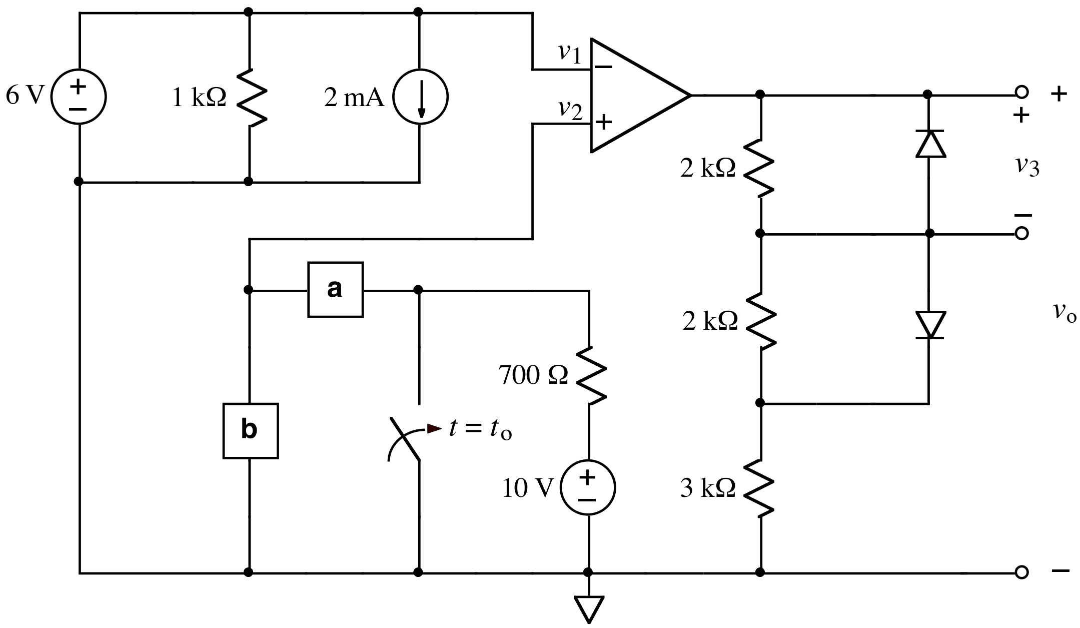
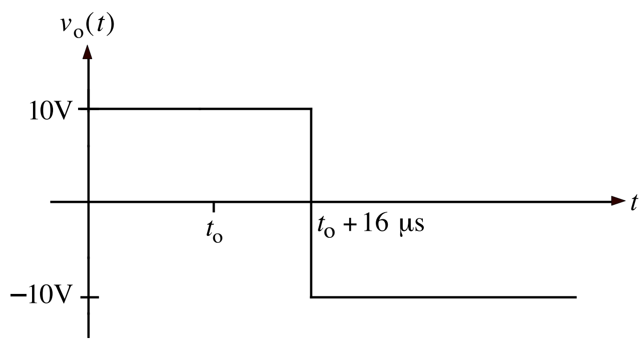
HOMEWORK #18 solution

Ex:



After being open for a long time, the switch closes at time $t = t_o$.

Rail voltages = ±10 V



a) Choose either an R or L to go in box **a** and either an R or L to go in box **b** to produce the v_o(t) shown above. (You will need one R and one L. Use an R value of 1.3 kΩ. Also, note that v_o stays low forever after t_o + 16 µs.) Specify which element goes in each box and its value.

b) Sketch $v_1(t)$, showing numerical values appropriately.

c) Sketch $v_2(t)$, showing numerical values appropriately.

d) Sketch $v_3(t)$. Show numerical values for $t < t_o$, for $t_o < t < t_o + 16$ µs, and for $t > t_o + 16$ µs. Use the ideal model of the diode: when forward biased, its resistance is zero; when reverse biased, its resistance is infinite.

sol'n: a) We first find v_1, which is constant. The 1kΩ resistor and 2mA source are across the 6V source. Because they are across the 6V source, they may be ignored. $v_1 = 6V$ regardless of what is across the 6V source.

