ANALYSIS  
 BASIC MATH  
 Roots and Powers  
 POWERS  
 example

**EX:** Find the value (in polar form) of  $(3e^{j40^\circ})^3$ .

**ANS:**  $27e^{j120^\circ}$

**SOL'N:** We take the power inside the parentheses and use the identity

$$(ab)^n = a^n b^n$$

giving the answer:

$$3^3 e^{j3 \cdot 40^\circ} = 27e^{j120^\circ}$$

**EX:** Find the polar form of  $2.5 - j3.2$ .

**ANS:**  $4.06e^{-j52^\circ}$

**SOL'N:** We express  $2.5 - j3.2$  in polar form  $Ae^{j\phi}$ .

Use the pythagorean theorem to find magnitude  $A$ :

$$A = \sqrt{2.5^2 + 3.2^2} \approx 4.06$$

Set the tangent of the phase angle equal to the side opposite (imaginary part) over the side adjacent (real part):

$$\tan \phi = \frac{\text{Im}[2.5 - j3.2]}{\text{Re}[2.5 - j3.2]} = \frac{-3.2}{2.5} = -1.28$$

$$\phi = \tan^{-1}\left(\frac{-3.2}{2.5}\right) \approx -52^\circ \text{ or } -0.9076 \text{ radians}$$

Our final answer:

$$2.5 - j3.2 \approx 4.06e^{-j52^\circ}$$

**NOTE:** When calculating the inverse tangent, if we use  $-1.28$  rather than both the imaginary and real parts, we have two possible values for  $\phi$  that differ by  $180$  degrees. The ratio of the imaginary and real parts is the same for  $1 + j$  and  $-1 - j$ , for example. Thus, it is necessary to keep track of which quadrant the complex number lies in if we wish to avoid confusion about the correct value of phase angle  $\phi$ .

**EX:** Find the rectangular form of  $6e^{-j47^\circ}$

**ANS:**  $4.09 - j4.39$

**SOL'N:** We must express  $6e^{-j47^\circ}$  in rectangular form  $a + jb$ .

We use Euler's formula for the complex exponential:

$$6e^{-j47^\circ} = 6 \cos(-47^\circ) + j6 \sin(-47^\circ)$$

Applying identities,  $\cos(-A) = \cos(A)$  and  $\sin(-A) = -\sin(A)$ , we have

$$= 6 \cos(47^\circ) - j6 \sin(47^\circ)$$

$$6e^{-j47^\circ} = 4.09 - j4.39$$



**EX:** Find the rectangular and polar forms of  $\frac{6 - j2}{4 + j5}$ .

**ANS:**  $\frac{14}{41} - j\frac{38}{41}$  and  $0.987e^{-j69.78^\circ}$

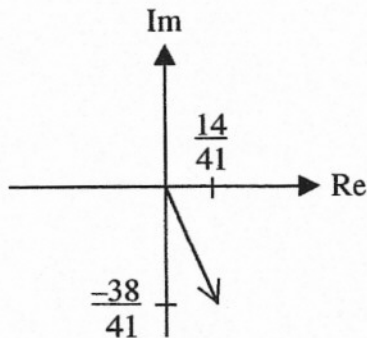
**SOL'N:**

$$\frac{6 - j2}{4 + j5} = \frac{6 - j2}{4 + j5} \frac{(4 + j5)^*}{(4 + j5)^*} = \frac{6 - j2}{4 + j5} \frac{(4 - j5)}{(4 - j5)}$$

$$= \frac{24 - 10 - j(30 + 8)}{16 + 25} = \frac{14}{41} - j\frac{38}{41}$$

$$A = \sqrt{\left(\frac{14}{41}\right)^2 + \left(\frac{38}{41}\right)^2} = 0.987$$

$$\phi = \tan^{-1}\left(\frac{-38}{14}\right) = -69.78^\circ$$



**TUTOR: THE PHASOR TRANSFORM**

All voltages and currents in linear circuits with sinusoidal sources are described by constant-coefficient linear differential equations of the form

$$(1) \quad a_n \frac{d^n f}{dt^n} + a_{n-1} \frac{d^{n-1} f}{dt^{n-1}} + \dots + a_0 f = C \cos(\omega t + \phi)$$

where  $f$  is a function of time, the  $a_n$  are constants,  $C$  is a constant,  $\omega$  is the radian frequency of the sinusoidal source, and  $\phi$  is the phase of the sinusoidal source. In (1),  $f$  represents any voltage or current in the circuit.

A particular solution to (1) can be found by an elegant procedure called the **phasor transform** method. This supplementary material outlines the mathematical basis of the method. The phasor transform is defined by

$$(2) \quad f(t) = \operatorname{Re}[\mathbf{F}(\omega)e^{j\omega t}]$$

where  $\mathbf{F}(\omega)$  is a function of  $\omega$  called the **phasor transform** of  $f(t)$ , and  $\operatorname{Re}$  means the real part of the quantity in the brackets.  $\mathbf{F}(\omega)$  is complex; it has a real and an imaginary part.

Two key mathematical relationships are used in finding a particular solution to (1). The first is

$$(3) \quad \operatorname{Re}[W] = \frac{W + W^*}{2}$$

where  $W$  is any complex number and  $W^*$  is the complex conjugate of  $W$ . Using (3) with (2) gives

$$(4) \quad f = \frac{\mathbf{F}e^{j\omega t} + \mathbf{F}^*e^{-j\omega t}}{2}$$

where  $f$  has been written for  $f(t)$  and  $\mathbf{F}$  for  $\mathbf{F}(\omega)$  for brevity. Note that  $\mathbf{F}$  is **not** a function of time. The second relationship is

$$(5) \quad \cos(\omega t + \phi) = \frac{e^{j(\omega t + \phi)} + e^{-j(\omega t + \phi)}}{2}$$

which is called Euler's formula.

Substituting (5) and (4) into (1), taking the derivatives with respect to time, and collecting terms gives

$$(6) \quad \left[ a_n (j\omega)^n \mathbf{F} + a_{n-1} (j\omega)^{n-1} \mathbf{F} + \dots + a_0 \mathbf{F} - C e^{j\phi} \right] e^{j\omega t} + \left[ a_n (-j\omega)^n \mathbf{F}^* + a_{n-1} (-j\omega)^{n-1} \mathbf{F}^* + \dots + a_0 \mathbf{F}^* - C e^{-j\phi} \right] e^{-j\omega t} = 0.$$

Now because  $e^{j\omega t}$  and  $e^{-j\omega t}$  are **linearly independent** functions (see, for example, C. R. Wylie, *Advanced Engineering Mathematics*, 3rd ed., New York: McGraw-Hill, 1966, p. 444), (6) can be true for all time only if

$$(7) \quad \left[ a_n (j\omega)^n \mathbf{F} + a_{n-1} (j\omega)^{n-1} \mathbf{F} + \dots + a_0 \mathbf{F} - C e^{j\phi} \right] = 0$$

and

$$(8) \quad \left[ a_n (-j\omega)^n \mathbf{F}^* + a_{n-1} (-j\omega)^{n-1} \mathbf{F}^* + \dots + a_0 \mathbf{F}^* - C e^{-j\phi} \right] = 0.$$

Equations (7) and (8) are identical because one is the complex conjugate of the other, so only one is needed. An expression for  $\mathbf{F}$  from (7) is

$$(9) \quad \mathbf{F} = \frac{C e^{j\phi}}{a_n (j\omega)^n + a_{n-1} (j\omega)^{n-1} + \dots + a_0}.$$

A particular solution to (1) can now be obtained from (9) and (2):

$$(10) \quad f = \operatorname{Re} \left[ \mathbf{F} e^{j\omega t} \right] = \operatorname{Re} \left[ \frac{C e^{j\phi} e^{j\omega t}}{a_n (j\omega)^n + a_{n-1} (j\omega)^{n-1} + \dots + a_0} \right].$$

Symbolically, the notation for a phasor transformation is

$$(11) \quad \mathbf{P}[f(t)] = \mathbf{F}(\omega)$$

where the bold  $\mathbf{P}$  means "phasor transform of". Thus,  $\mathbf{F}$  is the phasor transform of  $f$ . Taking the derivative of both sides of (2) gives

$$\frac{df}{dt} = \text{Re}[j\omega \mathbf{F}(\omega) e^{j\omega t}]$$

which corresponds to

$$\mathbf{P}\left[\frac{df}{dt}\right] = j\omega \mathbf{F}.$$

Similarly,

$$(12) \quad \mathbf{P}\frac{d^n f}{dt^n} = (j\omega)^n \mathbf{F}$$

and

$$(13) \quad \mathbf{P}[\cos(\omega t + \phi)] = e^{j\phi}$$

because

$$(14) \quad \cos(\omega t + \phi) = \text{Re}[e^{j\phi} e^{j\omega t}].$$

From the basic relation in (2) it can also be shown that

$$(15) \quad \mathbf{P}[f_1 + f_2] = \mathbf{F}_1 + \mathbf{F}_2$$

and

$$(16) \quad \mathbf{P}[af] = a\mathbf{F}$$

where

$$\mathbf{P}[f_1] = \mathbf{F}_1$$

and

$$\mathbf{P}[f_2] = \mathbf{F}_2$$

and "a" is a constant. The relation in (15) means that the phasor transform of a sum of functions can be found by taking the transform of each function and adding the transforms.

Equations (11), (12), (13), (15), and (16) describe phasor transforms. An inverse phasor transform relation is written as

$$(17) \quad f(t) = \mathbf{P}^{-1}[\mathbf{F}(\omega)].$$

Equations (11) and (17) are called a **transform pair**. Equation (11) states how to get  $\mathbf{F}$  when  $f$  is known; (17) how to get  $f$  when  $\mathbf{F}$  is known. Equation (2) is the inverse transform relation. The transform relation is derived as follows.  $f(t)$  will always be a sinusoid, because it is a particular solution to (1). Thus  $f(t)$  can be written as

$$(18) \quad f(t) = f_m \cos(\omega t + \alpha).$$

Substituting (18) into (2), using Euler's formula and (3) gives

$$\frac{f_m e^{j\omega t} e^{j\alpha} + f_m e^{-j\omega t} e^{-j\alpha}}{2} = \frac{\mathbf{F} e^{j\omega t} + \mathbf{F}^* e^{-j\omega t}}{2}.$$

Collecting terms and using the linear independence of  $e^{j\omega t}$  and  $e^{-j\omega t}$ , as before, gives

$$(19) \quad \mathbf{F} = f_m e^{j\alpha}$$

so the phasor transform of  $f_m \cos(\omega t + \alpha)$  is  $f_m e^{j\alpha}$ . The transform pairs are thus

$$(20) \quad \mathbf{F} = \mathbf{P}[f] = f_m e^{j\alpha}$$

and

$$(21) \quad f = \mathbf{P}^{-1}[\mathbf{F}] = \text{Re}[\mathbf{F}e^{j\omega t}].$$

With the phasor transform relations given in (12), (15), (16), (20), and (21), a particular solution to (1) can be found without going through the detailed derivation using (3) and linear independence. The phasor transform of (1) is taken term-by-term using (12), (13), (15), and (16) to get (7), which is then solved for  $\mathbf{F}$ . Having found  $\mathbf{F}$ ,  $f$  is found by taking the inverse transform according to (21).

