HOMEWORK #1 solution

Ex: a) Solve the following simultaneous equations for v_1 and v_2:

$$6v_1 - v_2 = 39$$

$$\frac{5(v_2 - v_1)}{9} + \frac{v_2}{3} = -6$$

b) Solve the following simultaneous equations for R_1 and R_2:

$$\sqrt{R_1 + R_2^2} = 5R_2$$

$$\frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} = \frac{24}{25}$$

Sol'n: a) We first decide which equation is simpler. Then we solve that equation for one variable in terms of the other. In the present case, the first equation is simpler.

$$6v_1 - v_2 = 39$$

or

$$-v_2 = 39 - 6v_1$$

or, if we multiply both sides by –1:

$$v_2 = -39 + 6v_1.$$

Now we turn to the second equation:

$$\frac{5(v_2 - v_1)}{9} + \frac{v_2}{3} = -6$$

Multiplying this equation by the common denominator, 9, simplifies the equation:

$$5(v_2 - v_1) + 3v_2 = -54$$

or

$$-5v_1 + 8v_2 = -54$$

Substituting for v_2 yields the following equation:

$$-5v_1 + 8(-39 + 6v_1) = -54$$

or

$$-5v_1 + 8(-39) + 48v_1 = -54$$

We factor out the constants multiplying v_1:

$$43v_1 = -54 + 8(39) = 258$$

or

$$v_1 = \frac{258}{43} = 6$$

Using this value in our earlier equation for v_2 in terms of v_1 yields the following result:

$$v_2 = -39 + 6v_1 = -39 + 6(6) = -3$$

b) There are various approaches one might take here. One choice is to solve the first equation for R_1 in terms of R_2:

$$\sqrt{R_1 + R_2^2} = 5R_2$$

or

$$R_1 + R_2^2 = (5R_2)^2$$

or

$$R_1 = 25R_2^2 - R_2^2 = 24R_2^2$$

Before substituting this into the second equation, we simplify the second equation:

$$\frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} = \frac{24}{25}$$

Inverting both sides leads to simplification:

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{25}{24}$$

Now we substitute for R_1:

$$\frac{1}{24R_2^2} + \frac{1}{R_2} = \frac{25}{24}$$

Multiplying by the common denominator, $24R_2^2$, yields a quadratic equation:

$$1 + 24R_2 = 25R_2^2$$

or

$$25R_2^2 - 24R_2 - 1 = 0$$

or

$$R_2^2 - \frac{24}{25}R_2 - \frac{1}{25} = 0$$

We have two roots:

$$R_2 = \frac{24}{50} \pm \sqrt{\left(\frac{24}{50}\right)^2 + \frac{1}{25}} = \frac{24}{50} \pm \sqrt{\left(\frac{12}{25}\right)^2 + \frac{1}{25}} = \frac{12}{25} \pm \frac{13}{25}$$

or

$$R_2 = 1 \text{ or } R_2 = -\frac{1}{25}$$

Going back to the original first equation, if the square root represents the positive square root, we discover that only the positive value of R_1 is valid. Thus, $R_2 = 1$.

Using our earlier equation for R_1 in terms of R_2, we have

$$R_1 = 24R_2^2 = 24(1)^2 = 24(1) = 24$$

NOTE: If the R's represent resistor values, as they often will, the values for R_1 and R_2 would have to be positive.