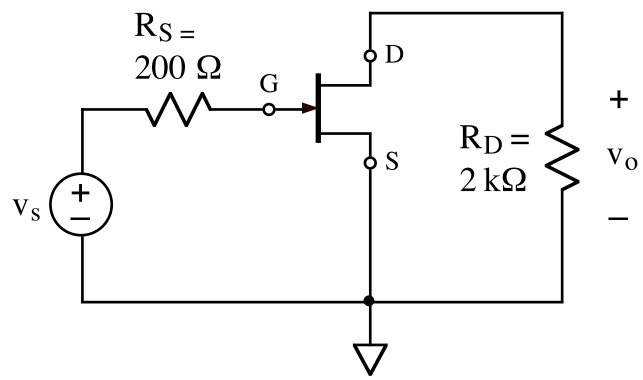
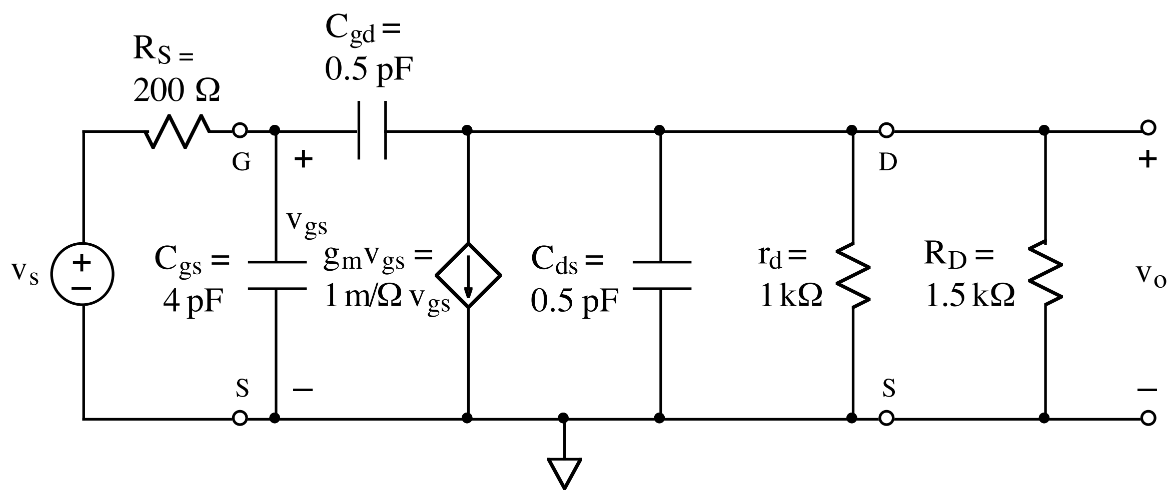
HOMEWORK #15 solution

Ex:





$v_s(t) = 2\cos(10kt)$ V

The above circuit diagrams show a common-source JFET amplifier and its high-frequency equivalent circuit. Find $v_o(t)$.

Sol'n: In this practical circuit, we have circuit values that allow us to make simplifying approximations.

We first calculate impedance values.

$\omega = 10k$ r/s from $v_s(t) = 2\cos(10kt)$ V

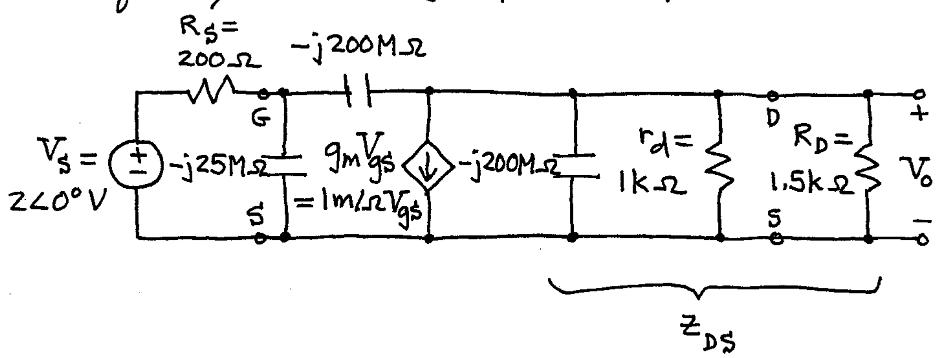
$$z_{C_{gs}} = \frac{-j}{\omega C_{gs}} = \frac{-j}{10k \cdot 4p}\,\Omega = -j\,25M\,\Omega$$

$$z_{C_{gd}} = \frac{-j}{\omega C_{gd}} = \frac{-j}{10k\,\frac{1}{2}p} = -j\,200M\,\Omega$$

$$z_{C_{ds}} = \frac{-j}{\omega C_{ds}} = \frac{-j}{10k\,\frac{1}{2}p} = -j\,200M\,\Omega$$

The phasor for $v_s(t)$ is $V_s = 2\angle 0°$ V.

