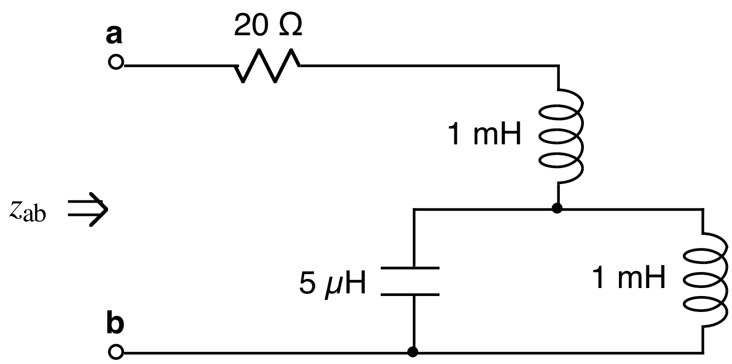
# HOMEWORK #15 solution prob 2

**Ex:**




Find a frequency, ω, that causes $z_{ab}$ to be real, (i.e., imaginary part equals zero).

Sol'n: $z_{ab} = 20\,\Omega + z_{L1} + z_C \| z_{L2}$

For $z_{ab}$ to be real, we must have

$$z_{L1} + z_C \| z_{L2} = \text{real}$$

One simple sol'n is to let $\omega = 0$ so both L's act like wires and C acts like open circuit.

Other potential sol'ns are $\omega = \infty$, (so L's act like opens, resulting in $z_{ab} = \infty$), and $\omega =$ frequency where $z_C = -z_{L2}$, (so C and L in parallel have equal but opposite impedances).

The latter case, where $z_C = -z_L$ gives the interesting result that $z_C \| z_L = \dfrac{L/C}{0} = \infty$

This means $z_{ab} = \infty\,\Omega$. In this case, (unlike $\omega \to \infty$), $z_{ab} \to \infty$ along real axis as $z_C \| z_L \to \infty$.

Another sol'n is that $z_C \| z_L$ has a value is minus $z_L$ of the top inductor.

In that case, $z_L + z_C \| z_L = 0$ and $z_{ab} = 0 =$ wire.

$$z_L = j\omega L$$

$$z_C \| z_L = \frac{-j}{\omega C} \| j\omega L = \frac{\frac{-j}{\omega C} \cdot j\omega L}{\frac{-j}{\omega C} + j\omega L} = \frac{L/C}{j(\omega L - \frac{1}{\omega C})}$$

$$= -j \frac{L/C}{\omega L - \frac{1}{\omega C}}$$

Thus, we want $j\omega L - \dfrac{jL/C}{\omega L - \frac{1}{\omega C}} = 0$

or $\omega L = \dfrac{L/C}{\omega L - \frac{1}{\omega C}}$

or $\omega L \left(\omega L - \frac{1}{\omega C}\right) = L/C$

or $\omega^2 L^2 - \dfrac{L}{C} = \dfrac{L}{C}$

or $\omega^2 L^2 = \dfrac{2L}{C}$ or $\omega^2 = \dfrac{2}{LC}$

or $\omega = \sqrt{\dfrac{2}{LC}}$ or $\omega = \sqrt{\dfrac{2}{5\mu F \cdot 1 mH}}$

or $\omega = \sqrt{\dfrac{2}{5} G}$ r/s $= \sqrt{400M}$ r/s

or $\omega = 20k$ r/s