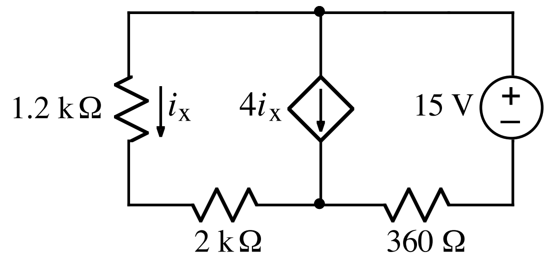
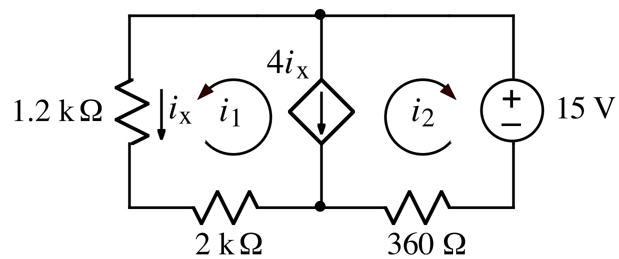
HOMEWORK #8 solution

Ex:



Calculate the power consumed (i.e., dissipated) by the $4i_x$ dependent source. **Note:** If a source supplies power, the power it consumes is negative.

SOL'N: The mesh-current method is one method of solution, and it works well here. The diagram below shows the definition of the mesh currents. Note the i_1 is defined in such a way that $i_x = i_1$.



We have a super-mesh, and we write an equation for the central current source in terms of the mesh currents:

$$4i_x = 4i_1 = -(i_1 + i_2)$$

or

$$i_2 = -5i_1$$

We also write a voltage-loop equation around the outside of the circuit:

$$i_1 \cdot 1.2\text{k}\Omega - 15\text{V} - i_2 \cdot 360\Omega + i_1 \cdot 2\text{k}\Omega = 0 \text{ V}$$

or

$$i_1 \cdot (1.2\text{k}\Omega + 2\text{k}\Omega) - i_2 \cdot 360\Omega = 15 \text{ V}$$

Substituting for i_2 yields the following result:

$$i_1 \cdot (1.2\text{k}\Omega + 2\text{k}\Omega) - (-5i_1) \cdot 360\Omega = 15 \text{ V}$$

or

$$i_1 \cdot (1.2\text{k}\Omega + 2\text{k}\Omega + 1.8\text{k}\Omega) = 15 \text{ V}$$

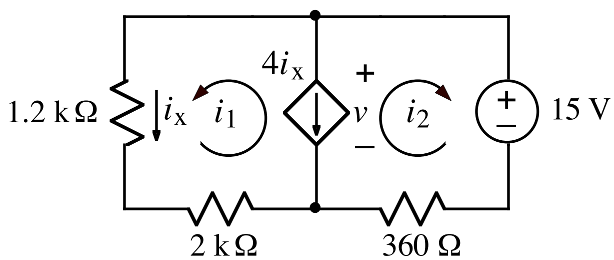
or

$$i_1 = \frac{15 \text{ V}}{5\text{k}\Omega} = 3 \text{ mA}$$

The current in the dependent source is $4i_1$:

$$4i_x = 12 \text{ mA}$$

A voltage loop on the left side yields the voltage, v, across the dependent source.



$$i_1 \cdot 2\text{k}\Omega + i_1 \cdot 1.2\text{k}\Omega - v = 0 \text{ V}$$

or

$$v = i_1 \cdot 2\text{k}\Omega + i_1 \cdot 1.2\text{k}\Omega = i_1 \cdot 3.2\text{k}\Omega = 9.6 \text{ V}$$

The power is the voltage times the current:

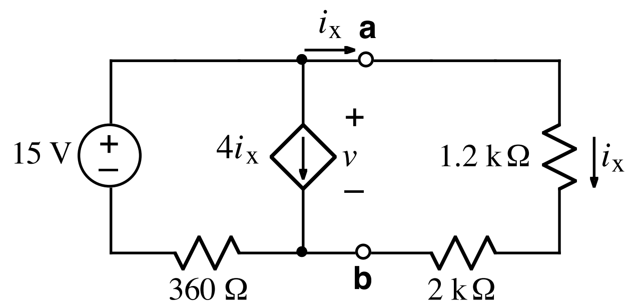
$$p = v \cdot 4i_x = 9.6\text{V} \cdot 12\text{mA} = 115.2 \text{ mW}$$

NOTE: Another way to approach this problem is to note that the dependent source carries four times the current in the parallel branch that has resistance 1.2 kΩ + 2 kΩ = 3.2 kΩ. Thus, the dependent source acts like a resistance of 3.2 kΩ/4 = 800 Ω.

