HOMEWORK #16 prob 1 solution

EX: Give numerical answers to each of the following questions:

a) Rationalize $\dfrac{5(j20)}{-10-j20}$. Express your answer in rectangular form.

b) Find the polar form of $\left(\dfrac{5(j20)}{-10-j20}\right)^*$. (Note: the asterisk means "conjugate".)

c) Given ω = 2k rad/sec, find the following inverse phasor:

$P^{-1}[j50\text{ V}]$

d) Find the magnitude of $\left(\dfrac{2j^2}{1+j}\right)\left(\dfrac{e^j\sqrt{j}}{1-j}\right)$.

e) Find the imaginary part of $\dfrac{e^{-j90°}}{1-j}$.

SOL'N: a) First, we cancel common factors in the numerator and denominator.

$$\frac{5(j20)}{-10-j20}=\frac{(j20)}{-2-j4}=\frac{j10}{-1-j2}$$

Second, we multiply by the conjugate of the denominator.

$$\frac{j10}{-1-j2}=-\frac{j10}{1+j2}\,\frac{1-j2}{1-j2}$$

Third, we simplify the numerator and denominator.

$$-\frac{j10}{1+j2}\,\frac{1-j2}{1-j2}=-\frac{j10+20}{1^2+2^2}=-\frac{20+j10}{5}$$

Fourth and last, we cancel common factors in the numerator and denominator once again.

$$\frac{5(j20)}{-10-j20}=-4-j2$$

b) We need only take the conjugate of the answer to part (a). To take the conjugate, we change each j to $-j$.

$$\left(\frac{5(j20)}{-10-j20}\right)^*=-4+j2$$

c) The inverse phasor for j is $-\sin(\omega t)$ or $\cos(\omega t+90°)$. We scale this by 50 for our answer.

$$\mathrm{P}^{-1}[j50] = 50\cos(2\mathrm{k}\,t + 90°)\,\mathrm{V}$$

d) The magnitude of a product or quotient is the product or quotient of the magnitudes.

$$\left|\left(\frac{2j^2}{1+j}\right)\left(\frac{e^j\sqrt{j}}{1-j}\right)\right| = \frac{|2||j|^2}{|1+j|}\frac{\left|e^j\right|\left|\sqrt{j}\right|}{|1-j|} = \frac{2\cdot 1^2\cdot 1\cdot\left|e^{j\pi/2}\right|^{1/2}}{\sqrt{2}\cdot\sqrt{2}} = \frac{2\cdot 1^{1/2}}{2} = 1$$

e) To find the imaginary part, we express the quantity in rectangular form.

$$\mathrm{Im}\left(\frac{e^{-j90°}}{1-j}\right) = \mathrm{Im}\left(\frac{-j}{1-j}\right) = \mathrm{Im}\left(\frac{-j}{1-j}\frac{1+j}{1+j}\right) = \mathrm{Im}\left(\frac{-j+1}{1^2+1^2}\right)$$

$$= \mathrm{Im}\left(\frac{1-j}{2}\right) = -\frac{1}{2}$$