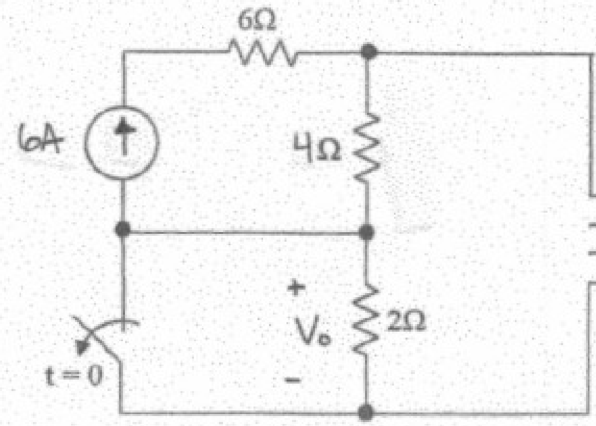
HW 6 Solution

Spring 2008



Step 5:
General Solution:

$V_o(t) = -8 + (-12+8)e^{-t/4/6}\ V$

$t > 0$

$\boxed{V_o(t) = -8 - 4e^{-3t/2\text{sec.}}\ V}$

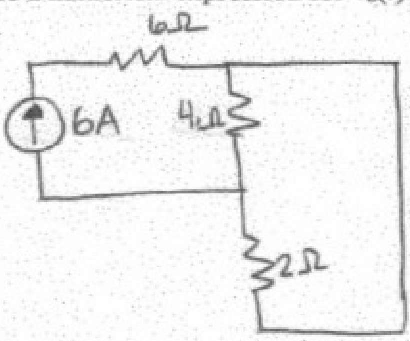
OR SAME

$V_o(t) = -12 + (-8+12)(1 - e^{-3t/2\text{sec.}})$

After being closed for a long time, the switch opens at t = 0.

1. Calculate the energy stored on the inductor as $t \to \infty$.
2. Write a numerical expression for $v_o(t)$ for $t > 0$.

(2) $t \to \infty$



$\downarrow i_L$

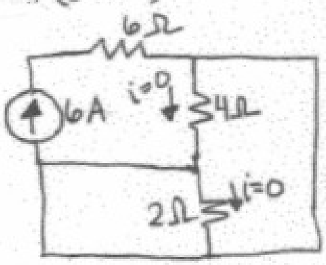
(1) $i_L = \frac{6(4)}{(4+2)} = \frac{24}{6} = 4A$

energy $= \frac{1}{2} L i^2$

$w_L(t \to \infty) = \frac{1}{2}(4)(4)^2$

$= \boxed{32\ J}$

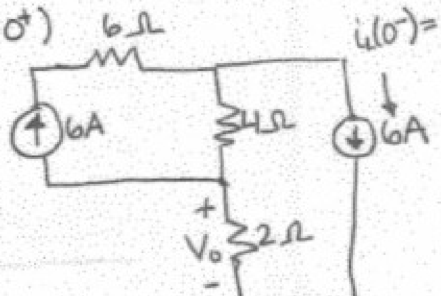
Step 1:
$(t = 0^-)$



$\downarrow i_L$

$\therefore i_L = 6A$

Step 2:
$(t = 0^+)$



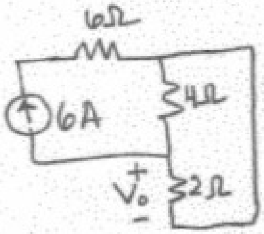
$i_L(0^-) = i_L(0^+)$

$V_o = -6(2)$

$= \boxed{-12V}$

Initial value

Step 3:
$(t \to \infty)$

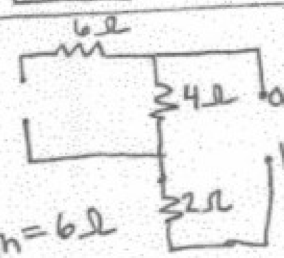


$\downarrow i_L = 4A$ (above)

$\therefore V_o = -4(2) = \boxed{-8V}$

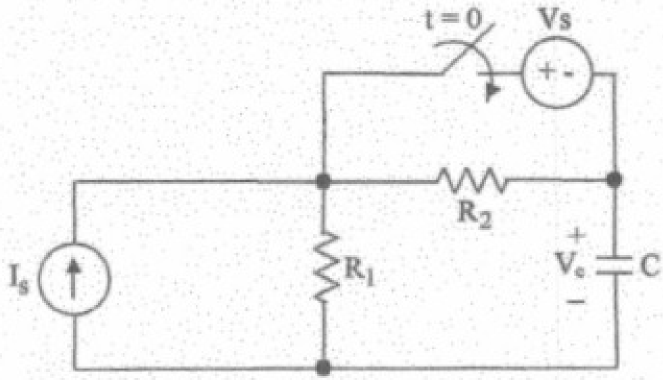
Final value

Step 4:



$R_{Th} = 6\Omega$

3.