**EX:** Let's find a particular solution to

$$(22) \quad \frac{d^3 f}{dt^3} + 3 \frac{d^2 f}{dt^2} + 50 \frac{df}{dt} - 60f = 500 \cos(10t + \pi/3).$$

Taking the phasor transform of this equation gives

$$(j10)^3 \mathbf{F} + 3(j10)^2 \mathbf{F} + 50(j10)\mathbf{F} - 60\mathbf{F} = 500e^{j\pi/3}.$$

Solving for  $\mathbf{F}$ ,

$$\mathbf{F} = \frac{500e^{j\pi/3}}{-j100 - 300 + j500 - 60}.$$

Converting  $\mathbf{F}$  to polar form gives

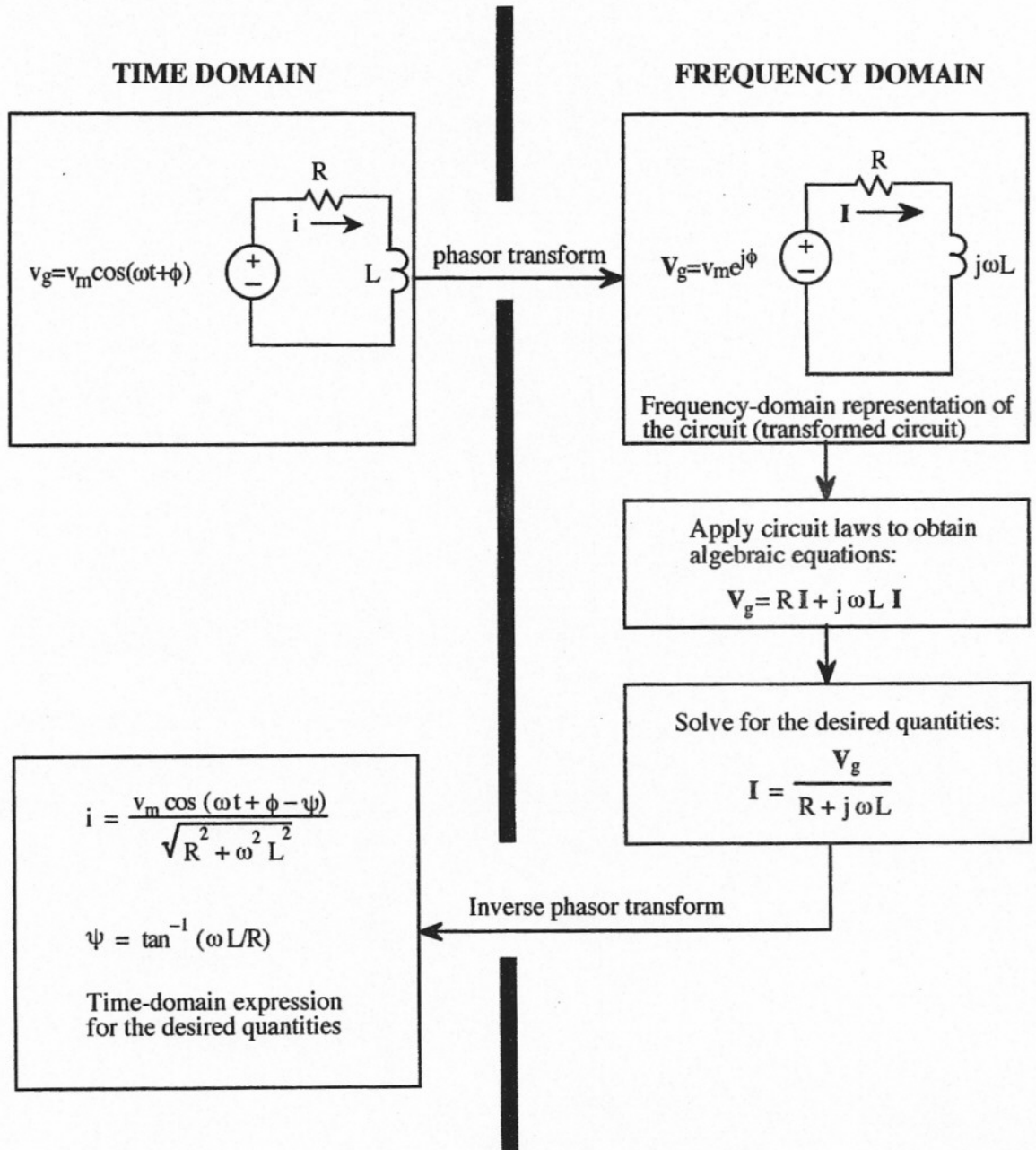
$$\mathbf{F} = 0.812e^{-j174.25^\circ}$$

and finding the inverse transform gives

$$f = \operatorname{Re}\left[\mathbf{F}e^{j\omega t}\right] = \operatorname{Re}\left[0.812e^{-j174.25^\circ}e^{j10t}\right] = 0.812\cos(10t - 174.25^\circ).$$

**COMMENT:** The phasor transform method is powerful because it transforms a differential equation (1) into an algebraic equation (7), which can be solved for the phasor  $\mathbf{F}$ , and then  $f$  can be found by taking the inverse transform.

Phasor voltages and currents satisfy Kirchhoff's laws, because of (15). Consequently, circuits can be transformed into the frequency domain, eliminating the need to write differential equations in the time domain and solve them by phasor transforms. The procedure for analyzing and designing circuits by transforming them into the frequency domain is summarized in the figure below. Note that impedance is defined as the ratio of a phasor voltage to a phasor current. Impedance is not defined in the time domain.





**EX:** If  $f(t) = 2 \sin(\omega t + \pi/3)$  find  $\mathbf{P}[f(t)]$ , (i.e., find the phasor)

**ANS:**  $\mathbf{P}[f(t)] \equiv \mathbf{F} = 2e^{-j\pi/6} \equiv 2\angle\pi/6$

**SOL'N:** If we have a cosine, we use the standard identity for phasors:

$$\mathbf{P}[A \cos(\omega t + \phi)] = Ae^{j\phi} \equiv A\angle\phi$$

For a sine, we multiply the standard identity by  $-j$  (which is the phasor for a sine of magnitude one and zero phase shift):

$$\mathbf{P}[\sin(\omega t)] = -j \equiv 1\angle -90^\circ$$

Thus, we have

$$\mathbf{P}[f(t)] \equiv \mathbf{F} = -2je^{j\pi/3}.$$

The above is mathematically correct and works properly in solving problems, but we will apply identities to express the answer in standard form:

$$-1 = e^{j180^\circ} = e^{-j180^\circ} = e^{j\pi} = e^{-j\pi}.$$

**NOTE:** (We use whichever of  $+180^\circ$  or  $-180^\circ$  is most convenient.)

$$j = e^{j90^\circ} = e^{j\pi/2}.$$

Applying the identities:

$$\mathbf{F} = -2je^{j\pi/3} = 2e^{-j\pi}e^{j\pi/2}e^{j\pi/3} = 2e^{-j\pi/6} \equiv 2\angle -\pi/6.$$

**EX:** If  $\mathbf{F} = (2.5 + j3.2)$  find  $\mathbf{P}^{-1}[\mathbf{F}]$ , (i.e., find the inverse phasor)

**ANS:**  $\mathbf{P}^{-1}[\mathbf{F}] = 4.06 \cos(\omega t + 52^\circ)$

**SOL'N:** We convert to polar form:

$$2.5 + j3.2 = \sqrt{2.5^2 + 3.2^2} e^{j \tan^{-1}\left(\frac{3.2}{2.5}\right)} \approx 4.06 e^{j52^\circ}$$

Now use the standard inverse phasor identity:

$$\mathbf{P}^{-1}[Ae^{j\phi}] = A \cos(\omega t + \phi)$$

**NOTE:** There is no math to do here—we just substitute the values of  $A$  and  $\phi$  into the  $\cos()$ .

**NOTE:** We don't know the value of  $\omega$  for this problem. Thus, we just use a symbolic variable for  $\omega$ . The value of  $\omega$  is *not* part of a phasor. (The value of  $\omega$  must be kept track of separately.)

Using the identity gives the answer:

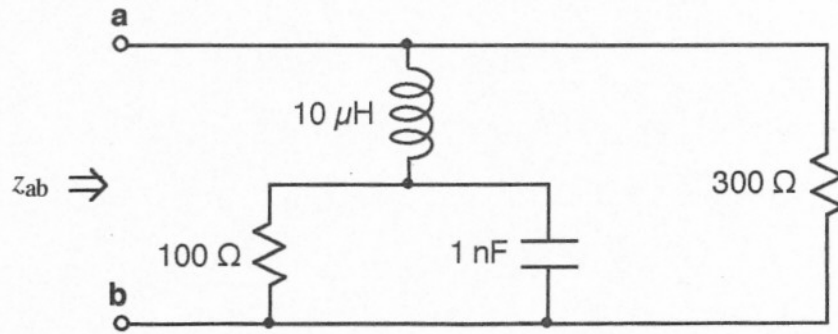
$$\mathbf{P}^{-1}[\mathbf{F}] = 4.06 \cos(\omega t + 52^\circ)$$

**NOTE:** Mathematically, it is also correct to invert the given phasor in two pieces, with the real part giving a cosine term having no phase shift and the imaginary part giving a (negative) sine term having no phase shift:

$$\mathbf{P}^{-1}[2.5 + j3.2] = 2.5 \cos(\omega t) - 3.2 \sin(\omega t).$$

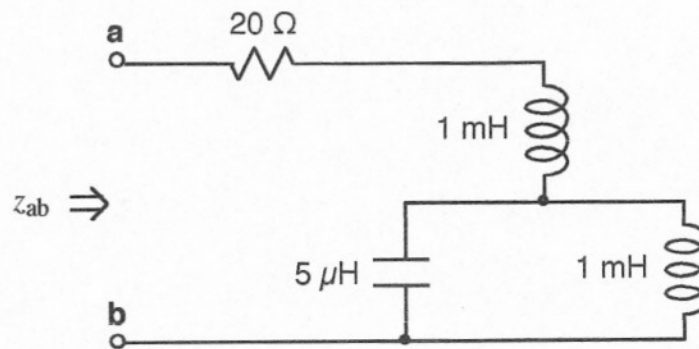
Although this answer is correct, it is usually easier to visualize a single sinusoid with a phase shift. The sum of the cos and sin terms is equal to the single cos with a phase shift given above. This follows from the observation that the sum of any number of sinusoids of the same frequency may be expressed as a single sinusoid of that frequency. (The challenging part is determining the magnitude and phase shift of the single sinusoid.)