NOTE: Another way to approach this problem is to first find the Thevenin equivalent of the circuit without the resistance, 1.2 kΩ + 2 kΩ = 3.2 kΩ, through which i_x flows. After the Thevenin equivalent is found, the 3.2 kΩ may be placed across its output, and current i_x flowing through it and voltage v

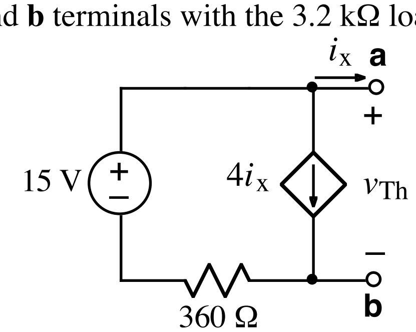
across it may be found. The power in the dependent source is $p = v \cdot 4i_X$ as before. This entire process is explained next.

First, flip the circuit horizontally so it looks more like a familiar form of a circuit with a resistance of 3.2 kΩ attached to terminals **a** and **b**:

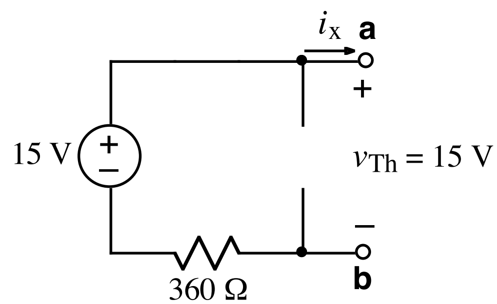


We may define i_X to be whatever current flows out of the **a** terminal, and we may change the load attached to the circuit at **a** and **b** as we find the Thevenin equivalent of the circuit to the left of **a** and **b**.

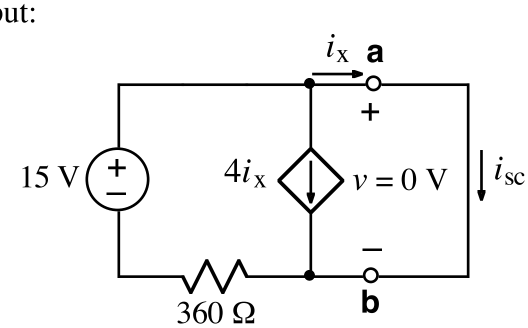
The voltage in the Thevenin equivalent is the voltage across the **a** and **b** terminals with the 3.2 kΩ load resistance removed:



Since i_X is zero, the dependent source has no current and acts like an open circuit. This, this circuit simplifies to one in which the current in the 360 Ω is zero and the voltage across **a** and **b** is the 15 V of the source.



To find the R_{Th}, we may take the ratio of v_{Th} to the short circuit current, i_{sc}, flowing in a wire connected to the circuit output:



Here, we have 0 V across the **a** and **b** terminals and $i_x = i_{sc}$. An outer voltage loop gives the current through the 360 Ω resistor:

$$i_{360\ \Omega} = \frac{15\text{ V}}{360\ \Omega} = \frac{1}{24}\text{ A}$$

By summing current out of the node to the right of the 360 Ω resistor, we obtain an equation for i_x:

$$5i_x = i_{360\ \Omega} = \frac{1}{24}\text{ A}$$

or

$$i_x = \frac{1}{120}\text{ A}$$

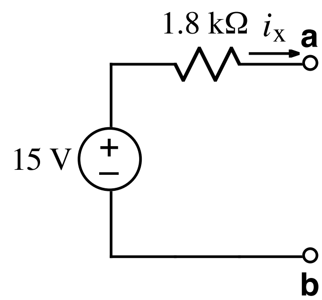
or

$$i_{sc} = \frac{1}{120}\text{ A}$$

We now calculate R_{Th}:

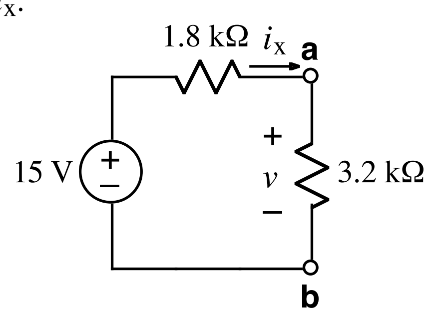
$$R_{Th} = \frac{v_{Th}}{i_{sc}} = \frac{15\ \text{V}}{\frac{1}{120}\ \text{A}} = 15 \cdot 120\ \Omega = 1.8\ \text{k}\Omega$$

We have the following Thevenin equivalent:



It may seem that we have gone to a great deal of trouble to avoid the dependent source, but the individual steps in this process are quite simple once the required circuit interpretations are mastered.

Now we attach the 3.2 kΩ resistance to the output and find v and i_x.



We have a voltage divider for v, and an application of Ohm's law for i_x:

$$v = 15\ \text{V} \frac{3.2\ \text{k}\Omega}{1.8\ \text{k}\Omega + 3.2\text{k}\Omega} = 9.6\ \text{V}$$

$$i_x = \frac{15\ \text{V}}{1.8\ \text{k}\Omega + 3.2\ \text{k}\Omega} = \frac{15\ \text{V}}{5\ \text{k}\Omega} = 3\ \text{mA}$$

Finally, we find the power for the dependent source:

$$p = v \cdot 4i_x = 9.6\text{V} \cdot 12\text{mA} = 115.2\ \text{mW}$$