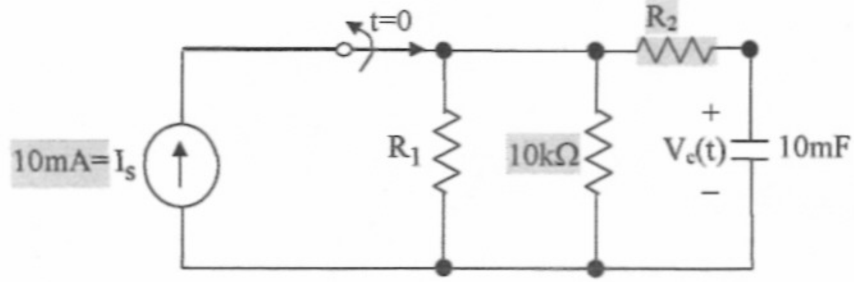
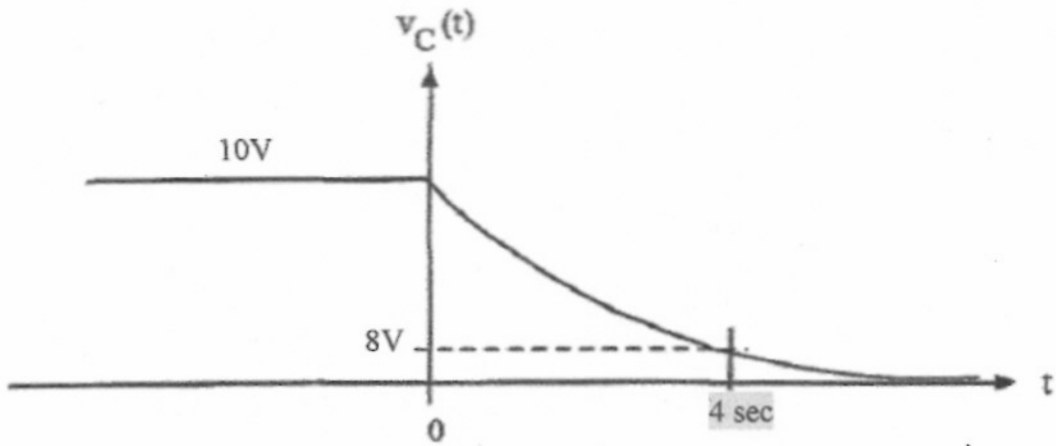
UNIVERSITY OF UTAH
ELECTRICAL AND COMPUTER ENGINEERING DEPARTMENT

ECE 1270 | HOMEWORK #5 | Spring 2008

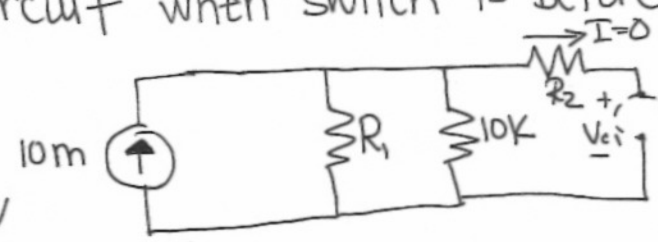
1.



After being closed for a long time, the switch becomes open at t=0. Find R1 and R2 that give the following plot for Vc(t):



Step 1. a. Draw circuit when switch is before $t=0$. This time is $t=0^-$.



Cap is open because switch has been at this position for a long time.
$\therefore\ i = C\left(\frac{dv}{dt}\right) = 0$
$\therefore\ i = 0$ (looks like an open)

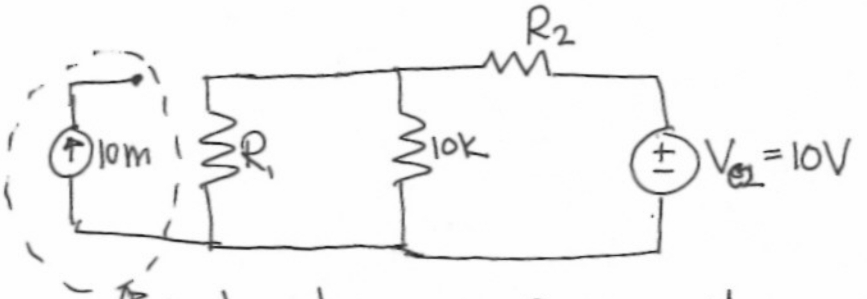
$$V_{c1} = \frac{10m(10k)R_1}{R_1 + 10k} = 10V$$

↑from graph

$$100\,R_1 = R_1(10) + 10k(10)$$

$$R_1(\cancel{100} - 10) = \frac{10k(10)}{90} \Rightarrow \boxed{R_1 \cong 1.11k\Omega}$$