1.



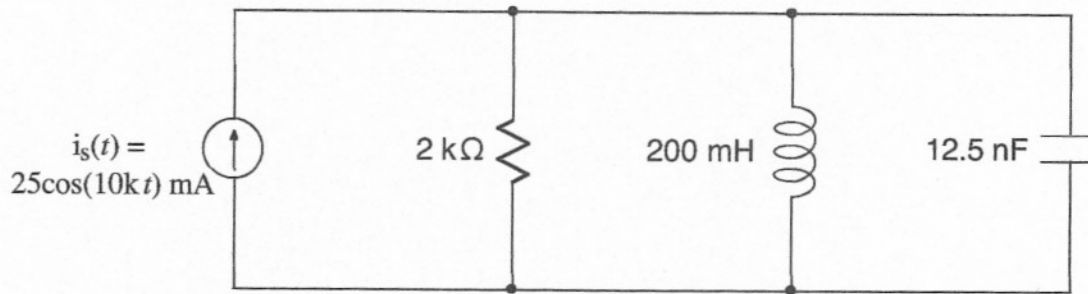
Given  $\omega = 10 \text{ M rad/s}$ , find  $z_{ab}$ .

2.



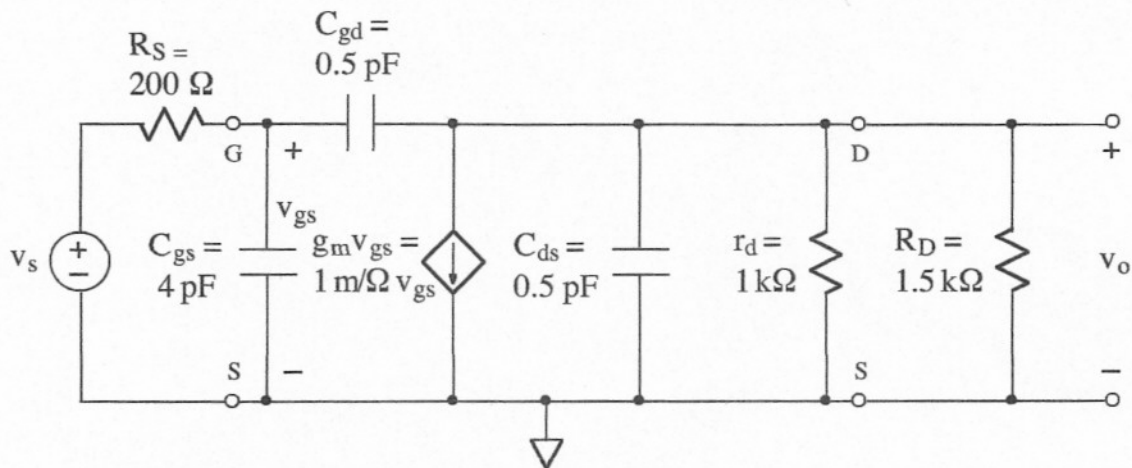
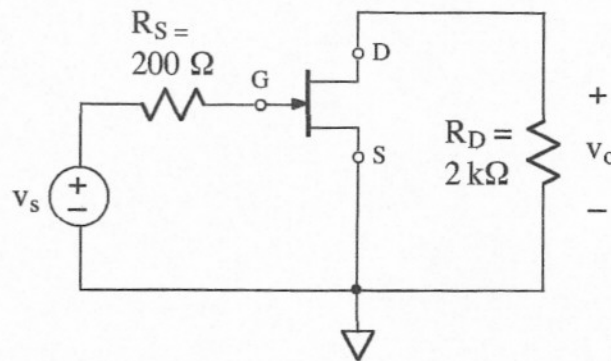
Find a frequency,  $\omega$ , that causes  $z_{ab}$  to be real, (i.e., imaginary part equals zero).

3.



- Find the phasor value for  $i_s(t)$ .
- Draw the frequency-domain circuit diagram, including the phasor value for  $i_s(t)$  and impedance values for components.
- Find the phasor value for  $i_L(t)$ .

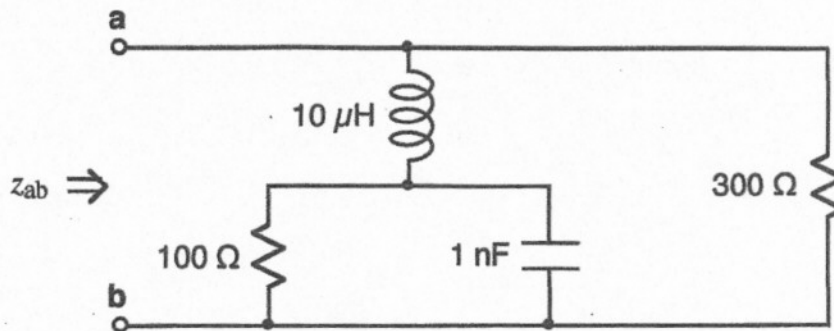
4.



$$v_s(t) = 2 \cos(10kt) \text{ V}$$

The above circuit diagrams show a common-source JFET amplifier and its high-frequency equivalent circuit. Find  $v_o(t)$ .

Ex:

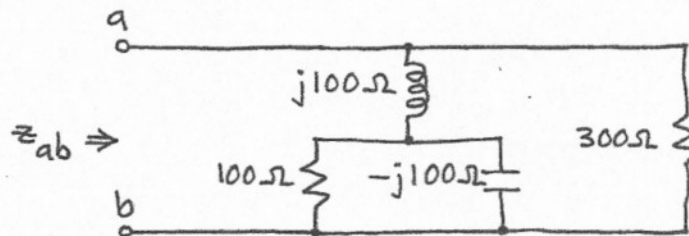


Given  $\omega = 10 \text{ M rad/s}$ , find  $z_{ab}$ .

Sol'n:  $z_L = j\omega L = j 10 \text{ Mr/s} \cdot 10 \mu\text{H} = j 100 \Omega$

$$z_C = \frac{-j}{\omega C} = \frac{-j}{10 \text{ Mr/s} \cdot 1 \text{ nF}} = \frac{-j \Omega}{10 \text{ m}} = -j 100 \Omega$$

frequency domain (or s-domain) model:



We first consider the R and C in parallel.

$$100 \Omega \parallel -j 100 \Omega = 100 \Omega \cdot 1 \parallel -j = 100 \Omega \cdot \frac{-j}{1-j}$$

Rationalizing this expression, we have

$$100 \Omega \parallel -j 100 \Omega = 100 \Omega \frac{-j}{1-j} \cdot \frac{1+j}{1+j}$$

or

$$100\Omega \parallel -j100\Omega = 100\Omega \frac{1-j}{2} = 100\Omega \cdot \left(\frac{1}{2} - \frac{j}{2}\right)$$

Now we add  $z_L = j100\Omega$

$$\begin{aligned} 100\Omega \parallel -j100\Omega + j100\Omega &= 100\Omega \left(\frac{1}{2} - \frac{j}{2} + j\right) \\ &= 100\Omega \left(\frac{1}{2} + \frac{j}{2}\right) \end{aligned}$$

To find  $z_{ab}$ , we use conductance  $g_{ab} = \frac{1}{z_{ab}}$ :

$$\begin{aligned} g_{ab} &= \frac{1}{100\Omega \parallel -j100\Omega + j100\Omega} + \frac{1}{300\Omega} \\ &= \frac{1}{100\Omega \left(\frac{1}{2} + \frac{j}{2}\right)} + \frac{1}{100\Omega} \cdot \frac{1}{3} \\ &= \frac{1}{100\Omega} \cdot \frac{2}{1+j} \cdot \frac{1-j}{1-j} + \frac{1}{100\Omega} \cdot \frac{1}{3} \\ &= \frac{1}{100\Omega} \cdot \frac{2-j2}{2} + \frac{1}{100\Omega} \cdot \frac{1}{3} \\ &= \frac{1}{100\Omega} (1-j) + \frac{1}{100\Omega} \cdot \frac{1}{3} \\ &= \frac{1}{100\Omega} \left(\frac{4}{3} - j\right) \end{aligned}$$

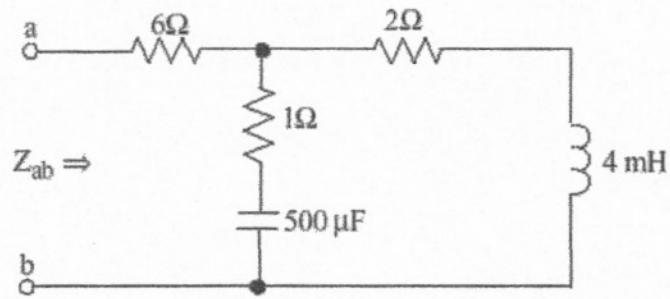
Now we calculate  $\frac{1}{g_{ab}} = z_{ab}$ .

$$\begin{aligned}z_{ab} &= \frac{1}{g_{ab}} = 100 \Omega \frac{1}{\frac{4}{3} - j} \\&= 100 \Omega \frac{3}{4 - j3} \\&= 100 \Omega \frac{3}{4 - j3} \cdot \frac{4 + j3}{4 + j3} \\&= 100 \Omega \frac{12 + j9}{4^2 + 3^2} \\&= 100 \Omega \frac{12 + j9}{25} \\&= 4 \cdot (12 + j9) \\z_{ab} &= 48 + j36\end{aligned}$$

# Homework #7 Examples

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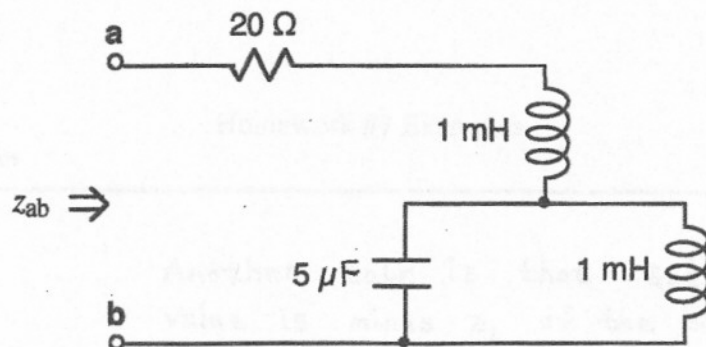
1.



