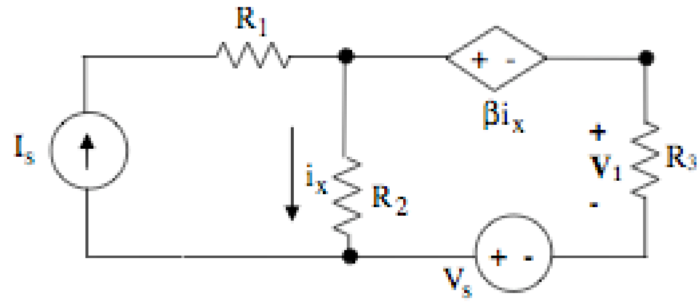
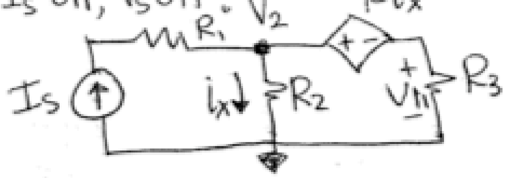
Ex:



Using superposition, derive an expression for v_1 that contains no circuit quantities other than i_S, v_S, R_1, R_2, R_3, and β, where $\beta < 0$.

Sol'n:

① I_s on, V_s off:



node-V:

$$-I_s + \frac{V_2}{R_2} + \frac{V_2 - \beta i_x}{R_3} = 0$$

$$i_x = \frac{V_2}{R_2}$$

$$-I_s + \frac{V_2}{R_2} + \frac{V_2}{R_3} - \frac{\beta \cdot V_2}{R_3 \cdot R_2} = 0$$

$$V_2\left(\frac{1(R_3)}{R_2R_3} + \frac{1(R_2)}{R_3R_2} - \frac{\beta}{R_2R_3}\right) = I_s$$

$$V_2 = \frac{I_s(R_2R_3)}{R_2+R_3-\beta} \Rightarrow +V_2 - \beta\left(\frac{V_2}{R_2}\right) - V_{1_1} = 0$$

$$V_{1_1} = \left(1 - \frac{\beta}{R_2}\right)\left(\frac{I_sR_2R_3}{R_2+R_3-\beta}\right)$$

② I_s off, V_s on:

V-loop: $-i_xR_3 + \beta i_x - i_xR_2 - V_s = 0$

$$i_x = \frac{-V_s}{R_2+R_3-\beta} \Rightarrow V_{1_2} = -i_xR_3$$

$$\therefore V_{1_2} = \frac{+V_sR_3}{R_2+R_3-\beta}$$

$$\boxed{V_1 = V_{1_1} + V_{1_2} = \left(1 - \frac{\beta}{R_2}\right)\left(\frac{I_sR_2R_3}{R_2+R_3-\beta}\right) + \frac{V_sR_3}{R_2+R_3-\beta}}$$