Step 1. b. Draw circuit when switch just changes at $t=0$. This time is $t=0^+$. Find INITIAL CONDITION at this time.



voltage on capacitor can not change instantaneously!

irrelevant part of circuit

$V_{c2} = V_{c1} = +10V$

Step 2. Draw circuit when switch has changed. Time is $t=\infty$. Find FINAL VALUE. cap open again because swith has been at that position for a long time $\Rightarrow$ $i = C\frac{dv}{dt} = 0$ (no change in voltage over time)

$V_{c3} = 0$

$$\tau = R_{eq}\cdot C = [R_2 + (1.11K \| 10K)]C = [R_2 + 999]\cdot C$$

Step 3. General equation $\Rightarrow$

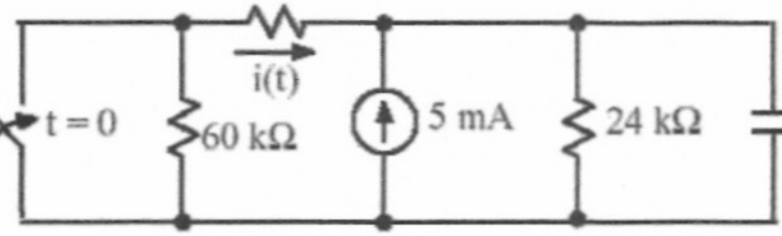
$$V_c(t) = 10e^{-t/[R_2+999]10m}$$

From graph $\Rightarrow$ $8 = 10e^{-4/(R_2 10m + 9.99)}$

$$\ln\frac{8}{10} = \frac{-4}{(R_2 10m + 9.99)}$$

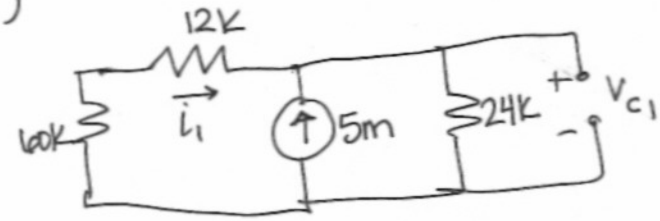
$$R_2 \cancel{10m} + \cancel{9.99} = \frac{\left[\frac{-4}{\ln(\frac{8}{10})} - 9.99\right]}{10m} = \boxed{794\Omega}$$

2.



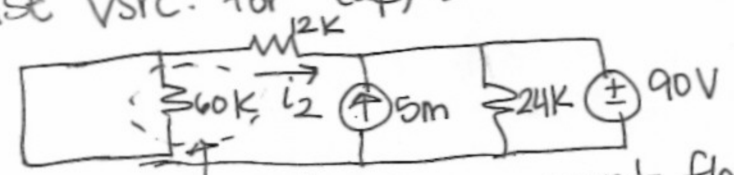
Step 1: After being open for a long time, the switch becomes closed at t=0. Find i(t) for t > 0.

a. $(t=0^-)$



$\therefore V_{C1} = \left[\frac{5m(72K)}{72K+24K}\right]24K = 90V$

b. $(t=0^+)$ ← Find $\boxed{\text{Initial Value}}$ at this time. (switch just changed, use Vsrc. for cap) ⇒



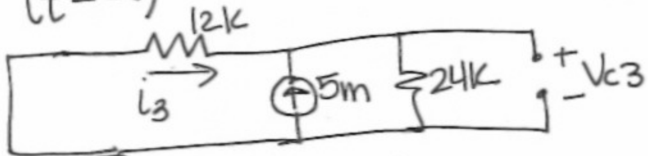
irrelevant : 0 current flow, 0volts

Use Ohm's Law: polarity is opposite +90V supply

$90 = -i_2(12K)$

$\therefore \; i_2 = -\frac{90}{12K} = \underline{\underline{7.5mA}}$ (Initial condition)

Step 2: $(t=\infty)$



$V_{C3} = \left[\frac{5m(12K)}{12K+24K}\right]24K = 40V$

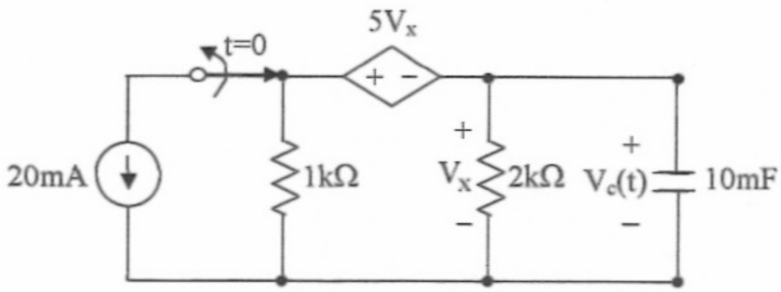
$i_3 = -\frac{5m(24K)}{36K} = \underline{\underline{-3.33mA}}$ Final