Given  $\omega = 1\text{ k rad/sec}$ , find  $Z_{ab}$ .



Ex:



Find a frequency,  $\omega$ , that causes  $z_{ab}$  to be real, (i.e., imaginary part equals zero).

Sol'n:  $z_{ab} = 20 \Omega + z_{L1} + z_C \parallel z_{L2}$

For  $z_{ab}$  to be real, we must have

$$z_{L1} + z_C \parallel z_{L2} = \text{real}$$

One simple sol'n is to let  $\omega = 0$  so both L's act like wires and C acts like open circuit.

Other potential sol'ns are  $\omega = \infty$ , (so L's act like opens, resulting in  $z_{ab} = \infty$ ), and  $\omega = \text{frequency where } z_C = -z_{L2}$ , (so C and L in parallel have equal but opposite impedances).

The latter case, where  $z_C = -z_L$  gives

the interesting result that  $z_C \parallel z_L = \frac{L/C}{0} = \infty$

This means  $z_{ab} = \infty \Omega$ . In this case, (unlike  $\omega \rightarrow \infty$ ),  $z_{ab} \rightarrow \infty$  along real axis as  $z_C \parallel z_L \rightarrow \infty$ .

or  $\omega = 20k \text{ r/s}$

Another sol'n is that  $z_c \parallel z_L$  has a value is minus  $z_L$  of the top inductor.

In that case,  $z_L + z_c \parallel z_L = 0$  and  $z_{ab} = 0 = \text{wire}$ .

$$z_L = j\omega L$$

$$\begin{aligned} z_c \parallel z_L &= \frac{-j}{\omega C} \parallel j\omega L = \frac{-j \cdot j\omega L}{-j + j\omega L} = \frac{L/C}{j(\omega L - \frac{1}{\omega C})} \\ &= -j \frac{L/C}{\omega L - \frac{1}{\omega C}} \end{aligned}$$

$$\text{Thus, we want } j\omega L - \frac{jL/C}{\omega L - \frac{1}{\omega C}} = 0$$

$$\text{or } \omega L = \frac{L/C}{\omega L - \frac{1}{\omega C}}$$

$$\text{or } \omega L \left( \omega L - \frac{1}{\omega C} \right) = L/C$$

$$\text{or } \omega^2 L^2 - \frac{L}{C} = \frac{L}{C}$$

$$\text{or } \omega^2 L^2 = \frac{2L}{C} \quad \text{or } \omega^2 = \frac{2}{LC}$$

$$\text{or } \omega = \sqrt{\frac{2}{LC}} \quad \text{or } \omega = \sqrt{\frac{2}{5\mu\text{F} \cdot 1\text{mH}}}$$

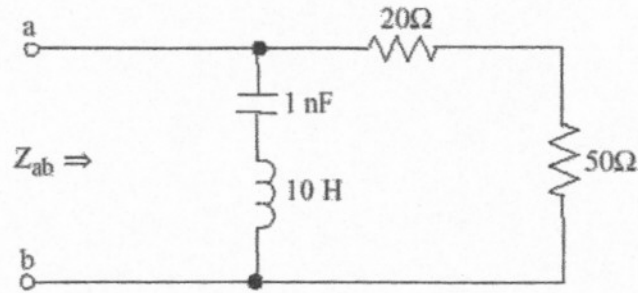
$$\text{or } \omega = \sqrt{\frac{2 \text{G}}{5}} \text{ r/s} = \sqrt{400\text{M}} \text{ r/s}$$

$$\text{or } \omega = 20\text{k} \text{ r/s}$$

Homework #7 Examples

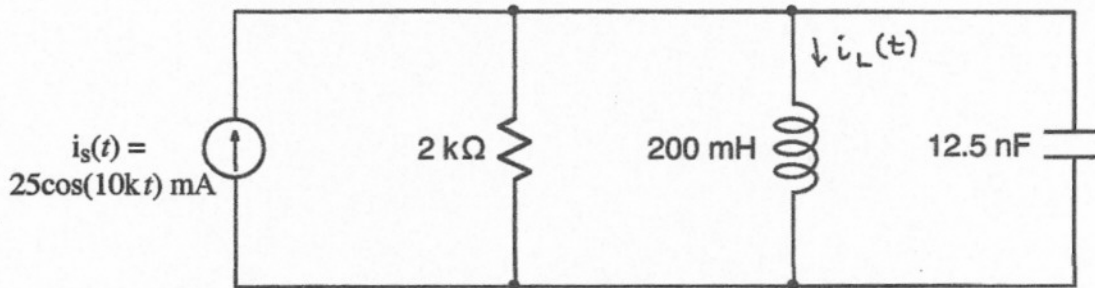
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2.



Find a frequency,  $\omega$ , that causes  $Z_{ab}$  to be real (i.e. imaginary part equals zero).  
 $\omega \neq 0$  or  $\omega \neq \infty$ .

EX:



- a) Find the phasor value for  $i_s(t)$ .
- b) Draw the frequency-domain circuit diagram, including the phasor value for  $i_s(t)$  and impedance values for components.
- c) Find the phasor value for  $i_L(t)$ .

Sol'n: a) The phasor for  $A\cos(\omega t + \phi)$  is  $Ae^{j\phi}$ .

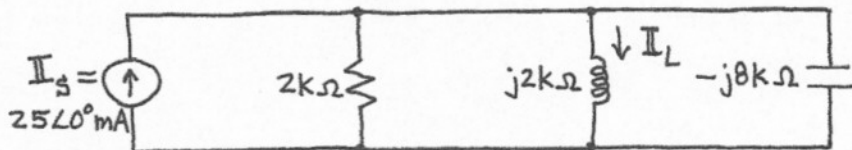
$$\therefore \mathbb{I}_s = 25 e^{j0^\circ} \text{ mA or } 25 \angle 0^\circ \text{ mA}$$

b) From  $i_s(t)$ , we see that  $\omega = 10\text{krad/s}$ .

$$\text{Impedance } z_L = j\omega L = j10\text{k} \cdot 200\text{mH} = j2\text{k}\Omega$$

$$z_C = \frac{-j}{\omega C} = \frac{-j}{10\text{k} \cdot 12.5\text{nF}} = \frac{-j}{125\mu}$$

$$z_C = \frac{-j \cdot 8\text{k}\Omega}{8\text{k} \cdot 125\mu} = -j8\text{k}\Omega$$



c) The value for  $I_L$  is given by the current divider formula:

$$I_L = I_s \cdot \frac{R \parallel z_c}{R \parallel z_c + z_L}$$

$$= I_s \frac{1}{1 + \frac{z_L}{R \parallel z_c}}$$

$$= I_s \frac{1}{1 + z_L \left( \frac{1}{R} + \frac{1}{z_c} \right)}$$

$$= 25 \angle 0^\circ \text{ mA} \frac{1}{1 + j2k\Omega \left( \frac{1}{2k\Omega} + \frac{1}{-j8k\Omega} \right)}$$

$$= 25 \angle 0^\circ \text{ mA} \frac{4}{4} \frac{1}{1 + j - \frac{1}{4}}$$

$$= 25 \angle 0^\circ \text{ mA} \frac{4}{3 + j4}$$

$$= 25 \angle 0^\circ \text{ mA} \frac{4}{3 + j4} \cdot \frac{3 - j4}{3 - j4}$$

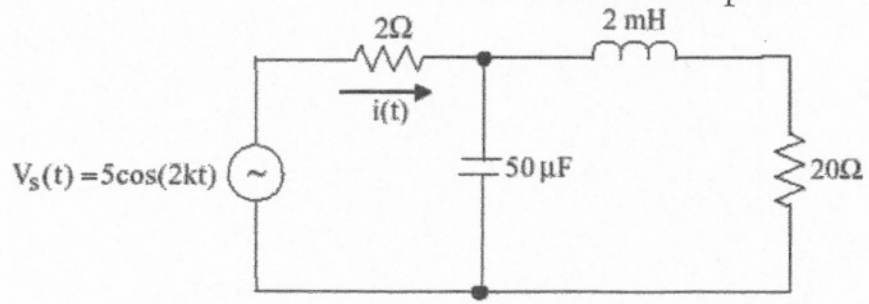
$$= 25 \angle 0^\circ \text{ mA} \frac{12 - j16}{3^2 + 4^2}$$

$$= 1 \angle 0^\circ \text{ mA} \cdot 20 \angle -53.1^\circ$$

$$I_L = 20 \text{ mA} \angle -53.1^\circ$$

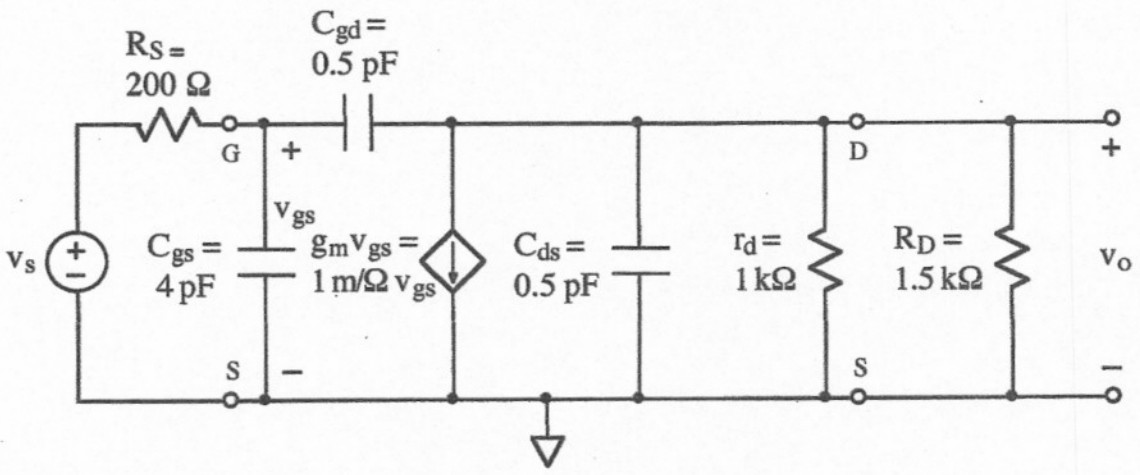
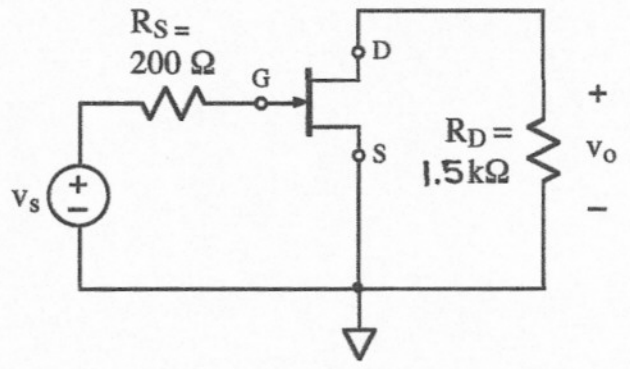
Homework #7 Examples

3.



