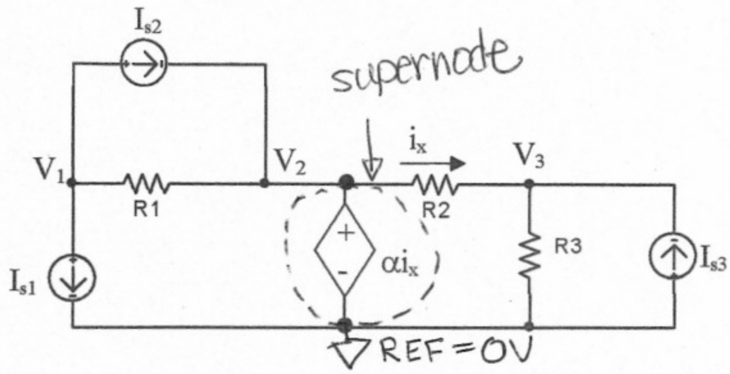
Exam 2 Solutions

1. (50 points)



20 pts a) For the circuit shown, write three independent equations for the node voltages V_1, V_2, and V_3. The quantity i_X must not appear in the equations. The equations must not contain more than the parameters α, I_{s1}, I_{s2}, I_{s3}, R1, R2, and R3.

10 pts b) Make a consistency check on your equations for part 1(a) by setting parameter α, resistors (R1, R2, R3) and/or sources (I_{s1}, I_{s2}, I_{s3}) to values for which the values of V_1, V_2, and V_3 are obvious. State the values of resistors, sources, and for your consistency check, and show that your equations for problem 1(a) are satisfied for these values. (In other words, plug the values into your equations for problem 1(a) and show that the left side and the right side of each equation are equal.)

- First, write i_x in terms of V_1, V_2, or V_3 ⟹

$$i_x = \frac{V_2 - V_3}{R_2}$$

- Second, check whether V_1 node is part of a supernode, (ie is it connected to another node by only a voltage source). V_1 is not part of a supernode, so we write a current-summation eq. for it:

① $$\boxed{+I_{S1} + \frac{(V_1 - V_2)}{R_1} + I_{S2} = 0A}$$

- Third, we observe that V_2 forms a supernode.

$$V_2 = \alpha i_x \Rightarrow ② \boxed{V_2 = \frac{\alpha (V_2 - V_3)}{R_2}}$$

· Fourth, check whether V_3 is part of a supernode. V_3 is not, so we write a current-summation eq. for it:

③ $$\boxed{\frac{(V_3 - V_2)}{R_2} + \frac{V_3}{R_3} - I_{S3} = 0}$$

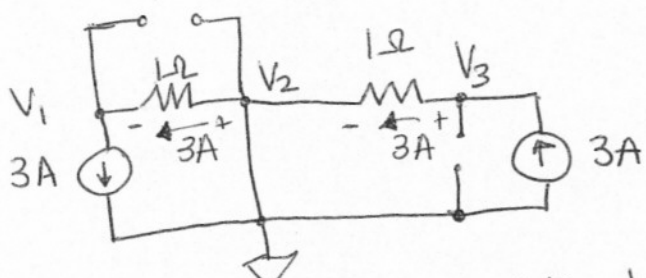
Note that this quantity is also $-i_x = \frac{-(V_2 - V_3)}{R_2} = \frac{(V_3 - V_2)}{R_2}$

1.b. There are many possible consistency checks. One example is as follows.

Let $\alpha = 0$, $I_{S2} = 0$, $R_3 = \infty$ (open), $R_1 = R_2 = 1\,\Omega$

$I_{S1} = I_{S3} = 3A$

Our circuit becomes:



From this circuit,

$(V_2 - V_1) = +3V$

$(V_3 - V_2) = +3V$

$V_2 = 0$ (tied to ref. node by wire)

Eq. 1 ⇒ $+3A + \frac{(V_1 - V_2)}{1\Omega} + 0 = 0$

$(V_1 - V_2) = -3A(1\Omega)$

SAME ✓ $\therefore (V_2 - V_1) = +3V$

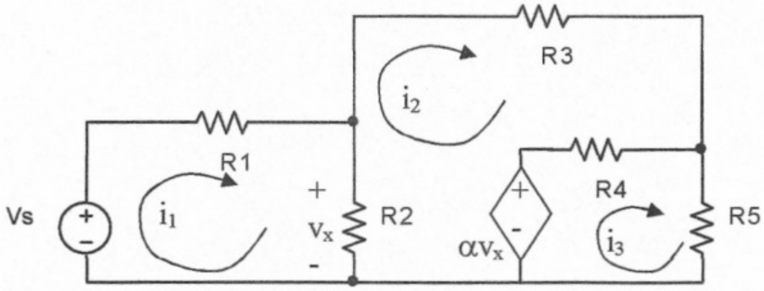
Eq. 3 ⇒ $\frac{(V_3 - V_2)}{1\Omega} + \cancel{\frac{V_3}{\infty}}^{\,0} - 3 = 0$

SAME ✓ $\therefore (V_3 - V_2) = +3V$

SAME ✓

Eq. 2 ⇒ $V_2 = 0\frac{(V_2 - V_3)}{1\Omega} = 0V$

1. cont.



20 pts c) For the circuit shown, write three independent equations for the three mesh currents i_1, i_2, and i_3. The quantity v_x must not appear in the equations.

