**Ex:** Find the voltage, \(v_C\), on the capacitor in the circuit below as a function of time if \(v_C(t = 0^+) = 12\ V\).

\[
C = 1 \ \mu F \quad v_C \quad R = 1 \ M\Omega
\]

**SOL’N:** We observe that voltage \(v_C\) appears across both the \(R\) and \(C\), as shown below.

\[
C = 1 \ \mu F \quad v_C \quad |i_C| \quad i_R \quad v_C \quad R = 1 \ M\Omega
\]

Using the defining equation of a capacitor and Ohm's law, we have the following results:

\[
i_C = C \frac{dv_C}{dt}
\]

and

\[
i_R = \frac{v_C}{R}
\]

Since the current must be the same everywhere in the loop, and since the currents are measured with opposite polarities, we have that \(i_C\) and \(i_R\) are equal but opposite.

\[
i_C = -i_R
\]

or

\[
C \frac{dv_C}{dt} = -\frac{v_C}{R}
\]

One way to solve this equation is to separate the variables:

\[
C \frac{1}{v_C} dv_C = -\frac{1}{R} dt
\]

Integrating both sides yields the following result:
\[ C \int_{V_C(t=0)}^{V_C(t)} \frac{1}{V_C} dV_C = -\frac{1}{R} \int_0^t dt \]

or

\[ C \ln V_C(t)|_{V_C(t=0)}^{V_C(t)} = -\frac{1}{R} t \]

or

\[ C[\ln V_C(t) - \ln V_C(0)] = -\frac{1}{R} t \]

or

\[ \ln \frac{V_C(t)}{V_C(0)} = -\frac{t}{RC} \]

or

\[ \frac{V_C(t)}{V_C(0)} = e^{-\frac{t}{RC}} \]

or

\[ V_C(t) = V_C(0)e^{-\frac{t}{RC}} \]

Substituting values given in the problem, we have the following answer:

\[ V_C(t) = 12V \cdot e^{-\frac{t}{1M\Omega \cdot 1\mu F}} = 12V \cdot e^{-\frac{t}{1s}} \]