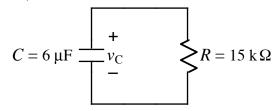
EX: Find the voltage, $v_{\rm C}$, on the capacitor in the circuit below as a function of time if $v_{\rm C}(t=0) = 2.4 \text{ V}.$



SOL'N: We observe that voltage v_C appears across both the *R* and *C*, as shown below.

Using the defining equation of a capacitor and Ohm's law, we have the following results:

$$i_C = C \frac{dv_C}{dt}$$

and

$$i_R = \frac{v_C}{R}$$

Since the current must be the same everywhere in the loop, and since the currents are measured with opposite polarities, we have that $i_{\rm C}$ and $i_{\rm R}$ are equal but opposite.

$$i_C = -i_R$$

or

$$C\frac{dv_C}{dt} = -\frac{v_C}{R}$$

One way to solve this equation is to separate the variables:

$$C\frac{1}{v_C}dv_C = -\frac{1}{R}dt$$

Integrating both sides yields the following result:

$$C\int_{v_C(t=0)}^{v_C(t)} \frac{1}{v_C} dv_C = -\frac{1}{R}\int_0^t dt$$

or

$$C \ln v_C(t) \Big|_{v_C(t=0)}^{v_C(t)} = -\frac{1}{R} t \Big|_0^t$$

or

$$C\left[\ln v_C(t) - \ln v_C(0)\right] = -\frac{1}{R}t$$

or

$$\ln \frac{v_C(t)}{v_C(0)} = -\frac{t}{RC}$$

or

$$\frac{v_C(t)}{v_C(0)} = e^{-\frac{t}{RC}}$$

or

$$v_C(t) = v_C(0)e^{-\frac{t}{RC}}$$

Substituting values given in the problem, we have the following answer:

$$v_C(t) = 2.4 \text{V} \cdot e^{-\frac{t}{15 \text{k} \Omega \cdot 6 \mu \text{F}}} = 2.4 \text{V} \cdot e^{-\frac{t}{90 \text{ms}}}$$