Frequency domain (or s-domain) model:



$$z_{DS} = -j\,200M \,\|\, 1k\Omega \,\|\, 1.5k\Omega$$

Starting with $1k\Omega \| 1.5k\Omega$ we have

$$1k\Omega \,\|\, 1.5\,k\Omega = 500\Omega \cdot 2\|3 = 500 \cdot \frac{6}{5}$$

$$= 600\Omega$$

Thus, $z_{DS} = -j\,200M \,\|\, 600\Omega = \dfrac{1}{\dfrac{1}{600} - \dfrac{1}{j200M}}$

Using $-\frac{1}{j} = j$ and rationalizing gives

$$z_{DS} = \frac{1}{\frac{1}{600} + \frac{j}{200M}} \; \frac{\frac{1}{600} - \frac{j}{200M}}{\frac{1}{600} - \frac{j}{200M}} \; \Omega$$

$$= \frac{\frac{1}{600} - \frac{j}{200M}}{\left(\frac{1}{600}\right)^2 + \left(\frac{1}{200M}\right)^2} \; \Omega$$

$$\approx \frac{\frac{1}{600} - \frac{j}{200M}}{\left(\frac{1}{600}\right)^2} \; \Omega \quad \text{Since } \frac{1}{200M} \ll \frac{1}{600}$$

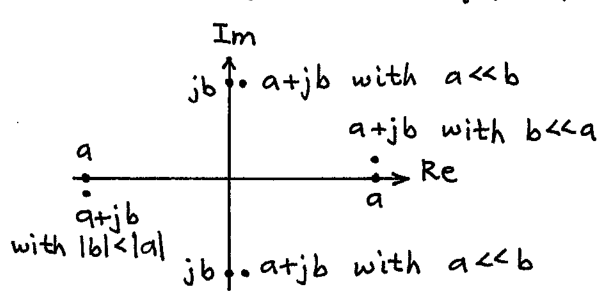
$$z_{DS} \approx \frac{\frac{1}{600}}{\left(\frac{1}{600}\right)^2} \; \Omega = 600 \, \Omega$$

In retrospect, we could have made the approximation that $-j200M \| 600\,\Omega \approx 600\,\Omega$.

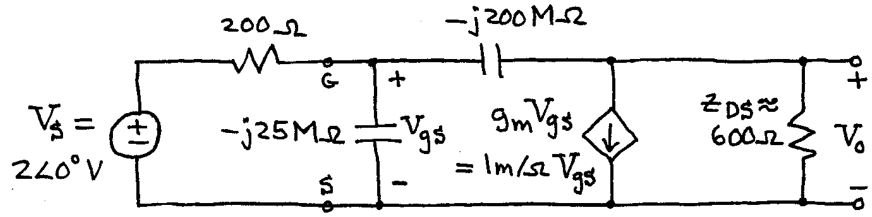
We may make this approximation despite the j in one of the quantities. In general, we may make the following approximations of complex values:

$$a + jb \approx a \quad \text{when } |b| \ll |a|$$

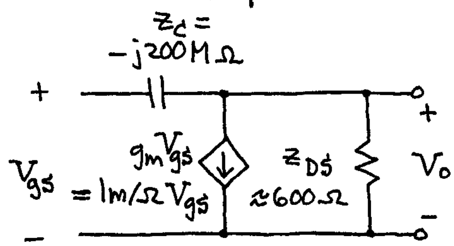
$$a + jb \approx jb \quad \text{when } |a| \ll |b|$$



With our Z_{DS} value, we have a simplified model:



We now analyze the dependent source so we can replace it with an impedance, Z_{eq}.



$$Z_{eq} = \frac{V_o}{g_m V_{gs}}$$ using Ohm's law to write $Z_{eq} = V/I$

Now we find a way to write V_o in terms of V_{gs}. We use a V-divider:

$$V_o = V_{gs} \frac{Z_{eq} \parallel Z_{DS}}{Z_{eq} \parallel Z_{DS} + Z_C}$$

Substituting for V_o in our Z_{eq} eq'n, we have

$$Z_{eq} = \frac{\cancel{V_{gs}} \dfrac{Z_{eq} \parallel Z_{DS}}{Z_{eq} \parallel Z_{DS} + Z_C}}{\cancel{V_{gs}} \cdot g_m}$$

$$Z_{eq} = \frac{1}{g_m} \frac{\dfrac{Z_{eq} Z_{DS}}{Z_{eq} + Z_{DS}}}{\dfrac{Z_{eq} Z_{DS}}{Z_{eq} + Z_{DS}} + Z_C}$$

$$z_{eq} = \frac{1}{g_m} \frac{z_{eq} z_{DS}}{z_{eq} z_{DS} + z_C (z_{eq} + z_{DS})}$$