- a. Find the phasor value for  $V_s(t)$ .
- b. Draw the frequency-domain circuit diagram, including the phasor value for  $V_s(t)$  and impedance values for components.
- b. Find the phasor value for  $i(t)$ .

EX:



$$v_s(t) = 2 \cos(10kt) \text{ V}$$

The above circuit diagrams show a common-source JFET amplifier and its high-frequency equivalent circuit. Find  $v_o(t)$ .

Sol'n: In this practical circuit, we have circuit values that allow us to make simplifying approximations.

We first calculate impedance values.

$$\omega = 10k \text{ r/s} \quad \text{from} \quad v_s(t) = 2 \cos(10kt) \text{ V}$$

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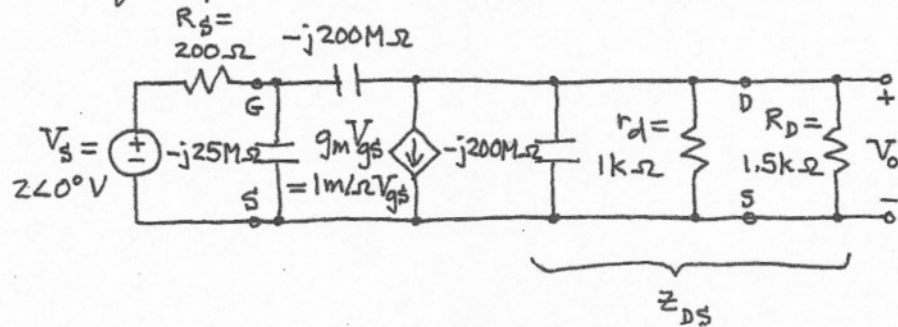
$$Z_{C_{gs}} = \frac{-j}{\omega C_{gs}} = \frac{-j \Omega}{10k \cdot 4p} = -j 25M \Omega$$

$$Z_{C_{gd}} = \frac{-j}{\omega C_{gd}} = \frac{-j}{10k \cdot \frac{1}{2} p} = -j 200M \Omega$$

$$Z_{C_{ds}} = \frac{-j}{\omega C_{ds}} = \frac{-j}{10k \cdot \frac{1}{2} p} = -j 200M \Omega$$

The phasor for  $v_s(t)$  is  $V_s = 2 \angle 0^\circ V$ .

Frequency domain (or s-domain) model:



$$Z_{DS} = -j 200M \parallel 1k\Omega \parallel 1.5k\Omega$$

Starting with  $1k\Omega \parallel 1.5k\Omega$  we have

$$1k\Omega \parallel 1.5k\Omega = 500\Omega \cdot 2 \parallel 3 = 500 \cdot \frac{6}{5}$$

$$= 600\Omega$$

$$\text{Thus, } Z_{DS} = -j 200M \parallel 600\Omega = \frac{1}{\frac{1}{600} - \frac{1}{j200M}}$$

Using  $-\frac{1}{j} = j$  and rationalizing gives



$$z_{DS} = \frac{1}{\frac{1}{600} + \frac{j}{200M}} \frac{\frac{1}{600} - \frac{j}{200M}}{\frac{1}{600} - \frac{j}{200M}} \Omega$$

$$= \frac{\frac{1}{600} - \frac{j}{200M}}{\left(\frac{1}{600}\right)^2 + \left(\frac{1}{200M}\right)^2} \Omega$$

$$\approx \frac{\frac{1}{600} - \frac{j}{200M}}{\left(\frac{1}{600}\right)^2} \Omega \quad \text{Since } \frac{1}{200M} \ll \frac{1}{600}$$

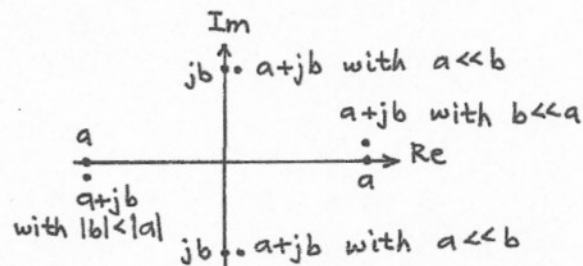
$$z_{DS} \approx \frac{1}{\frac{1}{600}} \Omega = 600 \Omega$$

In retrospect, we could have made the approximation that  $-j200M \parallel 600 \Omega \approx 600 \Omega$ .

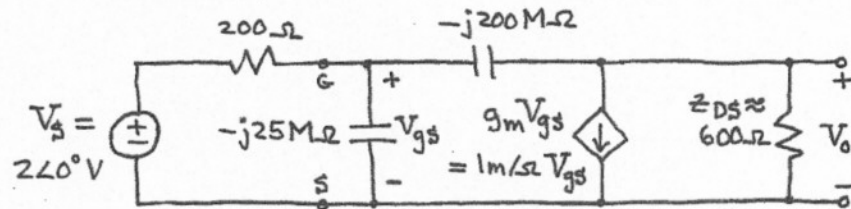
We may make this approximation despite the  $j$  in one of the quantities. In general, we may make the following approximations of complex values:

$$a + jb \approx a \quad \text{when } |b| \ll |a|$$

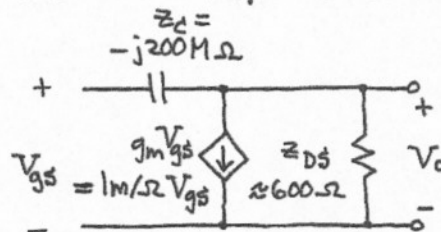
$$a + jb \approx jb \quad \text{when } |a| \ll |b|$$



With our  $z_{DS}$  value, we have a simplified model:



We now analyze the dependent source so we can replace it with an impedance,  $z_{eg}$ .



$$z_{eg} = \frac{V_o}{g_m V_{gs}} \quad \text{using Ohm's law to write } z_{eg} = V/I$$

Now we find a way to write  $V_o$  in terms of  $V_{gs}$ . We use a V-divider:

$$V_o = V_{gs} \frac{z_{eg} \parallel z_{DS}}{z_{eg} \parallel z_{DS} + z_c}$$

Substituting for  $V_o$  in our  $z_{eg}$  eq'n, we have

$$z_{eg} = \frac{V_{gs} \frac{z_{eg} \parallel z_{DS}}{z_{eg} \parallel z_{DS} + z_c}}{g_m V_{gs}}$$

$$z_{eg} = \frac{1}{g_m} \frac{\frac{z_{eg} z_{DS}}{z_{eg} + z_{DS}}}{\frac{z_{eg} z_{DS}}{z_{eg} + z_{DS}} + z_c}$$

$$z_{eg} = \frac{1}{g_m} \frac{z_{eg} z_{Ds}}{z_{eg} z_{Ds} + z_c (z_{eg} + z_{Ds})}$$

Dividing top and bottom by  $z_{eg}$  gives the following:

$$z_{eg} = \frac{1}{g_m} \frac{z_{Ds}}{z_{Ds} + z_c + \frac{z_c z_{Ds}}{z_{eg}}}$$

$$z_{eg} \left( z_{Ds} + z_c + \frac{z_c z_{Ds}}{z_{eg}} \right) = \frac{1}{g_m} z_{Ds}$$

$$z_{eg} (z_{Ds} + z_c) + z_c z_{Ds} = \frac{1}{g_m} z_{Ds}$$

$$z_{eg} \frac{(z_{Ds} + z_c)}{z_{Ds}} = \frac{1}{g_m} \frac{z_{Ds}}{z_{Ds}} - z_c \frac{z_{Ds}}{z_{Ds}}$$

$$z_{eg} = \frac{1}{g_m} - z_c = \frac{1 \Omega}{1 \text{ m}} - -j200 \text{ M}\Omega$$
$$\frac{1 + \frac{z_c}{z_{Ds}}}{1 + \frac{-j200 \text{ M}\Omega}{600 \Omega}}$$

$$z_{eg} = \frac{1 \text{ k} + j200 \text{ M} \Omega}{1 - j \frac{1}{3} \text{ M}}$$

The imaginary parts of the numerator and denominator are much larger than the real parts. Thus, we ignore the real parts.

$$z_{eg} \approx j200 \text{ M}\Omega / -j \frac{1}{3} \text{ M} \approx -600 \Omega$$

Now we have a problem:  $z_{eg} \parallel z_{DS} = -\frac{600^2 \Omega}{0}$ .

That means  $z_{eg} \parallel z_{DS} = \infty \Omega$ .

It is a good idea to try a more exact calculation to be sure that  $z_{eg} \parallel z_{DS}$  is much larger than  $z_c = -j200M\Omega$ .

