**EX:** a) Solve the following simultaneous equations for $v_1$ and $v_2$:

$$3v_1 - 4v_2 = 14$$

$$\frac{4(v_1 - v_2)}{7} + \frac{v_1}{2} = 29$$

b) Solve the following simultaneous equations for $R_1$ and $R_2$:

$$\sqrt{R_1^2 + R_2} = 3$$

$$\frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{10}{7}$$

**SOL'N:** a) We first decide which equation is simpler. Then we solve that equation for one variable in terms of the other. In the present case, the first equation is simpler.

$$3v_1 - 4v_2 = 14$$

or

$$3v_1 = 14 + 4v_2$$

or

$$v_1 = \frac{14}{3} + \frac{4}{3}v_2$$

Now we turn to the second equation:

$$\frac{4(v_1 - v_2)}{7} + \frac{v_1}{2} = 29$$

Multiplying this equation by the common denominator, $7 \cdot 2 = 14$, simplifies the equation:

$$8(v_1 - v_2) + 7v_1 = 29(14)$$

or

$$15v_1 - 8v_2 = 29(14)$$

Substituting for $v_1$ yields the following equation:

$$15\left(\frac{14}{3} + \frac{4}{3}v_2\right) - 8v_2 = 29(14)$$

or

$$5(14+4v_2)-8v_2=29(14)$$

We factor out the constants multiplying $v_2$:

$$12v_2+5(14)=29(14)$$

or

$$v_2=\frac{29(14)-5(14)}{12}=\frac{24(14)}{12}=2(14)=28$$

Using this value in our equation for $v_1$ in terms of $v_2$ yields the following result:

$$v_1=\frac{14}{3}+\frac{4}{3}(28)=\frac{14+8(14)}{3}=\frac{9(14)}{3}=3(14)=42$$

b) There are various approaches one might take here. One choice is to solve the first equation for $R_2$:

$$\sqrt{R_1^2+R_2}=3$$

or

$$R_1^2+R_2=3^2=9$$

or

$$R_2=9-R_1^2$$

Before substituting this into the second equation, we simplify the second equation:

$$\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}=\frac{10}{7}$$

Inverting both sides leads to simplification:

$$\frac{1}{R_1}+\frac{1}{R_2}=\frac{7}{10}$$

Now we substitute for $R_2$:

$$\frac{1}{R_1}+\frac{1}{9-R_1^2}=\frac{7}{10}$$

Multiplying by the common denominator yields a cubic equation:

$$R_1(9-R_1^2)\left(\frac{1}{R_1}+\frac{1}{9-R_1^2}\right)=\frac{7}{10}R_1(9-R_1^2)$$

or

$$(9-R_1^2)+R_1=\frac{7}{10}R_1(9-R_1^2)$$

or

$$\frac{10}{7}\left[(9-R_1^2)+R_1\right]=R_1(9-R_1^2)$$

or

$$R_1^3-\frac{10}{7}R_1^2+\left(\frac{10}{7}-9\right)R_1+\frac{10}{7}9=0$$

or

$$R_1^3-\frac{10}{7}R_1^2-\frac{53}{7}R_1+\frac{90}{7}=0$$

This cubic may be solved numerically by using a calculator such as the TI-83, or in closed form using the solution of the cubic equation, [1, 2]:

$$q=\frac{1}{3}\left(-\frac{53}{7}\right)-\frac{1}{9}\left(-\frac{10}{7}\right)^2$$

$$r=\frac{1}{6}\left[\left(-\frac{53}{7}\right)\left(-\frac{10}{7}\right)-3\left(\frac{90}{7}\right)\right]-\frac{1}{27}\left(-\frac{10}{7}\right)^3$$

$$s_1=\sqrt[3]{r+\sqrt{q^3+r^3}}$$

$$s_2=\sqrt[3]{r-\sqrt{q^3+r^3}}$$

The three roots are as follows:

$$z_1=(s_1+s_2)-\frac{a_2}{3}$$

$$z_2 = -\frac{1}{2}(s_1 + s_2) - \frac{a_2}{3} + j\frac{\sqrt{3}}{2}(s_1 - s_2)$$

$$z_3 = -\frac{1}{2}(s_1 + s_2) - \frac{a_2}{3} - j\frac{\sqrt{3}}{2}(s_1 - s_2)$$

Using a TI-83 calculator we use the "Y=" key and enter in the following equations:

\Y1=X^3–X^2*10/7–X*53/7+90/7

and

\Y2=0

Using the (2nd) CALC key and INTERSECTION, we respond to the "first curve" question by using the right arrow to position the cursor near the zero of the graphed function. That is, we position the cursor near $x = 2$. We press enter, and then we press enter twice more in response to the questions.

$$R_1^3 - \frac{10}{7}R_1^2 - \frac{53}{7}R_1 + \frac{90}{7} = 0$$

The calculator returns a value of $x = 2$ as the root. So $R_1 = 2$ is a solution.

For this value of $R_1$ we find the value of $R_2$:

$$R_2 = 9 - R_1^2 = 9 - 4 = 5$$

Factoring out this root, we have the following expression:

$$R_1^3 - \frac{10}{7}R_1^2 - \frac{53}{7}R_1 + \frac{90}{7} = (R_1 - 2)\left(R_1^2 + \frac{4}{7}R_1 - \frac{45}{7}\right) = 0$$

Solving the remaining quadratic equation yields the following result:

$$R_1 = -\frac{2}{7} \pm \sqrt{\left(\frac{2}{7}\right)^2 + \frac{45}{7}} = \frac{-2 \pm \sqrt{319}}{7} = 2.265 \text{ or } -2.837$$

If we rule out the negative resistance, we have a second solution for $R_2$ when $R_1 = 2.265$:

$$R_2 = 3.866$$

**REF:** [1] *The Cubic Formula*, http://www.sosmath.com/algebra/factor/fac11/fac11.html, accessed 19 May 2008.

[2] National Bureau of Standards, Milton Abramowitz and Irene A. Stegun Eds., *Handbook of Mathematical Functions,* US Dept. of Commerce, USGPO, 1964.