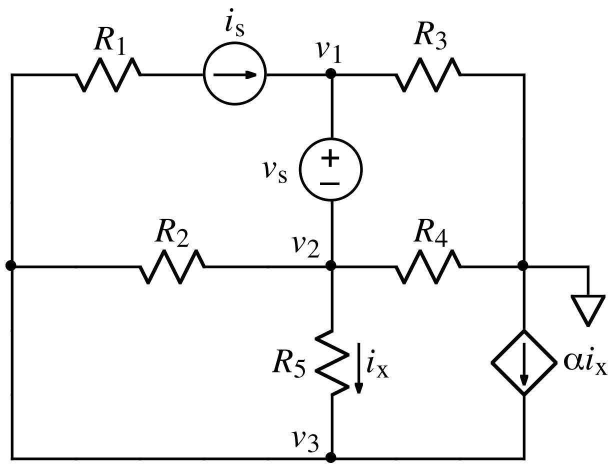
HOMEWORK #8 solution

Ex:



Make a consistency check on the following equations for the above circuit by setting resistors and sources to values for which the values of v_1, v_2, and v_3 are obvious. State the values of resistors, sources, and node voltages for your consistency check, and show that the circuit equations are satisfied for these values. (In other words, plug the values into the equations and show that the left side and the right side of each equation are equal.)

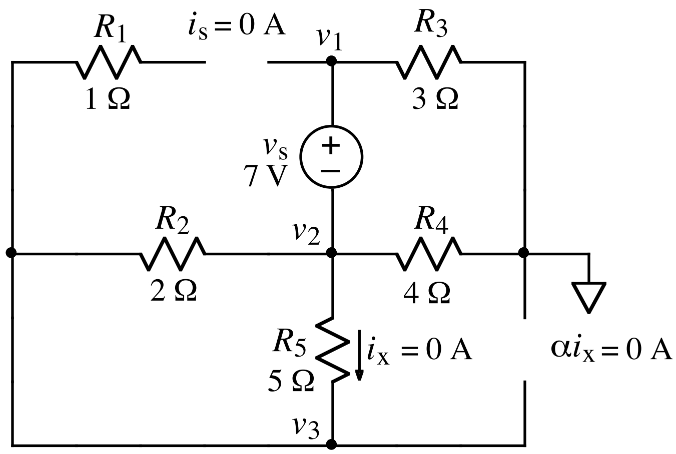
$$v_s = v_1 - v_2$$

$$-i_s + \frac{v_1}{R_3} + \frac{v_2 - v_3}{R_2} + \frac{v_2 - v_3}{R_5} + \frac{v_2}{R_4} = 0 \text{ A}$$

$$i_s + \frac{v_3 - v_2}{R_2} + \frac{v_3 - v_2}{R_5} - \alpha \frac{v_2 - v_3}{R_5} = 0 \text{ A}$$

SOL'N: Many checks are possible. One example is given here.

Suppose $i_s = 0$ A, $\alpha = 0$, $v_s = 7$ V, $R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$, $R_3 = 3\ \Omega$, $R_4 = 4\ \Omega$, and $R_5 = 5\ \Omega$. The resulting circuit is shown below. We see that turning off the two current sources causes R_1, R_2, and R_5 to dangle at the end of wires. No current flows in these resistors, as there is no complete circuit. By Ohm's law, it follows that there is no voltage drop across theses wires, implying that $v_3 = v_2$. We also have $v_1 = v_2 + 7$ V owing to the $v_s = 7$ V source.



We are left with the problem of finding v_2. For that, we observe that we have a voltage divider consisting of v_s, R_3, and R_4. Being careful to use the correct sign for voltage v_2 we have the following result:

$$v_2 = -v_s \frac{R_4}{R_3 + R_4} = -7\text{ V} \frac{4\ \Omega}{3\ \Omega + 4\ \Omega} = -4\text{ V}$$

Using earlier equations, we have the following node voltages:

$v_1 = 3$ V, $v_2 = -4$ V, and $v_3 = -4$ V

Now we plug in numerical values for all the terms in the node-voltage equations and check that the two sides are equal:

$$v_s = v_1 - v_2 \quad \text{or}$$

$$7\text{ V} = 3\text{ V} - -4\text{ V} \quad \surd\text{ (checks out)}$$

$$-i_s + \frac{v_1}{R_3} + \frac{v_2 - v_3}{R_2} + \frac{v_2 - v_3}{R_5} + \frac{v_2}{R_4} = 0\text{ A} \quad \text{or}$$

$$-0\text{A} + \frac{3\text{V}}{3\Omega} + \frac{-4\text{V} - -4\text{V}}{2\Omega} + \frac{-4\text{V} - -4\text{V}}{5\Omega} + \frac{-4\text{V}}{4\Omega} = 0\text{ A} \quad \surd\text{ (checks out)}$$

$$i_s + \frac{v_3 - v_2}{R_2} + \frac{v_3 - v_2}{R_5} - \alpha\frac{v_2 - v_3}{R_5} = 0\text{ A} \quad \text{or}$$

$$0\text{A} + \frac{-4\text{V} - -4\text{V}}{2\Omega} + \frac{-4\text{V} - -4\text{V}}{5\Omega} - 0\frac{-4\text{V} - -4\text{V}}{5\Omega} = 0\text{ A} \quad \surd\text{ (checks out)}$$