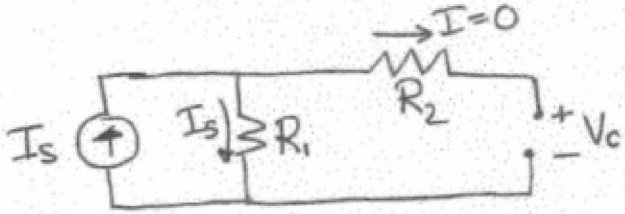
After being open for a long time, the switch closes at t = 0.

a) Write an expression for $V_c(t = 0^+)$

b) Write an expression for $V_c(t > 0)$ in terms of R_1, R_2, V_s, I_s, and C.

a.
$(t=0^-)$

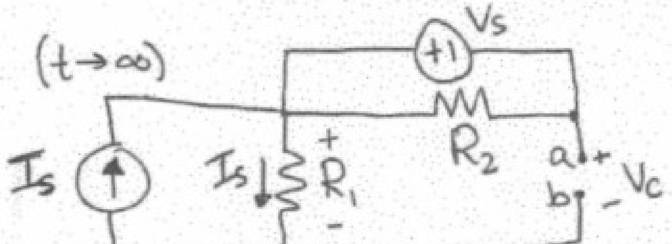
$V_c = I_s R_1$



Initial value

$V_c(t=0^-) = \boxed{V_c(t=0^+) = I_s R_1}$

b. $(t \to \infty)$

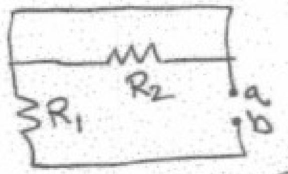


$+I_s R_1 - V_s - V_c = 0$

$\underline{V_c = I_s R_1 - V_s}$

Final value

$\tau \Rightarrow R_{Th} = R_1$:

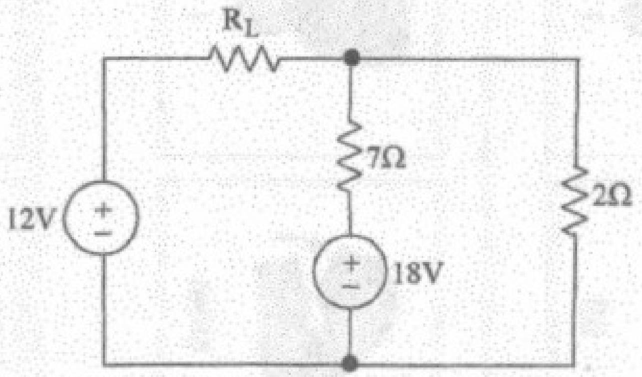


$v_c(t>0) = I_s R_1 - V_s + (I_s R_1 - I_s R_1 + V_s)e^{-t/R_1C}$

$\boxed{V_c(t>0) = I_s R_1 - V_s + V_s e^{-t/R_1C}}$

OR $V_c(t>0) = (I_s R_1) + [I_s R_1 - V_s - I_s R_1](1 - e^{-t/R_1C})$

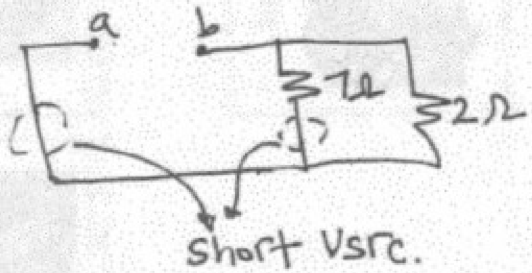
4.



a) Calculate the value of R_L that would absorb maximum power.

b) Calculate that value of maximum power R_L could absorb.

