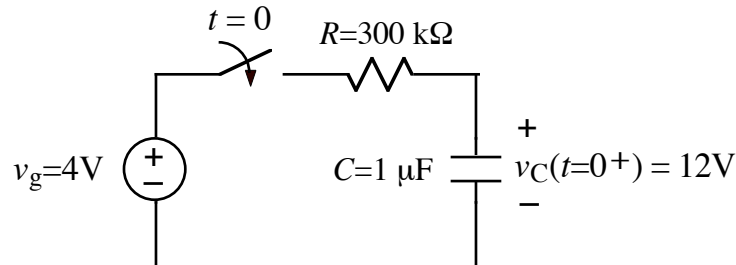


Ex: After being open for a long time, the switch closes at $t = 0$.



- Find an expression for $v_C(t)$ for $t \geq 0$.
- Find the energy stored in the capacitor at time $t = 10\text{ ms}$.

SOL'N: a) The following general form of solution applies to any RC circuit with a single capacitor:

$$v_C(t \geq 0) = v_C(t \rightarrow \infty) + [v_C(t = 0^+) - v_C(t \rightarrow \infty)]e^{-t/R_{\text{Th}}C}$$

The Thevenin resistance, R_{Th} , is for the circuit after $t = 0$ (with the C removed) as seen from the terminals where the C is connected. In the present case, we have $R_{\text{Th}} = 300\text{ k}\Omega$.

$$R_{\text{Th}}C = 300\text{ k}\Omega \cdot 1\text{ }\mu\text{F} = 300\text{ ms}$$

The value of $v_C(t=0)$ is given in the problem as 12 V. Note that the C could have any voltage before $t = 0$ in this circuit if the value were not specified. The voltage would stay on the ideal C indefinitely prior to $t = 0$.

As time approaches infinity, the C will charge to its final value, and current will cease to flow in the C . Thus, the C will become an open circuit. It follows that the current through the R , which is the same as the current through the C , will become zero. By Ohm's law, this in turn means that the voltage drop across the R will become zero, and the voltage across the C will be the same as the source voltage, 4 V.

$$v_C(t \rightarrow \infty) = 4\text{ V}$$

Substituting values, we have the following result:

$$v_C(t \geq 0) = 4\text{ V} + [12\text{ V} - 4\text{ V}]e^{-t/300\text{ms}} = 4\text{ V} + 8\text{ V} \cdot e^{-t/300\text{ms}}$$

b) The energy in a capacitor is given by the following formula:

$$w_C = \frac{1}{2} C v_C^2$$

We use the solution to (a) to evaluate $v_C(t)$ at $t = 10$ ms.

$$v_C(t = 10 \text{ ms}) = 4 \text{ V} + 8 \text{ V} \cdot e^{-10\text{ms}/300\text{ms}} = 11.74 \text{ V}$$

Using this voltage, we evaluate the energy on the capacitor.

$$w_C = \frac{1}{2} 1\mu\text{F} \cdot (11.74 \text{ V})^2 = 68.9 \mu\text{J}$$