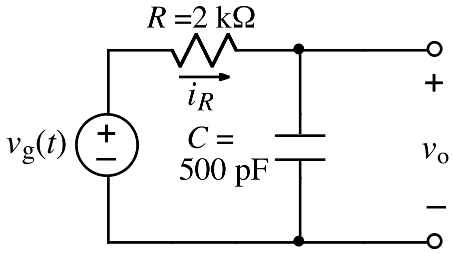
HOMEWORK #10 solution

Ex: After being zero for a long time, the value of $v_g(t)$ changes to 9 V at t = 0 (and remains at 9 V as time increases to infinity).



a) Find an expression for $v_o(t)$ for $t > 0$.

b) Find the current, i_R, in R as a function of time.

Sol'n: a) The following general form of solution applies to any RC circuit with a single capacitor:

$$v_C(t \geq 0) = v_C(t \to \infty) + [v_C(t = 0^+) - v_C(t \to \infty)]e^{-t/R_{Th}C}$$

The Thevenin resistance, R_{Th}, is for the circuit after t = 0 (with the C removed) as seen from the terminals where the C is connected. In the present case, we have R_{Th} = 2 kΩ.

$$R_{Th}C = 2\text{ k}\Omega \cdot 500\text{ pF} = 1\ \mu\text{s}$$

For time $t = 0^-$, the voltage source will be off and the capacitor will have discharged to zero volts. Since the voltage on the capacitor is an energy variable, it will not change instantly. Thus, the initial capacitor voltage is zero.

$$v_C(t = 0^+) = 0\text{ V}$$

For time approaching infinity, the capacitor will charge to a final value and no current will flow in the capacitor. Thus, the capacitor will act like an open circuit. It follows that no current will flow in the R as time approaches infinity, and the voltage drop across R will be zero. Thus, the voltage on C will be $v_g(t)$ = 9V:

$$v_C(t \to \infty) = v_g(t) = 9\text{ V}$$

Substituting values, we have the following result:

$$v_o(t \geq 0) = v_C(t \geq 0) = 9 \text{ V} + [0 \text{ V} - 9 \text{ V}]e^{-t/1\mu s} = 9 \text{ V} \cdot (1 - e^{-t/1\mu s})$$

b) The following general form of solution applies to any current in any RC circuit with a single capacitor:

$$i(t \geq 0) = i(t \to \infty) + [i(t = 0^+) - i(t \to \infty)]e^{-t/R_{Th}C}$$

In the present case, this applies to the resistor current:

$$i_R(t \geq 0) = i_R(t \to \infty) + [i_R(t = 0^+) - i_R(t \to \infty)]e^{-t/R_{Th}C}$$

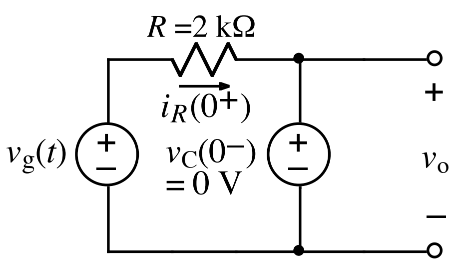
The Thevenin resistance, R_{Th}, is for the circuit after $t = 0$ (with the C removed) as seen from the terminals where the C is connected. In the present case, we have $R_{Th} = 2$ kΩ.

$$R_{Th}C = 2 \text{ k}\Omega \cdot 500 \text{ pF} = 1 \text{ μs}$$

For time $t = 0^-$, the voltage source will be off and the capacitor will have discharged to zero volts. Since the voltage on the capacitor is an energy variable, it will not change instantly. Thus, the initial capacitor voltage is zero.

$$v_C(t = 0^+) = 0 \text{ V}$$

At time $t = 0^+$, we model the capacitor as a voltage source, (with the voltage the capacitor had at time t = 0^-), and find $i_R(t = 0^+)$:



We find the current in the R by taking the voltage drop across the R and using Ohm's law:

$$i_R(t = 0^+) = \frac{v_g(t) - v_C(t = 0^+)}{R} = \frac{v_g(t) - v_C(t = 0^-)}{R}$$

or

$$i_R(t = 0^+) = \frac{9 \text{ V} - 0 \text{ V}}{2 \text{ k}\Omega} = 4.5 \text{ mA}$$

For time approaching infinity, the capacitor will charge to a final value and no current will flow in the capacitor. Thus, the capacitor will act like an open circuit. It follows that no current will flow in the R as time approaches infinity.

$$i_R(t \to \infty) = 0 \text{ A}$$

Substituting values, we have the following result:

$$i_R(t \geq 0) = 0 \text{ mA} + [4.5 - 0 \text{ mA}]e^{-t/1\mu\text{s}} = 4.5 \text{ mA} \cdot e^{-t/1\mu\text{s}}$$