Ex: After being 3 V for a long time, the value of $v_g(t)$ changes to 6 V at $t = 0$ (and remains at 6 V as time increases to infinity).

\[ R = 1 \text{ M}\Omega \]

\[ C = 10 \text{ nF} \]

\[ v_g(t) \]

\[ i_R \]

\[ v_o \]

\[ + \]

\[ - \]

\[ v_o \]

\[ + \]

\[ - \]

a) Find an expression for $v_o(t)$ for $t > 0$.
b) Find the current, $i_R$, in $R$ as a function of time.

SOL’N: a) The following general form of solution applies to any RC circuit with a single capacitor:

\[ v_C(t \geq 0) = [v_C(t = 0^+) - v_C(t \to \infty)]e^{-t/R_{Th}C} + v_C(t \to \infty) \]

The Thevenin resistance, $R_{Th}$, is for the circuit after $t = 0$ (with the $C$ removed) as seen from the terminals where the $C$ is connected. In the present case, we have $R_{Th} = 1 \text{ M}\Omega$.

\[ R_{Th}C = 1 \text{ M}\Omega \cdot 10 \text{ nF} = 10 \text{ ms} \]

For time $t = 0^-$, the voltage source will be 3 V and the capacitor will behave like an open circuit. Since no current will flow in the $R$, there will be no voltage drop across the $R$, and the voltage on the $C$ will be $v_g = 3 \text{ V}$. Since the voltage on the capacitor is an energy variable, it will not change instantly. Thus, the initial capacitor voltage is 3 V.

\[ v_C(t = 0^+) = 3 \text{ V} \]

For time approaching infinity, the capacitor will charge to a final value and no current will flow in the capacitor. Thus, the capacitor will act like an open circuit. It follows that no current will flow in the $R$ as time approaches infinity, and the voltage drop across $R$ will be zero. Thus, the voltage on $C$ will be $v_g(t) = 6 \text{ V}$:

\[ v_C(t \to \infty) = v_g(t) = 6 \text{ V} \]

Substituting values, we have the following result:
\[ v_o(t \geq 0) = [3 \text{ V} - 6 \text{ V}]e^{-t/10\text{ms}} + 6 \text{ V} \]

or

\[ v_o(t \geq 0) = -3 \text{ V} \cdot e^{-t/10\text{ms}} + 6 \text{ V} \]

b) The following general form of solution applies to any current in any RC circuit with a single capacitor:

\[ i(t \geq 0) = [i(t = 0^+) - i(t \to \infty)]e^{-t/R_{Th}C} + i(t \to \infty) \]

In the present case, this applies to the resistor current:

\[ i_R(t \geq 0) = [i_R(t = 0^+) - i_R(t \to \infty)]e^{-t/R_{Th}C} + i_R(t \to \infty) \]

The Thevenin resistance, \( R_{Th} \), is for the circuit after \( t = 0 \) (with the \( C \) removed) as seen from the terminals where the \( C \) is connected. In the present case, we have \( R_{Th} = 1 \text{ M}\Omega \).

\[ R_{Th}C = 1 \text{ M}\Omega \cdot 10 \text{ nF} = 10 \text{ ms} \]

For time \( t = 0^- \), the capacitor voltage is 3 V. Since the voltage on the capacitor is an energy variable, it will not change instantly.

\[ v_C(t = 0^+) = 3 \text{ V} \]

At time \( t = 0^+ \), we model the capacitor as a voltage source, (with the voltage the capacitor had at time \( t = 0^- \)), and find \( i_R(t = 0^+) \):

\[ i_R(t = 0^+) = \frac{v_g(t) - v_C(t = 0^+)}{R} \]

or
\[ i_R(t = 0^+) = \frac{6 \text{ V} - 3 \text{ V}}{1 \text{ M}\Omega} = 3 \mu\text{A} \]

For time approaching infinity, the capacitor will charge to a final value and no current will flow in the capacitor. Thus, the capacitor will act like an open circuit. It follows that no current will flow in the \( R \) as time approaches infinity.

\[ i_R(t \to \infty) = 0 \text{ A} \]

Substituting values, we have the following result:

\[ i_R(t \geq 0) = [3 \mu\text{A} - 0 \text{ A}]e^{-t/10\text{ms}} + 0 \text{ A} = 3 \mu\text{A} \cdot e^{-t/10\text{ms}} \]