Ex:

\[ 5v_x \quad + \quad 4V \quad + \quad 1k\Omega \quad v_x \quad - \quad a \]

\[ + \quad 3k\Omega \quad 2mA \quad 5v_x \quad + \quad 5v_x \quad - \quad b \]

a) Find the numerical Thevenin equivalent of the above circuit relative to terminals a and b.
b) If we attach \( R_L \) to terminals a and b, find the value of \( R_L \) that will absorb maximum power.
c) Calculate the value of that maximum power absorbed by \( R_L \).

**SOL’N:**
a) \( v_{Th} \) is the voltage seen at the a and b terminals with nothing connected to the a and b terminals. Because of the voltage source, \( 5v_x \), in the middle of the circuit, we effectively have two circuits that may be solved separately, (as though they each had their own \( 5v_x \) source). For the circuitry on the right, we have a voltage loop passing through the dependent source, the 1 kΩ resistor, the 4 V source, and across the gap from a to b, (which is \( v_{Th} \)).

\[ 5v_x - v_x - 4V - v_{Th} = 0V \]

To determine the value of \( v_x \), we observe that no current flows in the 1 kΩ resistor (since there is no complete circuit for the current to flow in). Thus, \( v_x = 0 \) V. It follows that \( v_{Th} \) equals the \(-4 \) V from the independent voltage source.

\[ v_{Th} = -4V \]

**NOTE:** The circuitry to the left of the dependent voltage source has no effect on the behavior of the circuit at a and b. The \( 5v_x \) source
effectively divides the circuit into two circuits, as mentioned above, and the circuitry on the other side of the dependent source is not "seen" from the a and b terminals.

To find $R_{Th}$, we may either either calculate the short-circuit current, $i_{sc}$, from a to b and use $R_{Th} = \frac{V_{Th}}{i_{sc}}$. Or we may turn off the independent $-4 \text{ V}$ (and irrelevant 2 mA) sources, connect a 1 V source to a and b, measure the current, $i_a$, flowing into the a terminal from the 1 V source, and use $R_{Th} = \frac{1 \text{ V}}{i_a}$. Here, we use the latter approach.

From a voltage loop on the right, we can determine the value of $v_x$:

$5v_x - v_x - 1 \text{ V} = 0 \text{ V}$

or

$4v_x = 1 \text{ V}$

or

$v_x = \frac{1}{4} \text{ V}$

From $v_x$ we find $i_a$.

$i_a = \frac{-v_x}{1 \text{ k} \Omega} = \frac{-1/4 \text{ V}}{1 \text{ k} \Omega} = -\frac{1}{4 \text{ k}} \text{ A}$

$R_{Th}$ is the inverse of $i_a$, (since the applied voltage is 1 V).
\[ R_{Th} = \frac{1 \text{V}}{i_a} = -4 \text{k}\Omega \]

Curiously, the circuit has both a negative \( v_{Th} \) and a negative \( R_{Th} \).

b) For the usual case of a circuit with positive \( R_{Th} \), the solution to the maximum power transfer problem is always the same: \( R_L = R_{Th} \). Here, the answer is \( R_L = -R_{Th} \).

c) This value of \( R_L \) causes the total resistance across \( v_{Th} \) to be zero. This, in turn, results in an infinite current flow and an infinite power in \( R_L \), as the power is \( p = i^2 R_L = \infty \text{W} \). (Compare this with the usual result that \( p_{max} = \frac{v_{Th}^2}{4R_{Th}} \).)