Dividing top and bottom by z_{eq} gives the following:

$$z_{eq} = \frac{1}{g_m} \frac{z_{DS}}{z_{DS} + z_C + \frac{z_C z_{DS}}{z_{eq}}}$$

$$z_{eq} \left(z_{DS} + z_C + \frac{z_C z_{DS}}{z_{eq}} \right) = \frac{1}{g_m} z_{DS}$$

$$z_{eq} (z_{DS} + z_C) + z_C z_{DS} = \frac{1}{g_m} z_{DS}$$

$$z_{eq} \frac{(z_{DS} + z_C)}{z_{DS}} = \frac{\frac{1}{g_m} \cancel{z_{DS}} - z_C \cancel{z_{DS}}}{\cancel{z_{DS}}}$$

$$z_{eq} = \frac{\frac{1}{g_m} - z_C}{1 + \frac{z_C}{z_{DS}}} = \frac{\frac{1\Omega}{1m} - -j200M\Omega}{1 + \frac{-j200M\Omega}{600\Omega}}$$

$$z_{eq} = \frac{1k + j200M}{1 - j\frac{1}{3}M} \Omega$$

The imaginary parts of the numerator and denominator are much larger than the real parts. Thus, we ignore the real parts.

$$z_{eq} \approx j200M\Omega / -j\tfrac{1}{3}M \approx -600\Omega$$

Now we have a problem: $z_{eq} \| z_{DS} = -\dfrac{600^2}{0}\,\Omega$.

That means $z_{eq} \| z_{DS} = \infty\ \Omega$.

It is a good idea to try a more exact calculation to be sure that $z_{eq} \| z_{DS}$ is much larger than $z_C = -j\,200\,M\Omega$.

We use conductance to simplify calculations.

$$\frac{1}{z_{eq} \| z_{DS}} = \frac{1}{z_{eq}} + \frac{1}{z_{DS}} = \frac{1 + \frac{z_C}{z_{DS}}}{\frac{1}{g_m} - z_C} + \frac{1}{z_{DS}}$$

$$= \frac{1}{z_{DS}} \left(\frac{z_{DS} + z_C}{\frac{1}{g_m} - z_C} + 1 \right)$$

$$= \frac{1}{z_{DS}} \left(\frac{z_{DS} + z_C}{\frac{1}{g_m} - z_C} + \frac{\frac{1}{g_m} - z_C}{\frac{1}{g_m} - z_C} \right)$$

$$= \frac{1}{z_{DS}} \left(\frac{z_{DS} + \frac{1}{g_m}}{\frac{1}{g_m} - z_C} \right)$$

$$= \frac{g_m + \frac{1}{z_{DS}}}{1 - g_m z_C}$$

$$= \frac{1m + \frac{1}{600}}{1 - 1m\left(-j\,200M\right)} \ /\Omega$$

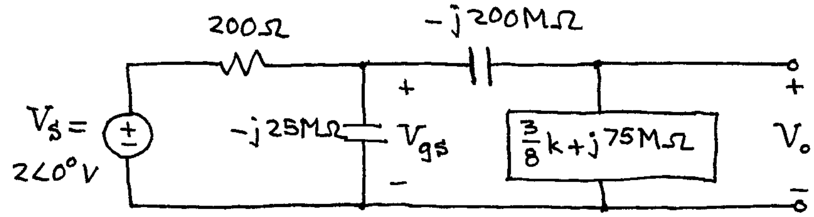
$$z_{eq} \| z_{DS} = \frac{1 + j\,200\,K}{\frac{1}{1K} + \frac{1}{600}}\ \Omega$$

$$z_{eq} \parallel z_{DS} = \frac{1 + j\,200k}{\frac{3+5}{3k}}\ \Omega = \frac{3}{8}k\,(1+j200k)\ \Omega$$

$$= \frac{3}{8}k + j\,75M\Omega$$

We see that the value is <u>smaller</u> than $z_C = -j\,200\ M\Omega$.

