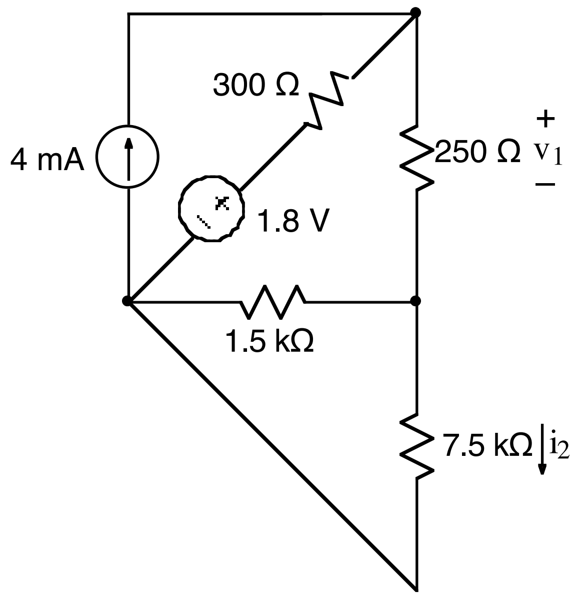
HOMEWORK #7 solution

Ex:

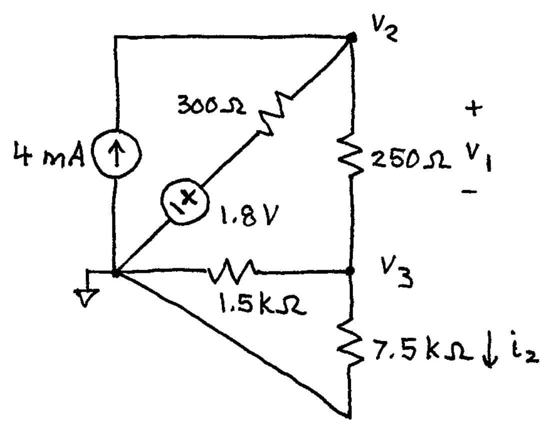


a) Use the node-voltage method to calculate v_1 and i_2.

b) Calculate the power in the 1.5 kΩ resistor.

sol'n: a) We first assign a reference node. The node on the left is convenient since it is connected to the − of the 1.8V supply.

We assign node voltages v_2 and v_3 on the right side of the circuit, (since v_1 is already used).



We check for dependent sources and super nodes, but neither is present in this circuit.

Now we write current summation eq'ns for each node.

$$v_2 \text{ node: } -4\,mA + \frac{v_2 - 1.8V}{300\Omega} + \frac{v_2 - v_3}{250\Omega} = 0A$$

$$v_3 \text{ node: } \frac{v_3 - v_2}{250\Omega} + \frac{v_3 - 0V}{1.5k\Omega} + \frac{v_3 - 0V}{7.5k\Omega} = 0A$$

We solve these eq'ns for v_2 and v_3. Grouping terms multiplying v_2 and v_3 and putting constant terms on the right side of the eq'n keeps things organized.

$$v_2\left(\frac{1}{300\Omega} + \frac{1}{250\Omega}\right) + v_3\left(\frac{-1}{250\Omega}\right) = 4mA + \frac{1.8V}{300\Omega}$$

$$v_2\left(\frac{-1}{250\Omega}\right) + v_3\left(\frac{1}{250\Omega} + \frac{1}{1.5k\Omega} + \frac{1}{7.5k\Omega}\right) = 0A$$

Multiplying both sides by $7.5k\Omega$ clears the denominators.

$$v_2\,(25 + 30) + v_3\,(-30) = 30V + 1.8V\,(25)$$

$$v_2\,(-30) + v_3\,(30 + 5 + 1) = 0V$$

The 2nd eq'n is easier to solve. (We may solve for either v_2 or v_3.)

$$v_2 = v_3 \frac{36}{30} = v_3 \cdot \frac{6}{5}$$

Substituting into the 1st eq'n, we have

$$v_3 \cdot \frac{6}{5}(55) + v_3(-30) = 30V + 45V = 75V$$

or $v_3 \cdot (66 - 30) = 75V$

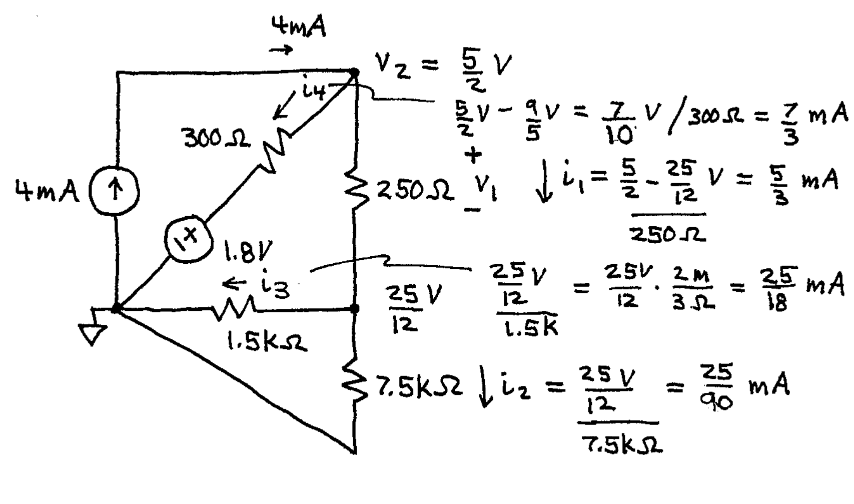
or $v_3 \cdot 36 = 75V$

or $v_3 = \frac{75}{36} V = \frac{25}{12} V$

Using or earlier eq'n for v_2, we have

$$v_2 = v_3 \cdot \frac{6}{5} = \frac{25}{12} V \cdot \frac{6}{5} = \frac{5}{2} V$$

Before going further we perform a consistency check on the currents to verify that they sum to zero at each node:



Check: $-4mA + \frac{7}{3} mA + \frac{5}{3} mA = 0 \ mA$ ✓

$-\frac{5}{3} mA + \frac{25}{18} mA + \frac{25}{90} mA = 0 \ mA$ ✓

For v_1 we have $v_1 = v_2 - v_3 = \frac{5}{2} - \frac{25}{12}\ V$

or $v_1 = \frac{30}{12} - \frac{25}{12}\ V = \frac{5}{12}\ V$

For i_2 we have $i_2 = \frac{v_3}{7.5\,k\Omega} = \frac{25/12}{7.5}\ mA$

$$i_2 = \frac{25}{12} \cdot \frac{10}{75} = \frac{5}{18}\ mA$$

(Note: v_1 and i_2 we actually found earlier in the consistency check.)

b) The power in the $300\,\Omega$ resistor is

$$p = v \cdot i = (v_2 - 1.8V) \cdot \frac{(v_2 - 1.8V)}{300\,\Omega}$$

$$= \frac{(2.5V - 1.8V)^2}{300\,\Omega} = \frac{(0.7)^2}{300}\ W$$

$$p \doteq 1.63\ mW$$

The power in the 1.5 kΩ resistor is

$$p = v \cdot i = \frac{(v_2 - v_1)^2}{1.5\ k\Omega} = \frac{\left(\frac{5}{2} - \frac{5}{12}\right)^2}{1.5\ k\Omega} = \frac{\left(\frac{25}{12}\right)^2}{1.5\ k\Omega} = 2.89\ \text{mW}$$