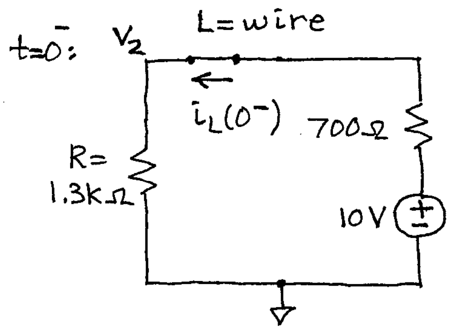
$$v_1 = 6\text{ V}$$

For boxes **a** and **b**, we observe that, if we had an L in **b**, that L will act like a wire at $t=0^-$. This would make $v_2(0^-) = 0V$. The op-amp acts like a comparator, with the output equal to $\pm 10V$ according to the sign of $v_2 - v_1$. For $v_2 = 0V$ and $v_1 = 6V$ we would have $v_2 - v_1 = 0 - 6V = -6V$ and $v_o = -10V$ at $t=0^-$. This differs from the plot of $v_o(t)$ given in the problem statement. Thus, **b** must contain an R rather than an L.

Because the output voltage, $v_o(t)$, changes 16 μs after the switch moves, box **a** must contain an L to give the circuit a time-varying behavior.

$$a = L$$
$$b = R \approx 1.3\text{K} \text{ (from prob statement)}$$

At $t=0^-$, we have the following equivalent circuit:



$$i_L(0^-) = \frac{10V}{1.3k\Omega + 700\Omega}$$

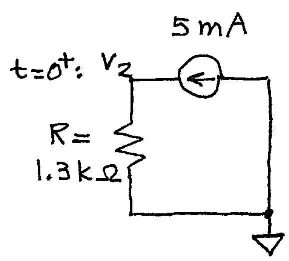
$$i_L(0^-) = 5mA$$

This is a voltage divider.

$$V_2(0^-) = 10V \cdot \frac{1.3k\Omega}{1.3k\Omega + 700\Omega} = 6.5V$$

We note that $V_2(0^-) = 6.5 > V_1(0^-) = 6V$ and $V_2 - V_1 = 6.5 - 6V = 0.5V > 0V$ so $V_o(0^-) = +10V$, as desired.

At $t=0^+$, we have the following equivalent circuit:

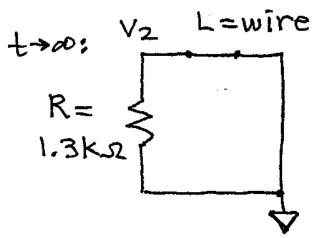


$$V_2(0^+) = 5mA \cdot 1.3k\Omega$$

$$V_2(0^+) = 6.5V$$

At $t=0^+$, $V_2 - V_1 = 6.5V - 6V = 0.5V > 0V$ so $V_o(0^+) = +10V$, as desired.

As $t \to \infty$, we have the following equivalent circuit:



Since there is no power source, $V_2(t \to \infty) = 0V$

The time constant of the circuit is

$$\tau = \frac{L}{R_{Th}} = \frac{L}{1.3k\Omega}$$

Using the general form of solution for RL circuits, we write an expression for $v_2(t>0)$:

$$v_2(t>0) = v_2(t \to \infty) + \left[v_2(0^+) - v_2(t \to \infty)\right] e^{-t/\tau}$$

or $V_2(t>0) = 0V + \left[6.5V - 0V\right] e^{-t/\tau}$

or $V_2(t>0) = 6.5V \, e^{-t/\tau}$

The op-amp output switches from high to low when $V_2(t) = V_1 = 6V$, which must occur at $t = t_0 + 16\mu s$.