We use conductance to simplify calculations.

$$\frac{1}{z_{eg} \parallel z_{DS}} = \frac{1}{z_{eg}} + \frac{1}{z_{DS}} = \frac{1 + \frac{z_c}{z_{DS}}}{\frac{1}{g_m} - z_c} + \frac{1}{z_{DS}}$$

$$= \frac{1}{z_{DS}} \left( \frac{z_{DS} + z_c}{\frac{1}{g_m} - z_c} + 1 \right)$$

$$= \frac{1}{z_{DS}} \left( \frac{z_{DS} + z_c}{\frac{1}{g_m} - z_c} + \frac{\frac{1}{g_m} - z_c}{\frac{1}{g_m} - z_c} \right)$$

$$= \frac{1}{z_{DS}} \left( \frac{z_{DS} + \frac{1}{g_m}}{\frac{1}{g_m} - z_c} \right)$$

$$= \frac{g_m + \frac{1}{z_{DS}}}{1 - g_m z_c}$$

$$= \frac{1m + \frac{1}{600}}{1 - 1m(-j200M)} \quad / \Omega$$

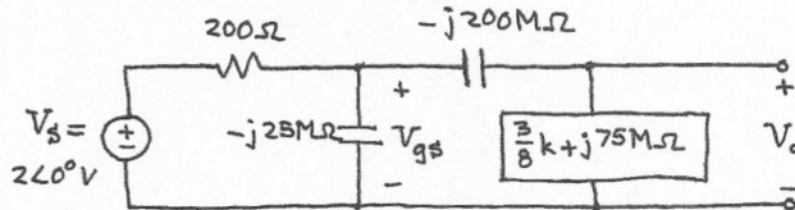
$$z_{eg} \parallel z_{DS} = \frac{1 + j200K}{\frac{1}{1K} + \frac{1}{600}} \Omega$$

$$z_{eg} \parallel z_{DS} = \frac{1 + j200k}{\frac{3+5}{3k}} \Omega = \frac{3k(1+j200k)}{8} \Omega$$

$$= \frac{3}{8}k + j75M\Omega$$

We see that the value is smaller than  $z_c = -j200M\Omega$ .

Our new, simplified model:



We use V-dividers to find  $V_o$ .

$$V_{gs} = V_s \cdot \frac{-j25M\Omega \parallel (-j200M + \overset{\text{small}}{\frac{3}{8}k + j75M\Omega})}{200 + -j25M\Omega \parallel (-j200M + \overset{\text{small}}{\frac{3}{8}k + j75M\Omega})}$$

$$V_{gs} \approx V_s \cdot \frac{-j25M\Omega \parallel -j125M\Omega}{200 - j25M\Omega \parallel -j125M\Omega}$$

$$\text{where } -j25M\Omega \parallel -j125M\Omega = -j25M\Omega \cdot 1 \parallel 5$$

$$= -j25M\Omega \cdot \frac{5}{6}$$

$$= -j \frac{125}{6} M\Omega$$

$$V_{gs} = V_s \cdot \frac{-j \frac{125}{6} M\Omega}{\overset{\text{small}}{200} - j \frac{125}{6} M\Omega} \approx V_s$$

$$V_o = V_{gs} \frac{\frac{3}{8} k + j 75 M\Omega}{\frac{3}{8} k + j 75 M\Omega - j 200 M\Omega}$$

$$V_o \approx V_{gs} \frac{j 75 M\Omega}{-j 125 M\Omega} = V_{gs} \left(-\frac{3}{5}\right)$$

$$V_o \approx 2 \angle 0^\circ V \left(-\frac{3}{5}\right) = -\frac{6}{5} \angle 0^\circ V = \frac{6}{5} \angle 180^\circ V$$

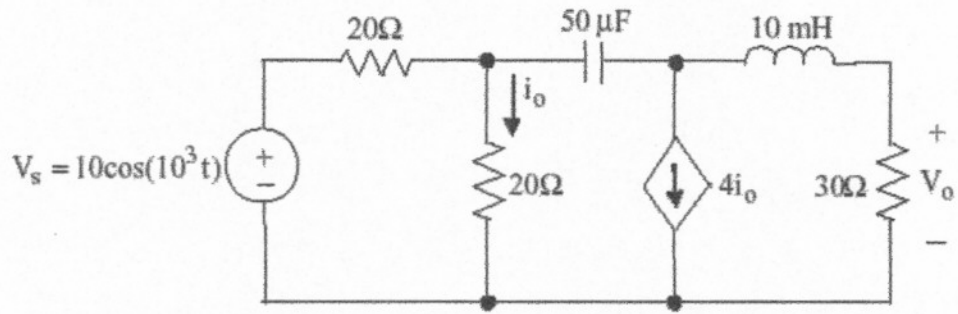
Note: a minus sign is the same as  $180^\circ$  of phase shift.

$$v_o(t) = \frac{6}{5} \cos(10kt + 180^\circ) V$$

# Homework #7 Examples

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4.

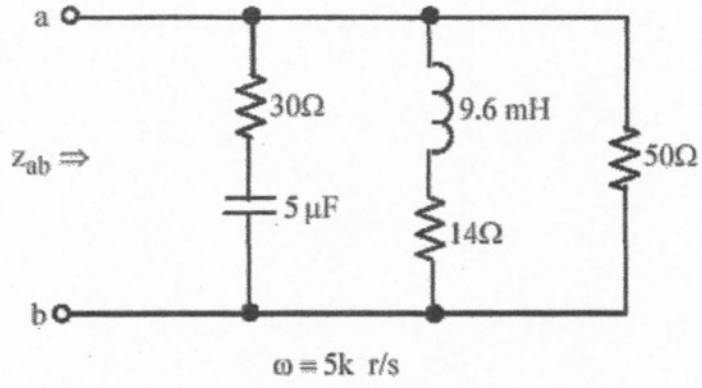


Find  $V_o(t)$ .

Homework #7 Example #2

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1.

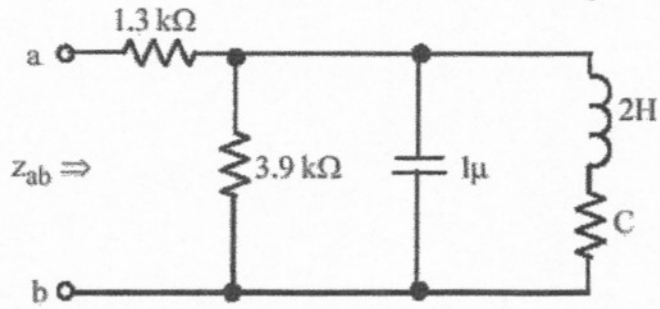


Find  $z_{ab}$ .



Homework #7 Example #2

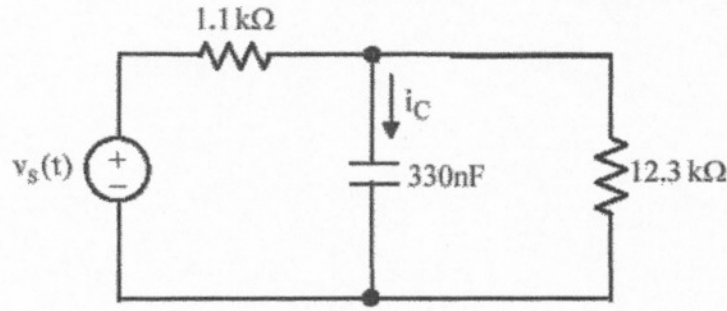
2.



Find a value for C that causes  $z_{ab}$  to be real, (i.e., imaginary part equals zero), for  $\omega = 1000\text{ r/s}$ .

Homework #7 Example #2

3.



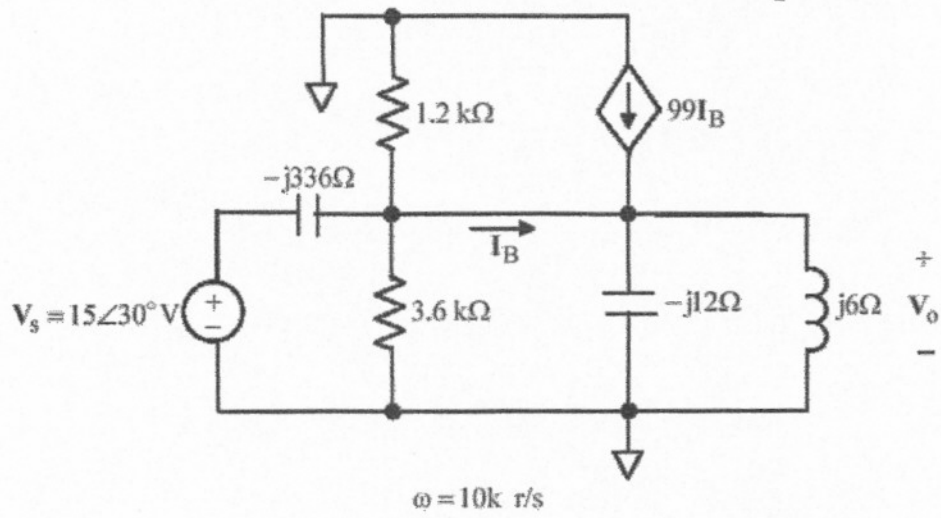
$$v_s(t) = 15 \cos(3 \text{ kt}) \text{ V}$$

- a. Find the phasor for  $v_s(t)$ .
- b. Draw the frequency-domain circuit diagram, including the phasor value for  $v_s(t)$  and impedance values for components.
- c. Find the phasor value for  $i_C(t)$ .
- d. Find  $i_C(t)$ .

Homework #7 Example #2

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4.



Find  $v_o(t)$ .

1.

Give numerical answers to each of the following questions:

pts

a. Rationalize  $\frac{3+j4}{7-j24}$ . Express your answer in rectangular form.

b. Find the rectangular form of  $\left[ \frac{1-\sqrt{2} e^{-j45^\circ}}{e^{j30^\circ}} \right]^*$ . Note the asterisk that means "conjugate".

c. Given  $\omega = 100\text{k rad/s}$ , find the following inverse phasor:  $P^{-1} [j30 \sin(-53^\circ)]$

d. Find the magnitude of  $\left( 2e^{j30^\circ} - j \right) \left( \frac{5-j12}{e^{j17^\circ}} \right)$ .

e. Find the real part of  $e^{-j30^\circ}$ .

sol'n: a)  $\frac{3+j4}{7-j24} \cdot \frac{7+j24}{7+j24} = \frac{21 - 4(24) + j3(24) + j4(7)}{7^2 + 24^2} = \frac{-75 + j100}{25^2} = \boxed{\frac{-3+j4}{25}}$

b)  $\left[ \frac{1-\sqrt{2} e^{-j45^\circ}}{e^{j30^\circ}} \right]^* = \frac{1-\sqrt{2} e^{j45^\circ}}{e^{-j30^\circ}} \cdot e^{j30^\circ} = \left[ 1 - \frac{\sqrt{2}(1+j)}{\sqrt{2}} \right] \cdot e^{j30^\circ} = -j e^{j30^\circ}$   
 $= e^{-j90^\circ} e^{j30^\circ} = e^{-j60^\circ} = \boxed{+\frac{1}{2} - j\frac{\sqrt{3}}{2}}$

c)  $P^{-1} [j30 \sin(-53^\circ)] = P^{-1} [j30 j \cdot 1 \angle -53^\circ] = -30 \angle -53^\circ = 30 \angle 137^\circ$   
 $= \boxed{30 \cos(100kt + 137^\circ) \text{ or } 30 \cos(100kt - 233^\circ)}$

d)  $\left| \left( 2e^{j30^\circ} - j \right) \frac{5-j12}{e^{j17^\circ}} \right| = \left| 2e^{j30^\circ} - j \right| \left| \frac{5-j12}{e^{j17^\circ}} \right| = \left| 2 \left( \frac{\sqrt{3}}{2} + \frac{j}{2} \right) - j \right| \cdot \frac{13}{1} = \boxed{\sqrt{3} \cdot 13}$

e)  $\text{Re} \left[ \frac{1}{e^{-j30^\circ}} \right] = \text{Re} [e^{j30^\circ}] = \text{Re} \left[ \frac{\sqrt{3}}{2} + j\frac{1}{2} \right] = \boxed{\frac{\sqrt{3}}{2}}$

