Use the node-voltage method to find $v_1$ and $v_2$.

**SOL’N:** We first observe that our dependent variable, $v_1$, is a node voltage. Thus, this variable is already defined in terms of node voltages.

We have only a voltage source between the nodes labeled $v_1$ and $v_2$. Thus, we have a supernode. We draw a bubble around these two nodes, (with the dependent source inside), and we set the sum of currents flowing out of the bubble equal to zero:

$$-i_a + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} + \frac{v_2 - v_1}{R_2} + \frac{v_2 - v_a}{R_3} = 0 \text{ A}$$

Note that the 3rd and 4th terms cancel out. It turns out that $R_2$ has no impact on this problem.

We put this equation in standard form:

$$\frac{v_1}{R_1} + \frac{v_2}{R_3} = i_a + \frac{v_a}{R_3}$$

We obtain a second equation for the supernode from the voltage relationship between the nodes.

$$v_2 - v_1 = \alpha v_1$$
**NOTE:** The node next to the plus sign of the dependent source, $v_2$, is added in this equation and the node next to the minus sign of the dependent source, $v_1$, is subtracted in this equation.

We solve these two equations in two unknowns. Here, we eliminate $v_2$ first.

$$v_2 = v_1 + \alpha v_1 = (1 + \alpha) v_1$$

Substituting into the first equation gives one equation with one unknown.

$$v_1 \left( \frac{1}{R_1} + (1 + \alpha) \frac{1}{R_3} \right) = i_a + \frac{v_a}{R_3}$$

or

$$v_1 \left( \frac{1}{R_1} + (1 + \alpha) \frac{1}{R_3} \right) = i_a + \frac{v_a}{R_3}$$

or

$$v_1 = \frac{i_a + \frac{v_a}{R_3}}{\frac{1}{R_1} + (1 + \alpha) \frac{1}{R_3}}$$

or

$$v_1 = \frac{R_1(R_3i_a + v_a)}{R_3 + (1 + \alpha)R_1}$$

Using the second of the original equations, we find $v_2$.

$$v_2 = (1 + \alpha)v_1 = \frac{(1 + \alpha)R_1(R_3i_a + v_a)}{R_3 + (1 + \alpha)R_1}$$