**Ex:** The following equation describes the voltage, $v_L$, across an inductor, $L = 10 \, \mu H$, as a function of time. Find an expression for the current, $i_L(t)$, through the inductor as a function of time. Assume that $i_L(t = 0) = 200 \, mA$. Hint: integrate $v_L$.

$v_L(t) = 0.1e^{-t/50\mu s} \, V$

**SOL’N:** We use the defining equation for an inductor and solve for $i$ in terms of $v$.

$v_L = L \frac{di_L}{dt}$

First, we multiply both sides by $dt$.

$v_L dt = Ldi_L$

Second, we integrate both sides and use limits that correspond to the variable of integration for each side and are evaluated at the same points in time for both sides.

\[ \int_0^t v_L dt = \int_{i_L(t=0)}^{i_L(t)} Ldi_L \]

The integral on the right side simplifies nicely.

\[ \int_0^t v_L dt = Li_L|_{i_L(t=0)}^{i_L(t)} = L[i_L(t) - i_L(t = 0)] \]

or

\[ i_L(t) = \frac{1}{L} \int_0^t v_L dt + i_L(t = 0) \]

The above expression applies to any inductor in any circuit.

We now substitute the formula given for $v_L(t)$ and the value given for $i_L(t = 0)$ to find $i_L(t)$:

\[ i_L(t) = \frac{1}{L} \int_0^t [0.1e^{-t/50\mu s} \, V] dt + 200 \, mA \]

or

\[ i_L(t) = \frac{1}{L} \left[ \frac{0.1e^{-t/50\mu s}}{-1/50\mu s} \right]_0^t V + 200 \, mA \]

or
\[ i_L(t) = \frac{1}{10\mu H} \left[ \begin{array}{c} 0.1 e^{-t/50\mu s} \\ -1 \end{array} \right] V + 200 \text{ mA} \]

or

\[ i_L(t) = -0.5 \left( e^{-t/50\mu s} - 1 \right) V + 200 \text{ mA} \]