HOMEWORK #16 solution prob 1

Ex: Give numerical answers to each of the following questions:

a) Rationalize $\dfrac{1-j\sqrt{3}}{1+j\sqrt{3}}$. Express your answer in rectangular form.

b) Find the polar form of $[(j+1)(-1-j)]^*$. (Note: the asterisk means "conjugate".)

c) Find the following phasor: $\mathrm{P}[-\sin(100t+45°)]$.

d) Find the magnitude of $\dfrac{(30-j40)}{(7-j24)e^{j30°}}$.

e) Find the imaginary part of $\dfrac{6-j}{3j}$.

Sol'n: a) To rationalize, we multiply top and bottom by the conjugate of the denominator:

$$\frac{1-j\sqrt{3}}{1+j\sqrt{3}} = \frac{1-j\sqrt{3}}{1+j\sqrt{3}}\cdot\frac{1-j\sqrt{3}}{1-j\sqrt{3}} = \frac{1-3-j2\sqrt{3}}{1^2+\sqrt{3}^2} = \frac{-2-j2\sqrt{3}}{4}$$

or

$$\frac{1-j\sqrt{3}}{1+j\sqrt{3}} = \frac{-1}{2} - j\frac{\sqrt{3}}{2}$$

b) The conjugate is obtained by changing j to $-j$ throughout the expression:

$$[(j+1)(-1-j)]^* = (-j+1)(-1+j) = (1-j)(-1+j)$$

Converting the rectangular forms to polar forms before multiplying yields an answer in polar form:

$$(1-j)(-1+j) = \sqrt{2}\angle-45°\cdot\sqrt{2}\angle135° = 2\angle90°$$

c) The phasor of $\sin(\omega t)$ is $-j$:

$$\mathrm{P}[-\sin(100t+45°)] = -(-j)\cdot1\angle45° = j\cdot1\angle45°$$

or

$$\mathrm{P}[-\sin(100t+45°)] = 1\angle90\cdot1\angle45° = 1\angle45°$$

d) The magnitude of a product (or quotient) is the product (or quotient) of the magnitudes:

$$\left|\frac{(30-j40)}{(7-j24)e^{j30^\circ}}\right| = \frac{|30-j40|}{|7-j24|\left|e^{j30^\circ}\right|} = \frac{\sqrt{30^2+40^2}}{\sqrt{7^2+24^2}\cdot 1}$$

or

$$\left|\frac{(30-j40)}{(7-j24)e^{j30^\circ}}\right| = \frac{50}{25} = 2$$

e) Note that the imaginary part is a *real* number:

$$\operatorname{Im}\left[\frac{6-j}{3j}\right] = \operatorname{Im}\left[\frac{-j(6-j)}{3}\right] = \operatorname{Im}\left[\frac{-1-j6}{3}\right] = \operatorname{Im}\left[-\frac{1}{3}-j2\right] = -2$$