$\tau = (24K \| 12K)\cdot 3\mu \cong 24m$

$t>0, \; i(t) = -3.33m + [7.5m - (-3.33m)]\,e^{-t/24m}$ A

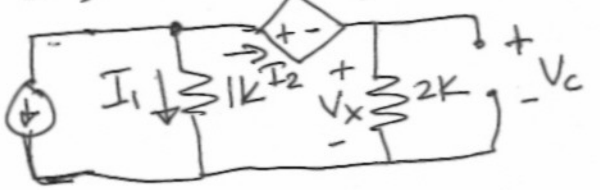
$\boxed{i(t) = -3.33m + 10.83m\,e^{-t/24m} \text{ A}}$

3.



After the switch has been closed for a long time, it <u>opens</u> at t=0. Find $V_c(t)$ for $t > 0$.

Step 1a. ($t=0^-$)



$\Sigma I: +20m + I_1 + I_2 = 0 \Rightarrow I_1 = (-I_2 - 20m)$

v-loop: $+I_1(1K) - 5(I_2)(2K) - I_2(2K) = 0$

$-(I_2 1K) - 20m(1K) - 10K I_2 - I_2(2K) = 0$

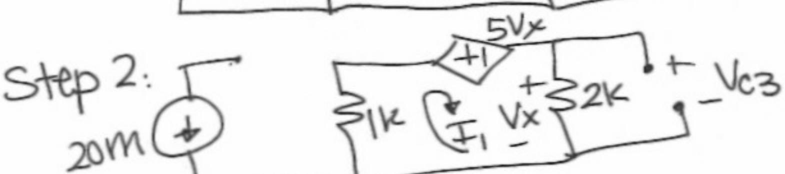
$I_2(-13K) = 20 \Rightarrow I_2 = \frac{-20}{13K}$

$V_c = I_2(2K) = -\frac{40}{13}$

Step 1b.

$V_{c2} = -\frac{40}{13}$ ⟸ Initial condition

Step 2:



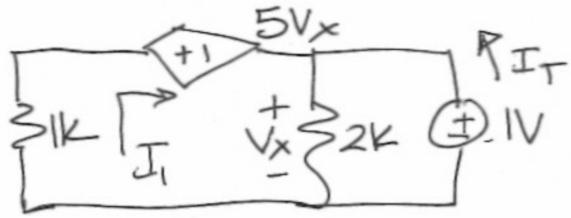
$-I_1(1K) - 5V_x - V_x = 0$

$-I_1(1K) - 6(I_1(2K)) = 0$

$I_1 = 0 \quad \therefore \; V_{c3} = 0$ Final value

$\tau \Rightarrow$ since $V_{c3} = 0$, need to use <u>test</u> source

3. (cont.)



$V_x = 1V$

$-I_1(1k) - 5(1) - 1 = 0$

$I_1 = \frac{-6}{1k} = -6mA$

$-I_1 + \frac{1}{2k} - I_T = 0$

$I_T = \frac{1}{2k} - (-6m) = 6.5m$

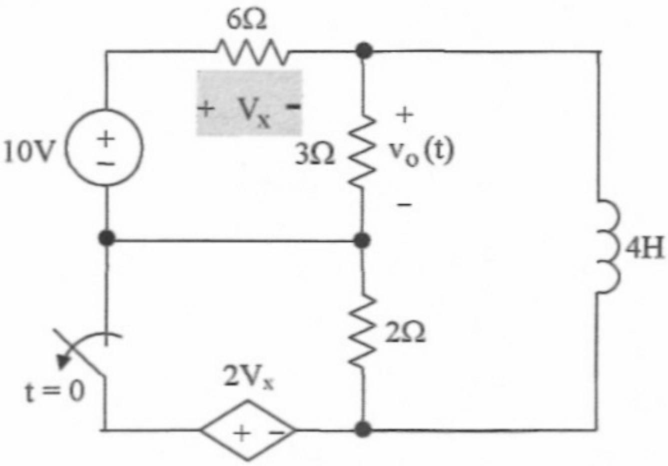
$R_{Th} = \frac{1V}{I_T} = \frac{1V}{6.5m} = 154\Omega$

$\tau = (154)(10m) = 1.54$

Step 3:
general eq: $t>0, \; V_c(t) = 0 + \left[-\frac{40}{13} - 0\right]e^{-t/1.54} \; V$

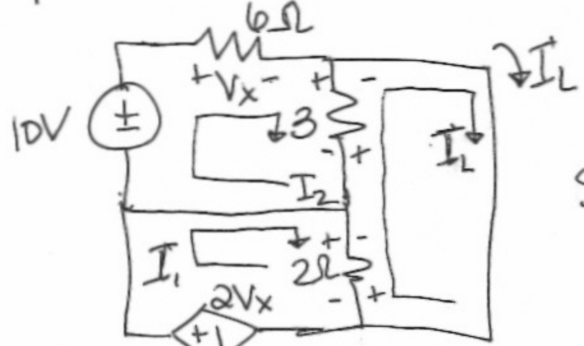
$$\boxed{t>0, \quad V_c(t) = -\frac{40}{13}e^{-t/1.54} \; V}$$

4.



