**Ex:**

\[
\begin{align*}
\text{15 V} & \quad \text{2kΩ} & \quad \text{3kΩ} & \quad \text{1kΩ} \\
\quad & \text{+} & \quad \text{−} & \quad \text{−} & \quad \text{+} \\
\end{align*}
\]

Find \(i_x\), \(v_3\), and the power dissipated by the dependent source.

**Sol’n:** Here, we use the resistor voltages and currents as shown in the diagram below.

\[
\begin{align*}
\text{15 V} & \quad \text{2kΩ} & \quad \text{3kΩ} & \quad \text{1kΩ} \\
\quad & \text{+} & \quad \text{−} & \quad \text{−} & \quad \text{+} \\
\end{align*}
\]

We have no components in series, so we move on to \(v\)-loops. On the left and right sides, we have voltage loops:

\[
\begin{align*}
15 \text{ V} - v_1 - v_2 &= 0 \text{ V} \\
v_4 - v_5 - 2 \text{ V} &= 0 \text{ V}
\end{align*}
\]

**Note:** Even though \(v_3\) is labeled, we avoid finding that voltage for now, since we always avoid finding voltages for current sources until we have solved the circuit.

We have current summation equations for the nodes on the left and right of the dependent source:

\[
\begin{align*}
-\text{\textit{i}}_1 + \text{\textit{i}}_2 + 3\text{\textit{i}}_x &= 0 \text{ A} \\
-3\text{\textit{i}}_x + \text{\textit{i}}_x + \text{\textit{i}}_5 &= 0 \text{ A} \text{ or } -2\text{\textit{i}}_x + \text{\textit{i}}_5 &= 0 \text{ A}
\end{align*}
\]

The bottom node is redundant, as it equals the sum of the above two equations, so we are done with current summations.
Now we write down Ohm’s law for every resistor:

\[ v_1 = i_1 R_1 \]
\[ v_2 = i_2 R_2 \]
\[ v_4 = i_x R_4 \]
\[ v_5 = i_5 R_5 \]

Now we solve the equations. We may proceed along any one of many different paths, but one to proceed is to use the Ohm’s law equations to substitute for all resistor voltages:

\[ 15 \text{ V} - i_1 R_1 - i_2 R_2 = 0 \text{ V} \]
\[ i_x R_4 - i_5 R_5 - 2 \text{ V} = 0 \text{ V} \]

These equations, along with the current summations from earlier, give us four equations in four unknowns. Using the second current summation equation, we can solve for \( i_5 \) in terms of \( i_x \):

\[ i_5 = 2i_x \]

Using this in the second voltage-loop equation (2nd equation above) yields a value for \( i_x \):

\[ i_x R_4 - 2i_x R_5 - 2 \text{ V} = 0 \text{ V} \]

or

\[ i_x (R_4 - 2R_5) = 2 \text{ V} \]

or

\[ i_x = \frac{2 \text{ V}}{R_4 - 2R_5} = \frac{2 \text{ V}}{3 \text{ k}\Omega - 2(1 \text{ k}\Omega)} = 2 \text{ mA} \]

It follows that we have

\[ i_5 = 2i_x = 4 \text{ mA}. \]

The equations for the current summation and voltage loop on the left are as follows:

\[ 15 \text{ V} - i_1 R_1 - i_2 R_2 = 0 \text{ V} \]
\[-i_1 + i_2 + 3i_x = 0 \text{ A}\]

Substituting for \(i_x\) and rearranging to solve the second equation for \(i_1\) yields the following:

\[-i_1 + i_2 + 3 \cdot 2 \text{ mA} = 0 \text{ A}\]

or

\[i_1 = i_2 + 6 \text{ mA}\]

Now we replace \(i_1\) in the first voltage-loop equation:

\[15 \text{ V} - (i_2 + 6 \text{ mA})R_1 - i_2R_2 = 0 \text{ V}\]

or

\[-i_2(R_1 + R_2) = -15 \text{ V} + 6 \text{ mA}R_1\]

or

\[-i_2 = \frac{-15 \text{ V} + 6 \text{ mA}R_1}{R_1 + R_2} = \frac{-15 \text{ V} + 6 \text{ mA} \cdot 1 \text{ k}\Omega}{1 \text{ k}\Omega + 2 \text{ k}\Omega}\]

or

\[i_2 = \frac{9 \text{ V}}{3 \text{ k}\Omega} = 3 \text{ mA}\]

For the middle \(v\)-loop, we have

\[v_2 - v_3 - v_4 = 0 \text{ V}\]

or

\[v_3 = v_2 - v_4 = 3 \text{ mA} \cdot 2 \text{ k}\Omega - 2 \text{ mA} \cdot 3 \text{ k}\Omega = 0 \text{ V}\]

Finally, we calculate the power as the current times the voltage (for the dependent source):

\[p = 3 \cdot 2 \text{ mA} \cdot 0 \text{ V} = 0 \text{ W}\]

The dependent source has current but no voltage or power! (Wires behave the same way.)