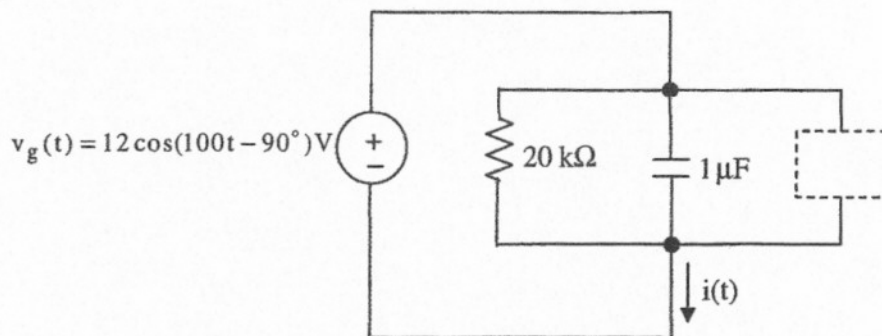
## Homework #8 Examples

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Give numerical answers to each of the following questions:

1.
  - a. Rationalize  $\frac{23+j7}{15-j8}$ . Express your answer in rectangular form.
  - b. Find the polar form of  $(2+j3)(3+j2)+[3+j16]^*$ . Note the asterisk that means "conjugate".
  - c. Find the following phasor:  $P[-5\sin(100t-30^\circ)]$ .
  - d. Find the magnitude of  $\frac{100(3+j4)(4+j3)}{(7+j)(7-j)}$ .
  - e. Find the imaginary part of  $(1+j)e^{-j45^\circ}(j2)$ .

2.



pts

a. Choose an R, an L, or a C to be placed in the dashed-line box to make

$$i(t) = I_0 \cos(100t - 45^\circ) \text{ A}$$

where  $I_0$  is a real constant. State the value of the component you choose.

b. With your component from (a) in the circuit, calculate the resulting value of

$I_0$ .

soln: a) Use conductance:  $\mathbf{I} = I_0 \angle -45^\circ \text{ A} = V_g \cdot \overbrace{\left( \frac{1}{20 \text{ k}\Omega} + j \frac{100 \cdot 1\mu}{\Omega} + \frac{1}{Z_{\text{box}}} \right)}^{G_{\text{tot}}}$   
 (and phasors)

Note:  $\omega = 100$  from  $v_g(t)$  where  $V_g = 12 \angle -90^\circ \text{ V}$

We have  $\angle \mathbf{I} = \angle V_g + \angle G_{\text{tot}}$  from phasor multiplication

$$-45^\circ = -90^\circ + \angle G_{\text{tot}}$$

$$\therefore \angle G_{\text{tot}} = 45^\circ \text{ or } \text{Re}[G_{\text{tot}}] = \text{Im}[G_{\text{tot}}]$$

$$G_{\text{tot}} = \frac{50\mu}{\Omega} + j \frac{100\mu}{\Omega} + \frac{1}{Z_{\text{box}}}$$

we can choose  $\frac{1}{Z_{\text{box}}} = \frac{50\mu}{\Omega} \Rightarrow Z_{\text{box}} = 20 \text{ k}\Omega \text{ resistor}$

or  $\frac{1}{Z_{\text{box}}} = \frac{-j50\mu}{\Omega} = \frac{-j}{\omega L} = \frac{-j}{100 \cdot L}$

Note: Either answer accepted

but 20 kΩ R is more sensible.

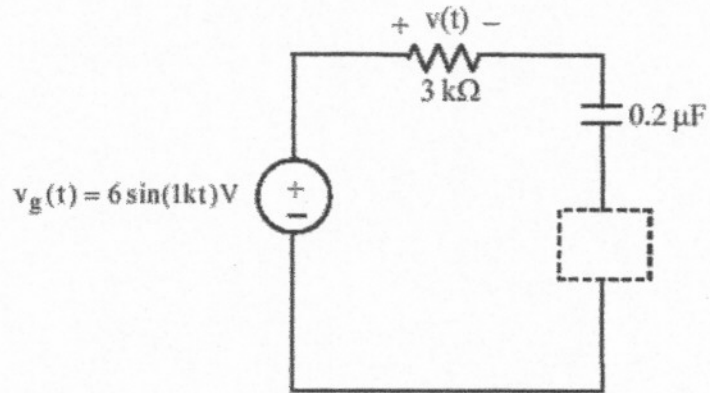
$\Rightarrow Z_{\text{box}} = 200 \text{ H inductor}$

b)  $I_0 = |\mathbf{I}| = |V_g| \cdot |G_{\text{tot}}| = 12 \cdot \sqrt{2} \cdot 100 \mu\text{A} = \sqrt{2} \cdot 12 \text{ mA for } 20 \text{ k}\Omega \text{ R}$   
 or  $12 \cdot \sqrt{2} \cdot 50 \mu\text{A} = \sqrt{2} \cdot 600 \mu\text{A for } 200 \text{ H L}$

Homework #8 Examples

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2.

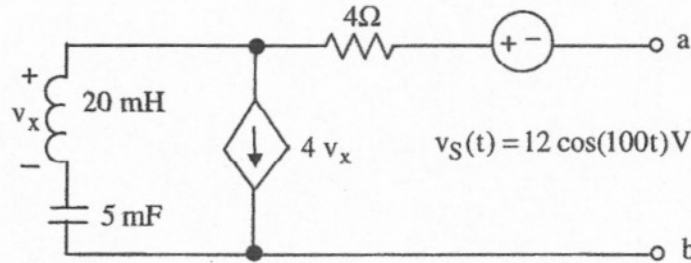


Choose an R, an L, or a C to be placed in the dashed-line box to make

$$v(t) = V_0 \cos(1kt - 45^\circ) \text{ V}$$

where  $V_0$  is a real constant. State the value of the component you choose.

3.

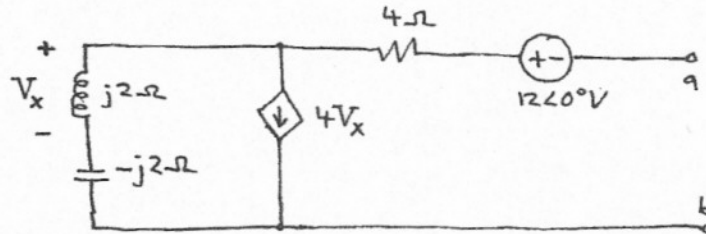


pts

- Draw a frequency-domain equivalent of the above circuit. Show a numerical phasor value for  $v_S(t)$ , and show numerical impedance values for R, L, and C. Label the dependent source appropriately.
- Find the Thevenin equivalent (in the frequency domain) for the above circuit. Give the numerical phasor value for  $V_{Th}$  and the numerical impedance value of  $z_{Th}$ .

sol'n: a)  $\omega = 100$  from  $v_S(t)$   $j\omega L = j100 \cdot 20 \text{ m}\Omega = j2\Omega$   
 $\frac{-j}{\omega C} = \frac{-j}{100 \cdot 5 \text{ m}} \Omega = \frac{-j}{500 \text{ m}} = -j2\Omega$

phasor  $V_S \equiv P [12 \cos(100t)] V = 12 \angle 0^\circ V$



b)  $V_{Th} = V_{a,b}$  with no load.

We have  $z_L + z_C = 0 \Omega$  so 0V across L & C together.

Also, no current in  $4\Omega \Rightarrow 0V$  across  $4\Omega$ .

Add the -12V for v-src to get  $V_{Th} = -12V$

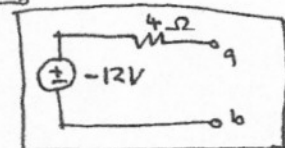
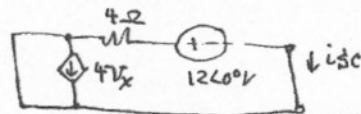
For  $z_{Th}$ , short a,b and measure i out of a terminal.

Circuit model:

$4V_x$  irrelevant

$i_{sc} = \frac{-12V}{4\Omega} = -3A$

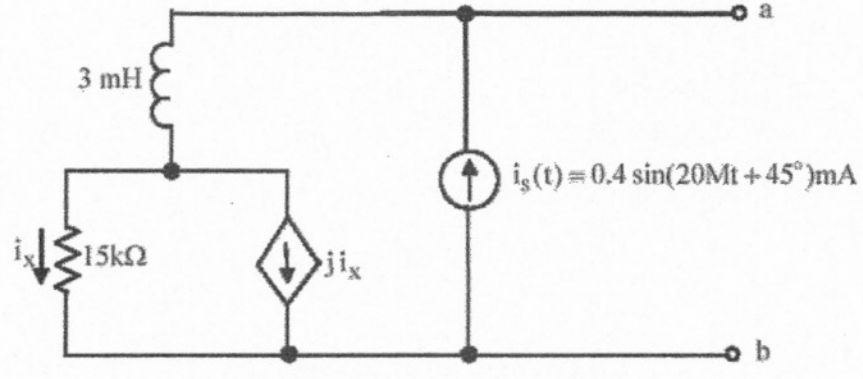
$z_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{-12V}{-3A} = 4\Omega$





Homework #8 Examples

3.



Find the Thevenin equivalent (in the frequency domain) for the above circuit. Give the numerical phasor value for  $V_{Th}$  and the numerical impedance value of  $Z_{Th}$ .

## Homework #8 Example #2

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Give numerical answers to each of the following questions:

1.

a) Rationalize  $\frac{-25j}{3-4j}$ . Express your answer in rectangular form.

b) Find the rectangular form of  $\left[ \frac{(1+j)}{e^{j30^\circ}} \right] \left[ \frac{(1+j)}{e^{-j60^\circ}} \right]^*$ . (Note the asterisk that means "conjugate".)

