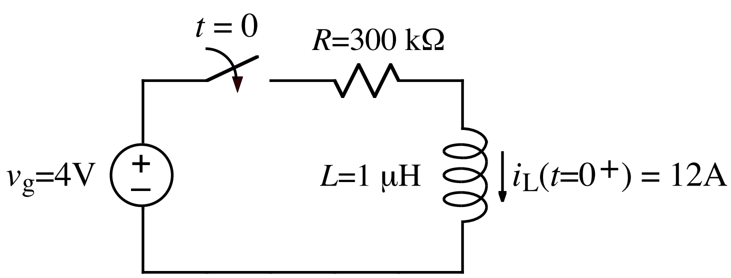
HOMEWORK #10 solution

Ex:



a) Find an expression for $i_L(t)$ for $t \geq 0$. Note: Assume the initial current in the L is created by circuitry not shown in the diagram.

b) Find the energy stored in the inductor at time t = 10 ms.

SOL'N: a) The following general form of solution applies to any RL circuit with a single inductor:

$$i_L(t \geq 0) = i_L(t \to \infty) + [i_L(t = 0^+) - i_L(t \to \infty)]e^{-t/(L/R_{Th})}$$

The Thevenin resistance, R_{Th}, is for the circuit after t = 0 (with the L removed) as seen from the terminals where the L is connected. In the present case, we have R_{Th} = 300 kΩ.

$$L/R_{Th} = 1\ \mu\text{H}/300\ \text{k}\Omega = 3.33\ \text{ps}$$

The value of $i_L(t{=}0)$ is given in the problem as 12 A, (created by circuitry not shown).

As time approaches infinity, the L current will converge to its final value, and the voltage across the L will cease to change. Thus, di_L/dt = 0 and $v_L = 0$, meaning the L will act like a wire. It follows that the current through the L will equal the current through the R, which will equal 4 V/300 kΩ = 13.3 μA.

$$i_L(t \to \infty) = 13.3\ \mu\text{A}$$

Substituting values, we have the following result:

$$i_L(t \geq 0) = 13.3\ \mu\text{A} + [12 - 13.3\ \mu\text{A}]e^{-t/3.33\text{ps}}$$
$$= 13.3\ \mu\text{A} - 1.3\ \mu\text{A}e^{-t/3.33\text{ps}}$$

b) The energy in an inductor is given by the following formula:

$$w_L = \frac{1}{2} L i_L^2$$

We use the solution to (a) to evaluate $i_L(t)$ at t = 10 ms.

$$i_L(t = 10\text{ms}) = 13.3\ \mu\text{A V} - 1.3\ \mu\text{A} \cdot e^{-10\text{ms}/3.33\text{ps}} \approx 13.3\ \mu\text{A}$$

NOTE: 10 ms is 3 billion time constants, (where the time constant is τ = 3.33 ps), after time zero. By that time, the inductor current is extremely close to its final value. That is to say $e^{-3,000,000,000}$ is extremely close to zero.

Using the current at t = 10 ms, we evaluate the energy on the inductor.

$$w_L = \frac{1}{2} 1\mu\text{H} \cdot (13.3\mu\text{A})^2 = 88.9\ \text{fJ}$$