Ex: Give numerical answers to each of the following questions:

a) Rationalize $\dfrac{175-j600}{-3+j4}$. Express your answer in rectangular form.

b) Find the polar form of $\dfrac{1}{2}+j\dfrac{\sqrt{3}}{2}$.

c) Find the rectangular form of $5\angle 25°\cdot 8\angle 35°$

d) Find the magnitude of $\left(\dfrac{j^3}{2+j4}\right)\left(\dfrac{30e^{j129°}}{2-j}\right)$.

e) Find the real part of $\dfrac{(1+j)^4}{1+j\sqrt{3}}$.

Sol'n: a) To rationalize, we multiply the numerator and denominator by the conjugate of the denominator.

$$\frac{175-j600}{-3+j4}\cdot\frac{-3-j4}{-3-j4}=\frac{175(-3)-600(4)-j175(4)-j600(-3)}{(-3)^2+4^2}$$

$$\frac{175-j600}{-3+j4}=\frac{-2925+j1100}{25}=-117+j44$$

b) We think of the complex number as a vector and find its length and its angle relative to the real axis.

$$\frac{1}{2}+j\frac{\sqrt{3}}{2}=\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\,e^{j\tan^{-1}\frac{\sqrt{3}/2}{1/2}}=\sqrt{\frac{1}{4}+\frac{3}{4}}e^{j60°}=1e^{j60°}$$

or

$$\frac{1}{2}+j\frac{\sqrt{3}}{2}=e^{j60°}$$

c) We first multiply the numbers in polar form.

$$5\angle 25°\cdot 8\angle 35°=5(8)\angle 25°+35°=40\angle 60°=40e^{j60°}$$

Now we convert to rectangular form using Euler's formula.

$$5\angle 25^\circ \cdot 8\angle 35^\circ = 40\cos(60^\circ) + j40\sin(60^\circ) = 40\cdot\frac{1}{2} + j40\frac{\sqrt{3}}{2}$$

or

$$5\angle 25^\circ \cdot 8\angle 35^\circ = 20 + j20\sqrt{3}$$

d) We take the magnitude of each term of a product or quotient. We must keep sums as is, however.

$$\left|\left(\frac{j^3}{2+j4}\right)\left(\frac{30e^{j129^\circ}}{2-j}\right)\right| = \frac{\left|j^3\right|}{|2+j4|}\frac{\left|30e^{j129^\circ}\right|}{|2-j|} = \frac{1^3\cdot 30}{\sqrt{2^2+4^2}\sqrt{2^2+1^2}}$$

or

$$\left|\left(\frac{j^3}{2+j4}\right)\left(\frac{30e^{j129^\circ}}{2-j}\right)\right| = \frac{30}{\sqrt{20}\sqrt{5}} = 3$$

e)

$$\mathrm{Re}\left[\frac{(1+j)^4}{1+j\sqrt{3}}\right] = \mathrm{Re}\left[\frac{\left(\sqrt{2}e^{j45^\circ}\right)^4}{2e^{j60^\circ}}\right] = \mathrm{Re}\left[\frac{4ej^{180^\circ}}{2e^{j60^\circ}}\right] = \mathrm{Re}\left[2e^{j(180^\circ-60^\circ)}\right]$$

or

$$\mathrm{Re}\left[\frac{(1+j)^4}{1+j\sqrt{3}}\right] = \mathrm{Re}\left[2e^{j120^\circ}\right] = \mathrm{Re}[2\cos(120^\circ) + j2\sin(120^\circ)]$$

or

$$\mathrm{Re}\left[\frac{(1+j)^4}{1+j\sqrt{3}}\right] = 2\cos(120^\circ) = 2\left(-\frac{1}{2}\right) = -1$$