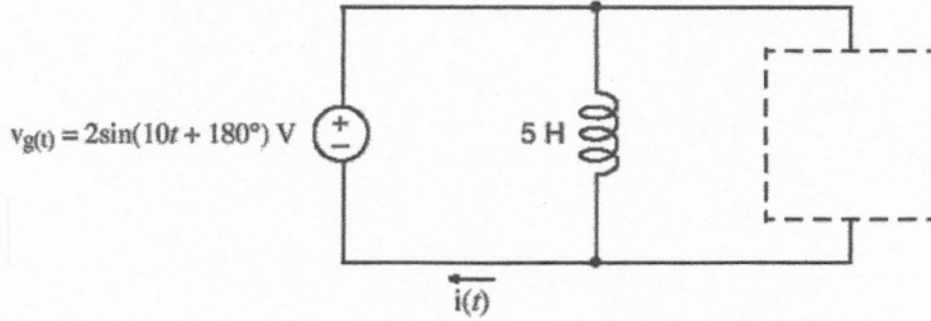
c) Given  $\omega = 2\pi$  rad/s, find the following inverse phasor:  
 $P^{-1} = [10(-0.866 - 0.5j)]$

d) Find the magnitude of  $\frac{\left(4e^{j30^\circ} - \frac{1}{2}j\right)(-1-j)}{\sqrt{2}e^{j10^\circ}}$ .

e) Find the real part of  $\frac{e^{-2}}{e^{-j45^\circ}}$ .

Homework #8 Example #2

2.



Choose an R, an L, or a C to be placed in the dashed-line box to make

$$i(t) = I_o \cos(10t + 45^\circ) \text{ A}$$

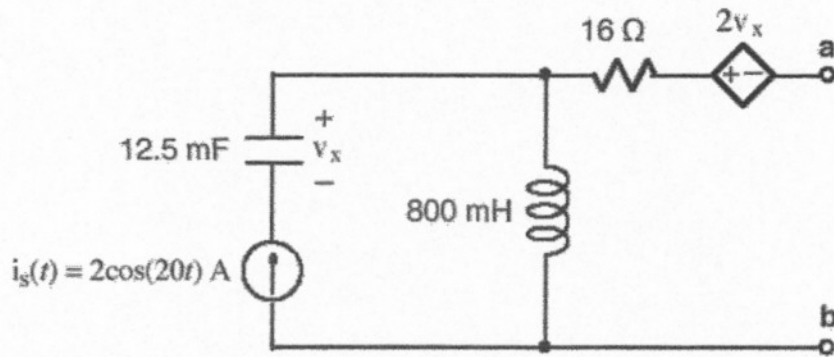
where  $I_o$  is a real constant. State the value of the component you choose.

With your component from Problem 2 in the circuit, calculate the resulting value of  $I_o$ .

Homework #8 Example #2

183

3.



- Draw a frequency-domain equivalent of the above circuit. Show a numerical phasor value for  $i_s(t)$ , and show numerical impedance values for  $R$ ,  $L$ , and  $C$ . Label the dependent source appropriately.
- Find the Thevenin equivalent (in the frequency domain) for the above circuit. Give the numerical phasor value for  $V_{Th}$  and the numerical impedance value of  $z_{Th}$ .

## Homework #8 Example #3

184

1. Give numerical answers to each of the following questions:

a. Rationalize  $\frac{42 - j26}{6 + j5}$ . Express your answer in rectangular form.

b. Find the rectangular form of  $\frac{1 - e^{j90^\circ}}{e^{-j90^\circ}} \left[ \frac{1 - j}{e^{j45^\circ}} \right]^*$ . Note the asterisk that means "conjugate".

c. Given  $\omega = 2\pi$  rad/s, find the following inverse phasor:  $P^{-1}[(-1 + j) \cdot 30]$ .

d. Find the magnitude of  $\left[ (-1 - j)e^{j135^\circ} + j\sqrt{2} \right] e^{j45^\circ}$ .

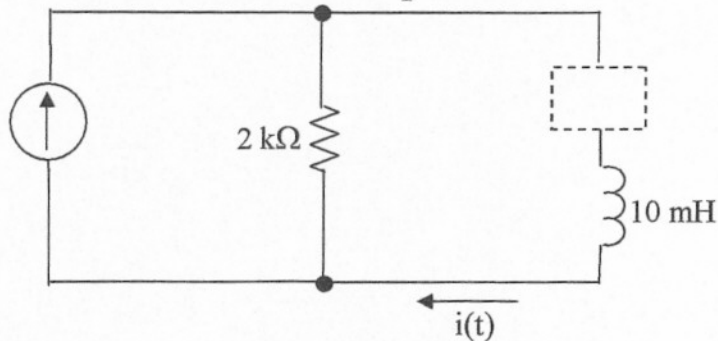
e. Find the real part of  $e^{-3 + j180^\circ}$ .

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### Homework #8 Example #3

2.

$$i_g(t) = 4 \cos(20kt + 60^\circ) \text{ mA}$$



- a. Choose an R, an L, or a C to be placed in the dashed-line box to make

$$i(t) = I_0 \cos(20kt) \text{ A}$$

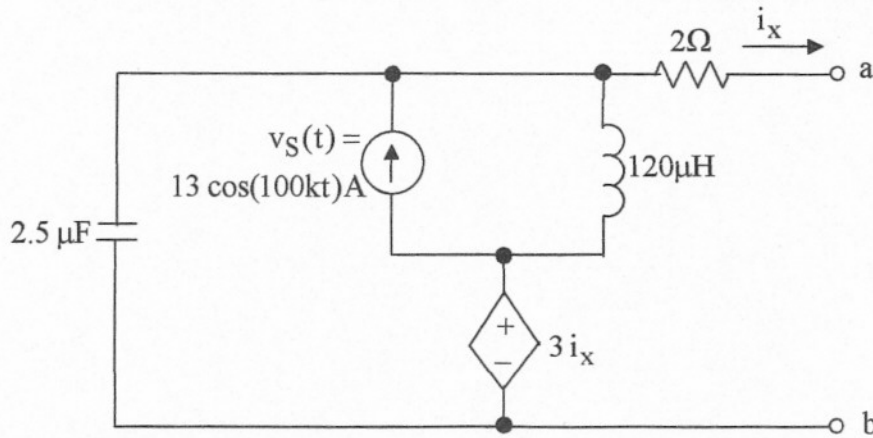
where  $I_0$  is a real constant. State the value of the component you choose.

- b. With your component from (a) in the circuit, calculate the resulting value of  $I_0$ .

### Homework #8 Example #3

(186)

3.



- a. Draw a frequency-domain equivalent of the above circuit. Show a numerical phasor value for  $v_S(t)$ , and show numerical impedance values for R, L, and C. Label the dependent source appropriately.
- b. Find the Thevenin equivalent (in the frequency domain) for the above circuit. Give the numerical phasor value for  $V_{\text{TH}}$  and the numerical impedance value of  $z_{\text{TH}}$ .

## Homework #8 Example #4

187

1. Give numerical answers to each of the following questions:

a) Rationalize  $\frac{120 - j22}{-11 + j60}$ . Express your answer in rectangular form.

b) Find the polar form of  $j(1 + j) * e^{j30^\circ}$ . (Note the asterisk that means "conjugate".)

c) Find the following phasor:  $P[-7\cos(49t + 135^\circ)]$ .

d) Find the magnitude of  $\left(\frac{24 + j7}{3 - j4}\right) \left(\frac{-1}{e^{j10^\circ}}\right)$ .

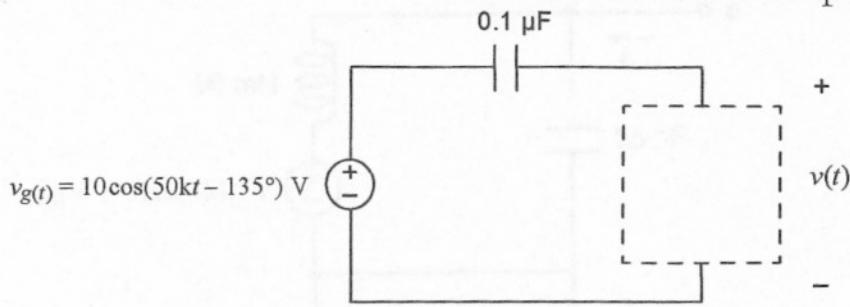
e) Find the imaginary part of  $\frac{e^{j45^\circ}}{e^{-j225^\circ}}$ .



# Homework #8 Example #4

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2.



- a) Choose an R, an L, or a C to be placed in the dashed-line box to make  $v(t) = V_0 \sin(50kt)$

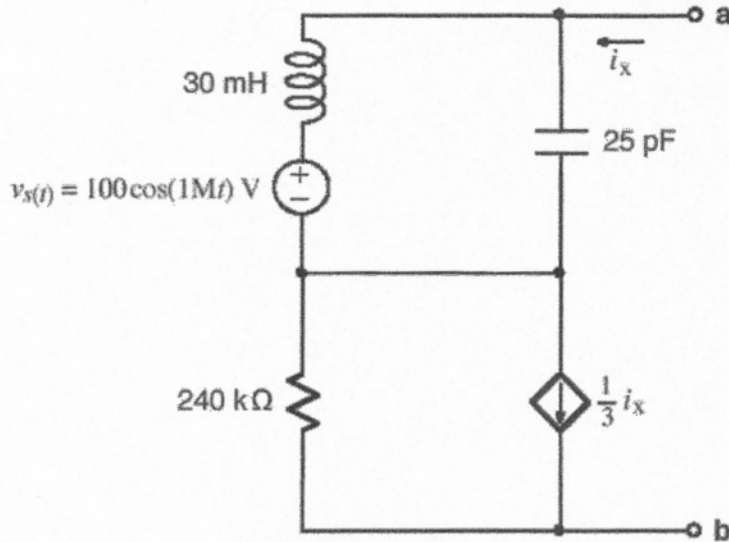
where  $V_0$  is a positive real constant (with units of Volts). State the value of the component you choose.

- b) With your component from (a) in the circuit, calculate the resulting value of  $V_0$ .

# Homework #8 Example #4

189

3.



- Draw a frequency-domain equivalent of the above circuit. Show a numerical phasor value for  $v_s(t)$ , and show numerical impedance values for R, L, and C. Label the dependent source appropriately.
- Find the Thevenin equivalent (in the frequency domain) for the above circuit. Give the numerical phasor value for  $V_{Th}$  and the numerical impedance value of  $z_{Th}$ .