· First, we write V_x in terms of mesh currents:

$$V_x = (i_1 - i_2) R_2$$

[Note: i_1 has a + sign because it has same polarity as V_x, whereas i_2 has a negative sign because it flows in the opposite direction]

· Second, observe any super meshes – none. Use v-loop eq.

possibly loops ⇒

V_s to R_1 to R_2 → $\boxed{+V_s - i_1 R_1 - i_1 R_2 + i_2 R_2 = 0}$

V_s to R_1 to R_3 to R_5 → $\boxed{+V_s - i_1 R_1 - i_2 R_3 - i_3 R_5 = 0}$

R_2 to R_3 to R_5 → $\boxed{+i_1 R_2 - i_2 R_3 - i_2 R_2 - i_3 R_5 = 0}$

R_2 to R_3 to R_4 to αV_x → (1.) $+i_1 R_2 - i_2 R_2 - i_2 R_3 + i_3 R_4 - i_2 R_4 - \alpha V_x =$

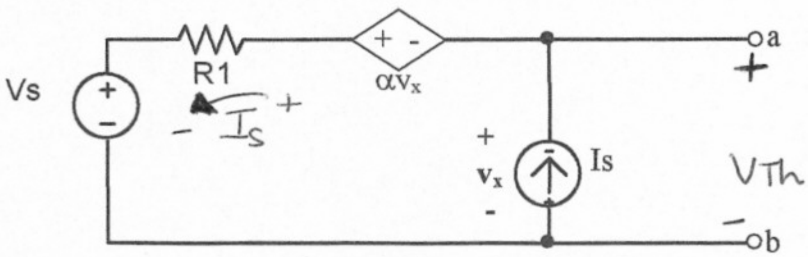
αV_x to R_4 to R_5 → (2.) $+\alpha V_x - i_3 R_4 + i_2 R_4 - i_3 R_5 = 0$

Substituting V_x into eq. (1) and (2):

$$\boxed{\begin{aligned} &+i_1 R_2 - i_2 R_2 - i_2 R_3 + R_4 (i_3 - i_2) - \alpha (i_1 - i_2) R_2 = 0 \\ &\alpha (i_1 - i_2) R_2 - R_4 (i_3 - i_2) - i_3 R_5 = 0 \end{aligned}}$$

Any three marked eq. are valid

2. (25 points)



25 pts Find the Thevenin equivalent circuit at terminals a-b. v_x must not appear in your solution. The expression must not contain more than circuit parameters α, Vs, R1, R2, and R3. **Note:** $0 < \alpha < 1$.

$$V_x = V_{Th}$$

$$+V_s + I_s R_1 - \alpha V_{Th} - V_{Th} = 0$$

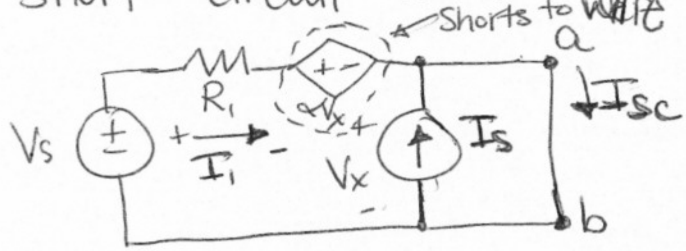
$$\frac{V_s + I_s R_1}{\alpha + 1} = V_{Th}$$

$$R_{Th} = \frac{V_{Th}}{I_{sc}}$$

short-circuit from a to b $\Rightarrow$

Shorts to wire

$V_x = 0$



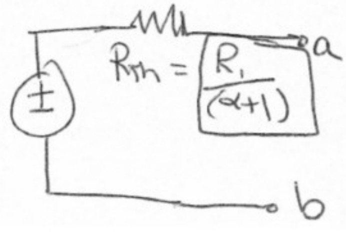
$$+V_s - I_1 R_1 = 0$$

$$\therefore \; I_1 = \frac{V_s}{R_1}$$

$$-I_1 - I_s + I_{sc} = 0$$

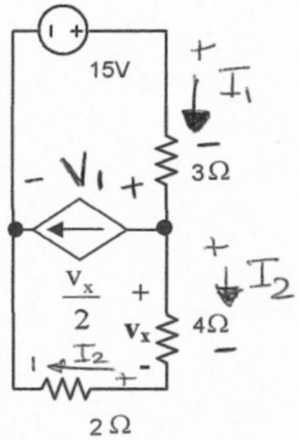
$$I_{sc} = I_1 + I_s = \frac{V_s}{R_1} + \frac{I_s R_1}{R_1} = \frac{V_s + I_s R_1}{R_1}$$

$$V_{Th} = \boxed{\frac{V_s + I_s R_1}{\alpha + 1}}$$



$$R_{Th} = \boxed{\frac{R_1}{(\alpha+1)}}$$

3. (25 points)



25 pts Calculate the power in the $\frac{V_x}{2}$ dependent source. State whether the source is **absorbing** or **supplying** the power.

Ohm's Laws ⇒

$V_x = I_2(4\Omega)$

V-loop ⇒ $+15 - I_1(3) - I_2(4) - I_2(2) = 0$

$15 - I_1(3) - I_2(6) = 0$

$I_1(3) = 15 - I_2(6)$

① $I_1 = 5 - I_2(2)$

Current-summation ⇒

$-I_1 + \frac{V_x}{2} + I_2 = 0$

$-I_1 + I_2(2) + I_2 = 0$

plugging in ① ⇒

$-5 + I_2(2) + I_2(2) + I_2 = 0$

$I_2(5) = +5$

$I_2 = 1A$

$I_2 = 1A$

$I_1 = 5 - (1)(2) = +3A$

$V_x = I_2(4) = 4V$

$\frac{V_x}{2} = \frac{4}{2} = 2$

$+15 - I_1(3) - V_1 = 0$

$V_1 = 15 - 9 = +6V$

OR

$+V_1 - I_2(6) = 0$

$V_1 = 6V$

$\therefore$ power $= (+2)(6) = \boxed{+12W}$

$\boxed{\text{absorbing}}$