Setting $t_0 = 0$, we solve for L in $V_1 = V_2$ at $t = 16\mu s$.

$$6V = 6.5V \, e^{-16\mu s/\tau}$$

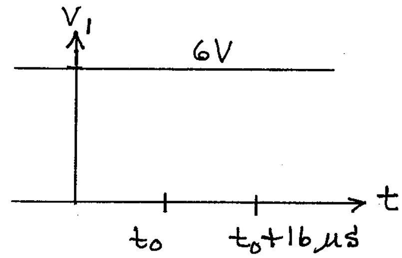
or $\ln(6/6.5) = -16\mu s/\tau$

or $\tau = \dfrac{-16\,\mu s}{\ln(6/6.5)} \doteq 200\,\mu s$

$L = \tau \cdot R_{Th} = 200\,\mu s \cdot 1.3\,k\Omega = 260\,mH$

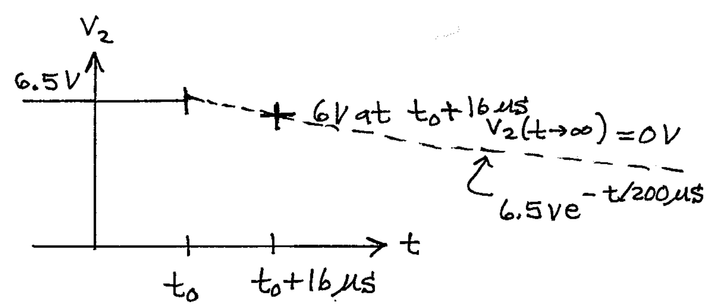
$$\boxed{\begin{array}{l} R = 1.3\ k\Omega \\ L = 260\ mH \end{array}}$$

b) From above, $v_1(t) = 6V$ at all times

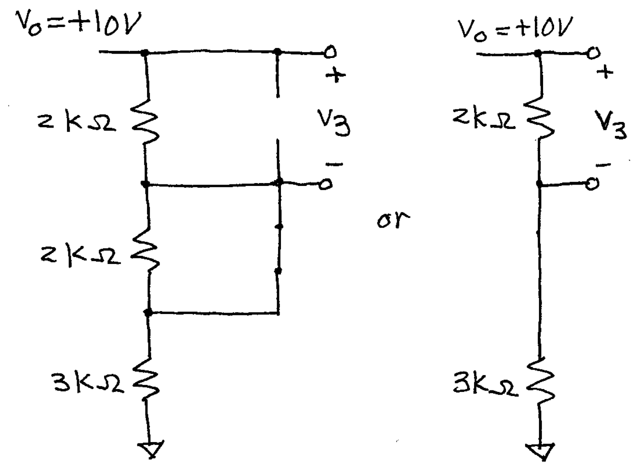


c) From above, $V_2(0^-) = 6.5V$ and
$v_2(t>0) = 6.5\,e^{-t/200\mu s}\ V$

Also, $v_2(t=16\,\mu s) = v_1 = 6V$



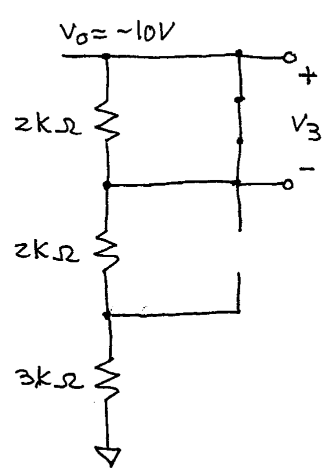
d) When $V_o = +10V$, the equivalent circuit on the right side is as follows:



We have a voltage divider: $V_3 = \dfrac{10V \cdot 2k\Omega}{2k\Omega + 3k\Omega}$

$$V_3 = 4V$$

When $V_o = -10V$, the equivalent circuit on the right side is as follows:



Since we have a short across V_3, $V_3 = 0V$.