Our new, simplified model:



We use V-dividers to find V_o.

$$V_{gs} = V_s \cdot \frac{-j25M\Omega \parallel \left(-j200M + \overset{\text{small}}{\cancel{\tfrac{3}{8}k}} + j\,75M\Omega\right)}{200 + -j25M\Omega \parallel \left(-j200M + \overset{\text{small}}{\cancel{\tfrac{3}{8}k}} + j75M\Omega\right)}$$

$$V_{gs} \doteq \frac{V_s \cdot (-j25M\Omega \parallel -j\,125\,M\Omega)}{200 - j25M\Omega \parallel -j125M\Omega}$$

where $-j25M\Omega \parallel -j125M\Omega = -j25M\Omega \cdot 1 \parallel 5$

$$= -j25M\Omega \cdot \frac{5}{6}$$

$$= -j\,\frac{125}{6}\ M\Omega$$

$$V_{gs} = \frac{V_s \cdot \left(-j\,\frac{125}{6} M\Omega\right)}{\overset{\text{small}}{\cancel{200}} - j\,\frac{125}{6} M\Omega} \approx V_s$$

$$V_o = V_{gs}\,\frac{\overset{\text{small}}{\cancel{\tfrac{3}{8}k}} + j\,75\,M\Omega}{\overset{\text{small}}{\cancel{\tfrac{3}{8}k}} + j\,75\,M\Omega - j\,200\,M\Omega}$$

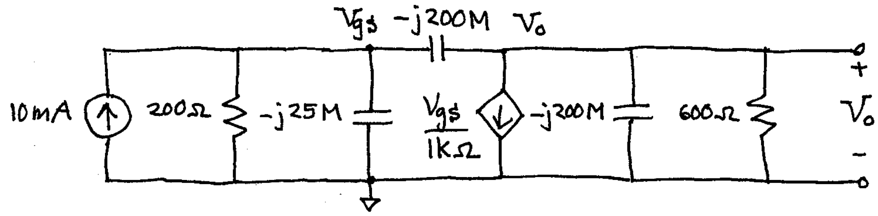
$$V_o \approx V_{gs}\,\frac{j\,75\,M\Omega}{-j\,125\,M\Omega} = V_{gs}\left(-\frac{3}{5}\right)$$

$$V_o \approx 2\angle 0^\circ V\left(-\frac{3}{5}\right) = -\frac{6}{5}\angle 0^\circ V = \frac{6}{5}\angle 180^\circ\ V$$

Note: a minus sign is the same as 180° of phase shift.

$$v_o(t) = \frac{6}{5}\cos\left(10k t + 180^\circ\right)\ V$$

Alternate approach to solution below.



Using exact Node-Voltage method:

1) $0A = -10mA + V_{gs}\left(\frac{1}{200\Omega} + \frac{1}{-j25M\Omega} + \frac{1}{-j200M\Omega}\right) - V_o\frac{1}{-j200M\Omega}$

2) $0A = V_{gs}\left(\frac{-1}{-j200M\Omega} + \frac{1}{1k\Omega}\right) + V_o\left(\frac{1}{-j200M\Omega} + \frac{1}{-j200M\Omega} + \frac{1}{600\Omega}\right)$

Solve eq'n (2) for V_{gs} and substitute in eq'n (1).

2') $V_{gs} = -V_o\frac{\left(\frac{1}{-j100M\Omega} + \frac{1}{600\Omega}\right)}{\frac{1}{j200M\Omega} + \frac{1}{1k\Omega}} = V_o\frac{\left(\frac{-j}{100M\Omega} - \frac{1}{600\Omega}\right)}{\frac{-j}{200M\Omega} + \frac{1}{1k\Omega}}$

Multiply eq'n (1) by $200M\Omega$ to clear denominator:

1') $10mA \cdot 200M\Omega = V_{gs}(1M + j8 + j1) - V_o j$

Multiply eq'n (2') by $\frac{200M\Omega}{200M\Omega}$:

2'') $V_{gs} = V_o\left(\frac{-j2 - \frac{1}{3}M}{-j + 200k}\right) = V_o\frac{\frac{1}{3}M + j2}{-200k + j}$

1'') $2MV = V_{gs}(1M+j9) - V_o j = V_o\left[\left(\frac{\frac{1}{3}M + j2}{-200k+j}\right)(1M+j9) - j\right]$

1''') $V_o \approx \frac{2\cancel{M}V(-200\cancel{k})}{\frac{1}{3}\cancel{M}\cdot 1\cancel{M}_{k}} = -1.2V$ Agrees with previous answer