After the switch has been closed for a long time, it opens at t=0. Find the output voltage $v_o(t)$ for $t > 0$.

Step 1a. $(t=0^-)$



① $+2(I_2)(6) - I_1(2) + I_L(2) = 0$

② $2(I_1 - I_L) + 3(I_2 - I_L) = 0$

③ $+10 - I_2(6) + 3(I_L - I_2) = 0$

Solve ③ for I_2:

$10 - 6I_2 - 3I_2 + 3I_L = 0$

$10 - 9I_2 + 3I_L = 0$

④ $I_2 = \frac{10 + 3I_L}{9}$

plug ④ into ①

$12\left(\frac{10}{9}\right) + 12\left(\frac{I_L}{3}\right) - I_1 2 + 2I_L = 0$

$6I_L + \frac{120}{9} = 2I_1$

$3I_L + \frac{60}{9} = I_1$ ⑤

plug ⑤④ into ②

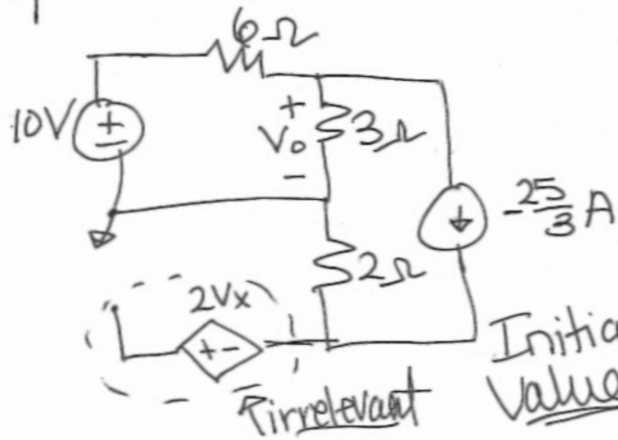
$2\left(3I_L + \frac{60}{9}\right) - 2I_L + 3\left(\frac{10}{9}\right) + 3\left(\frac{3I_L}{9}\right) - 3I_L = 0$

$I_L(6 - 2 + 1 - 3) = -\frac{120}{9} - \frac{30}{9} = -\frac{150}{9}$

$I_L = \frac{-150}{9(2)} = \frac{-25}{3}$ A

4. (cont.)

Step. 1b. $(t=0^+)$



Initial Value

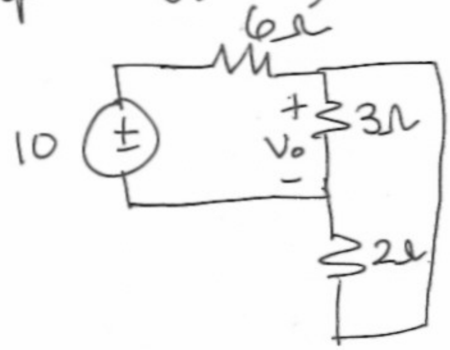
$$\frac{V_o-10}{6}+\frac{V_o}{3}+\left(-\frac{25}{3}\right)=0$$

$$V_o\left(\frac{1}{6}+\frac{2}{6}\right)=\frac{25}{3}+\frac{5}{3}=\frac{30}{3}$$

$$V_o=\frac{30}{3}\cdot\frac{6}{3}=\frac{180}{9}\,V$$

$$V_o=20V$$

Step 2: $(t=\infty)$



$$V_o=\frac{10(3\|2)}{\underbrace{(3\|2)}_{\frac{2(3)}{5}=1.2}+6}=\frac{5}{3}\ V$$

Final value

$\tau \Rightarrow$

$$R_{eq}=(3\|6)+2=\frac{3(6)}{9}+2$$

$$R_{eq}=4\,\Omega$$

$$\tau=4/R_{eq}=\frac{4}{4}=1$$

$$t>0,\ V_o(t)=\frac{5}{3}+\left[20-\frac{5}{3}\right]e^{-t/1}\ V$$

$$\boxed{V_o(t)=\frac{5}{3}+\frac{55}{3}e^{-t}\ V}$$