1. (50 points)

20 pts a) For the circuit shown, write three independent equations for the node voltages $V_1$, $V_2$, and $V_3$. The quantity $i_x$ must not appear in the equations. The equations must not contain more than the parameters $\alpha$, $I_{s1}$, $I_{s2}$, $I_{s3}$, $R_1$, $R_2$, and $R_3$.

10 pts b) Make a consistency check on your equations for part 1(a) by setting parameter $\alpha$, resistors (R1, R2, R3) and/or sources ($I_{s1}$, $I_{s2}$, $I_{s3}$) to values for which the values of $V_1$, $V_2$, and $V_3$ are obvious. State the values of resistors, sources, and for your consistency check, and show that your equations for problem 1(a) are satisfied for these values. (In other words, plug the values into your equations for problem 1(a) and show that the left side and the right side of each equation are equal.)

First, write $i_x$ in terms of $V_1$, $V_2$, or $V_3$ \( \Rightarrow \)

\[ i_x = \frac{V_2 - V_3}{R_2} \]

Second, check whether $V_i$ node is part of a supernode, (it is it connected to another node by only a voltage source). $V_i$ is not part of a supernode, so we write a current-summation eq. for it:

\[ i_s + (V_i - V_2) = 0 \]

Third, we observe that $V_2$ forms a supernode.

\[ V_2 = \alpha i_x \Rightarrow \]

\[ V_2 = \alpha \left( \frac{V_2 - V_3}{R_2} \right) \]
Fourth, check whether \( V_3 \) is part of a supernode. \( V_3 \) is not, so we write a current-summation eq. for it:

\[
\begin{align*}
3 \left( \frac{V_3 - V_2}{R_2} \right) + \frac{V_3}{R_3} - I_{S3} &= 0 \\
\text{note that this quantity is also } t'_x &= -\frac{(V_2 - V_3)}{R_2} = \frac{(V_3 - V_2)}{R_2}
\end{align*}
\]

1.b. There are many possible consistency checks. One example is as follows.

Let \( \alpha = 0, I_{S2} = 0, R_3 = \infty \text{ (open)}, R_1 = R_2 = 1 \Omega \)

\( I_{S1} = I_{S3} = 3A \)

Our circuit becomes:

![Circuit Diagram]

From this circuit,

\[
\begin{align*}
\text{Eq. 1} & \Rightarrow 3A + (V_1 - V_2) + 0 = 0 \\
(V_2 - V_1) &= +3V \quad \text{SAME} \quad \therefore (V_2 - V_1) = +3V \\
(V_3 - V_2) &= +3V \quad \text{SAME} \quad \therefore (V_3 - V_2) = +3V \\
V_2 &= 0 \text{ (tied to ref. node by wire)} \quad \text{SAME} \
\text{Eq. 2} & \Rightarrow V_2 = 0 \left( \frac{V_2 - V_3}{1\Omega} \right) = 0V
\end{align*}
\]
20 pts  c) For the circuit shown, write three independent equations for the three mesh currents $i_1$, $i_2$, and $i_3$. The quantity $v_x$ must not appear in the equations.

- First, we write $v_x$ in terms of mesh currents:
  
  $$v_x = (i_1 - i_2)R_2$$

- Second, observe any super meshes—none. Use V-loop eq.

  possibly loops $\Rightarrow$ $v_s$ to $R_1$ to $R_2$ $\rightarrow$ $v_s - i_1R_1 - i_2R_2 + i_2R_2 = 0$
  $v_s$ to $R_1$ to $R_3$ to $R_5$ $\rightarrow$ $v_s - i_1R_1 - i_2R_3 - i_3R_5 = 0$
  $R_2$ to $R_3$ to $R_5$ $\rightarrow$ $i_2R_3 - i_2R_2 - i_3R_5 = 0$
  $R_2$ to $R_3$ to $R_4$ to $\alpha v_x$ (1) $\rightarrow$ $i_1R_2 - i_2R_2 - i_3R_3 + i_3R_4 - i_2R_4 - \alpha v_x = 0$
  $\alpha v_x$ to $R_4$ to $R_5$ (2) $\rightarrow$ $V_x - i_3R_4 + i_2R_4 - i_3R_5 = 0$

Substituting $v_x$ into eq. (1) and (2):

$$i_1R_2 - i_2R_2 - i_2R_3 + R_4(i_3 - i_2) - \alpha(i_1 - i_2)R_2 = 0$$

$$\alpha(i_1 - i_2)R_2 - R_4(i_3 - i_2) - i_3R_5 = 0$$

Any three marked eq. are valid
2. (25 points)

25 pts Find the Thevenin equivalent circuit at terminals a-b. \( v_x \) must not appear in your solution. The expression must not contain more than circuit parameters \( \alpha, V_s, R_1, R_2, \) and \( R_3 \). Note: \( 0 < \alpha < 1 \).

\[
\begin{align*}
V_x &= V_{Th} \\
+V_s + I_s R_1 - \alpha V_{Th} - V_{Th} &= 0 \\
\frac{V_s + I_s R_1}{\alpha + 1} &= V_{Th} \\
R_{Th} &= \frac{V_{Th}}{I_{Sc}}
\end{align*}
\]

Short-circuit from a to b implies:

\[
\begin{align*}
V_x &= 0 \\
+V_s - I_1 R_1 &= 0 \\
\therefore I_1 &= \frac{V_s}{R_1} \\
-I_1 - I_s + I_{Sc} &= 0 \\
I_{Sc} &= I_1 + I_s = \frac{V_s}{R_1} + \frac{I_s R_1}{R_1} = \frac{V_s + I_s R_1}{\alpha + 1}
\end{align*}
\]

\[
V_{Th} = \frac{V_s + I_s R_1}{\alpha + 1}
\]

\[
R_{Th} = \frac{R_1}{\alpha + 1}
\]
3. (25 points)

25 pts  Calculate the power in the \( \frac{V_x}{2} \) dependent source. State whether the source is

absorbing or supplying the power.

\[ \text{Ohm's law: } V_x = I_2(4\Omega) \]

\[ V\text{-loop: } +15 - I_1(3) - I_2(4) - I_2(2) = 0 \]

\[ 15 - I_1(3) - I_2(6) = 0 \]

\[ I_1(3) = 15 - I_2(6) \]

1. \( I_1 = 5 - I_2(2) \)

Current summation:

\[ -I_1 + \frac{V_x}{2} + I_2 = 0 \]

\[ -I_1 + I_2(2) + I_2 = 0 \]

Plugging in 1 ⇒

\[ -5 + I_2(2) + I_2(2) + I_2 = 0 \]

\[ I_2(5) = +5 \]

\[ I_2 = 1A \]

\[ I_2 = 1A \]

\[ I_1 = 5 - (1)(2) = +3A \]

\[ V_x = I_2(4) = 4V \]

\[ \frac{V_x}{2} = \frac{4}{2} = 2 \]

\[ +15 - I_1(3) - V_1 = 0 \]

\[ V_1 = 15 - 9 = +6V \]

or

\[ +V_1 - I_2(6) = 0 \]

\[ V_1 = 6V \]

Power = \( (+2)(6) = +12W \)

absorbing