Ex: Plot each of the following complex numbers as vectors in the complex plane:

a) $1 + j2$

b) $2e^{j\pi/3}$

c) $\left[ \frac{1}{2} + j\frac{\sqrt{3}}{2} \right] + \left[ -1 - j\frac{\sqrt{3}}{2} \right]$

d) $\sqrt{j}$

e) $\frac{-3 + j4}{3 + j4}$

Sol.'N: a) We think of the complex numbers as vectors specified in either rectangular form, $a + jb$, or polar form, $Ae^{j\Phi}$.

b) The vector length is 2 and the angle relative to the real axis is $\pi/3$, or we may use Euler's formula.

$$2e^{j\pi/3} = 2\cos(\pi/3) + j2\sin(\pi/3) = 1 + j\sqrt{3}$$

c) $\left[ \frac{1}{2} + j\frac{\sqrt{3}}{2} \right] + \left[ -1 - j\frac{\sqrt{3}}{2} \right] = -\frac{1}{2}$
d) Note that \( j = e^{j\pi/2} \).

\[
\sqrt{j} = (e^{j\pi/2})^{1/2} = e^{j\pi/4} = \cos(\pi/4) + j\sin(\pi/4) = \frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}
\]

**NOTE:** We take the square root of the magnitude, \( \sqrt{1} = 1 \), and half the angle, \( \frac{\pi}{2} \cdot \frac{1}{2} = \frac{\pi}{4} = 45^\circ \).


e) We can rationalize the value. We do this by multiplying the numerator and denominator by the conjugate of the denominator.

\[
\frac{-3 + j4}{3 + j4} \cdot \frac{3 - j4}{3 - j4} = \frac{-3^2 - j^2 \cdot 4^2 + j(-3 \cdot (-4) + 3 \cdot 4)}{3^2 + 4^2} = \frac{7 + j24}{25}
\]