HOMEWORK #13 solution prob 3

Ex: Give numerical answers to each of the following questions:

a) Rationalize $\dfrac{25 - j60}{-12 + j5}$. Express your answer in rectangular form.

b) Find the polar form of $\dfrac{\sqrt{3}}{2} - j\dfrac{1}{2}$.

c) Find the rectangular form of $4\angle 5° \cdot \sqrt{2}\angle 40°$

d) Find the magnitude of $\left(\dfrac{j^j}{1+j}\right)\left(\dfrac{6e^{j3.14°}}{1-j}\right)$.

e) Find the real part of $\dfrac{(1-j)^2}{\sqrt{2} + j\sqrt{2}}$.

Sol'n: a) To rationalize, we multiply the numerator and denominator by the conjugate of the denominator.

$$\frac{25 - j60}{-12 + j5} \cdot \frac{-12 - j5}{-12 - j5} = \frac{-25(12) - 60(5) - j25(5) + j60(12)}{(-12)^2 + 5^2}$$

$$\frac{25 - j60}{-12 + j5} = \frac{-600 + j595}{169} \approx -3.55 + j3.52$$

b) We think of the complex number as a vector and find its length and its angle relative to the real axis.

$$\frac{\sqrt{3}}{2} - j\frac{1}{2} = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2}\, e^{j\tan^{-1}\frac{-1/2}{\sqrt{3}/2}} = \sqrt{\frac{3}{4} + \frac{1}{4}}\, e^{-j30°} = 1e^{-j30°}$$

or

$$\frac{1}{2} + j\frac{\sqrt{3}}{2} = e^{-j30°}$$

c) We first multiply the numbers in polar form.

$$4\angle 5° \cdot \sqrt{2}\angle 40° = 4\sqrt{2}\angle(5° + 40°) = 4\sqrt{2}\angle 45° = 4\sqrt{2}e^{j45°}$$

Now we convert to rectangular form using Euler's formula.

$$4\sqrt{5}\angle 5^\circ \cdot \sqrt{2}\angle 40^\circ = 4\sqrt{2}\cos(45^\circ) + j4\sqrt{2}\sin(45^\circ)$$

$$= 4\sqrt{2} \cdot \frac{\sqrt{2}}{2} + j4\sqrt{2}\frac{\sqrt{2}}{2}$$

or

$$4\angle 5^\circ \cdot \sqrt{2}\angle 40^\circ = 4 + j4$$

d) We take the magnitude of each term of a product or quotient. We must keep each sum as is, however. When we take the magnitude, we may ignore exponentials with purely imaginary exponents, as they always have magnitude equal to unity.

$$\left|\left(\frac{j^j}{1+j}\right)\left(\frac{6e^{j3.14^\circ}}{1-j}\right)\right| = \frac{\left|j^j\right|}{|1+j|}\frac{\left|6e^{j3.14^\circ}\right|}{|1-j|} = \frac{\left|e^{\left(j\frac{\pi}{2}\right)^j}\right| \cdot 6}{\sqrt{1^2+1^2}\sqrt{1^2+(-1)^2}}$$

or

$$\left|\left(\frac{j^j}{1+j}\right)\left(\frac{6e^{j3.14^\circ}}{1-j}\right)\right| = \frac{e^{-\frac{\pi}{2}} \cdot 6}{\sqrt{2}\cdot\sqrt{2}} = \frac{6e^{-\frac{\pi}{2}}}{2} = 3e^{-\frac{\pi}{2}}$$

e) Use polar to find the quotient. Then use rectangular form to find Re[ ].

$$\mathrm{Re}\left[\frac{(1-j)^2}{\sqrt{2}+j\sqrt{2}}\right] = \mathrm{Re}\left[\frac{\left(\sqrt{2}e^{-j45^\circ}\right)^2}{2e^{j45^\circ}}\right] = \mathrm{Re}\left[\frac{2e^{-j90^\circ}}{2e^{j45^\circ}}\right] = \mathrm{Re}\left[e^{-j90^\circ-45^\circ)}\right]$$

or

$$\mathrm{Re}\left[\frac{(1-j)^2}{\sqrt{2}+j\sqrt{2}}\right] = \mathrm{Re}\left[e^{-j135^\circ}\right] = \mathrm{Re}\left[\cos(-135^\circ) + j\sin(-135^\circ)\right]$$

or

$$\mathrm{Re}\left[\frac{(1-j)^2}{\sqrt{2}+j\sqrt{2}}\right] = \cos(-135^\circ) = \frac{-\sqrt{2}}{2}$$