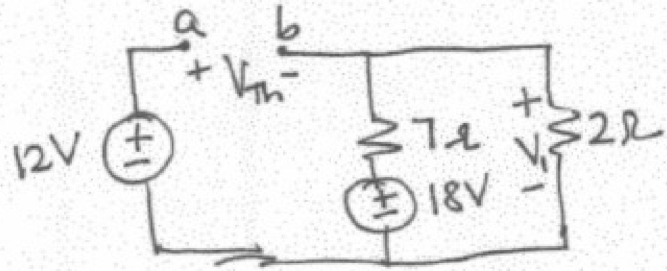
a. Maximum power achieved when $R_L = R_{Th}$



$R_{Th} = R_L = 7\Omega \| 2\Omega$

$R_L = \frac{7(2)}{9} = \boxed{\frac{14}{9}\Omega}$

b. maximum power $= \frac{V_{Th}^2}{4R_{Th}}$



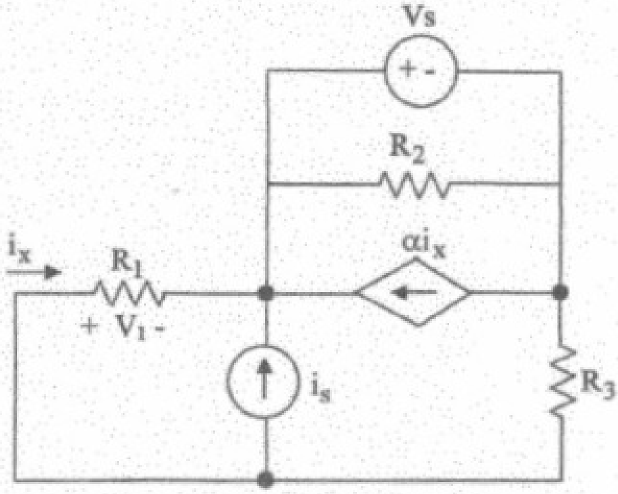
$V_1 = \frac{18(2)}{9} = 4V$

$+12 - V_{Th} - V_1 = 0$

$V_{Th} = 12 - 4 = 8V$

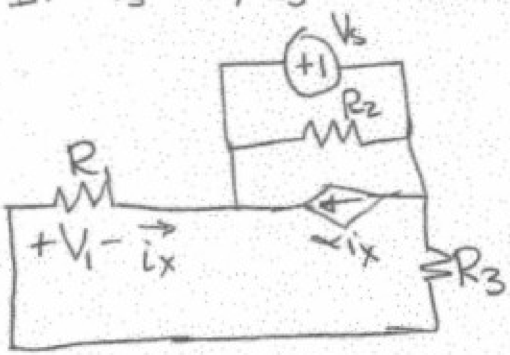
$\frac{V_{Th}^2}{4R_{Th}} = \frac{(8)^2}{4(\frac{14}{9})} \cong \boxed{10.3W}$

5.



Using superposition, derive an expression for V_1 that contains no circuit quantities other than i_s, V_S, R_1, R_2, R_3, and α, where $\alpha > 0$.

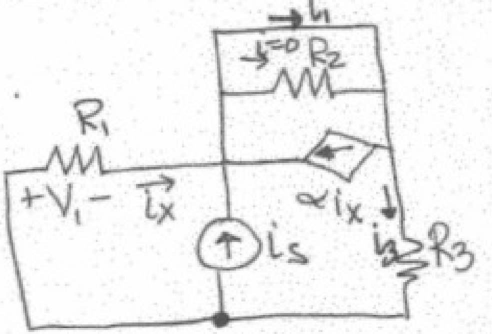
Case I: V_s on, i_s off ($i_s = 0$)



V_1 is found by a V-divider:

$$V_1 = \frac{-V_s R_1}{R_1 + R_3}$$

Case II: V_s off ($V_s = 0$), i_s ON



$V_1 = i_x R_1$

$-i_3 + i_s + i_x = 0$

$i_3 = (i_s + i_x)$

V-loop: $-i_x R_1 - i_3 R_3 = 0$

$-i_x R_1 - i_s R_3 - i_x R_3 = 0$

$i_x = \frac{-i_s R_3}{R_1 + R_3}$ [I-divider]

$\therefore V_1 = \frac{-i_s R_1 R_3}{R_1 + R_3}$

Total = sum both cases

$$\boxed{V_1 = \frac{-V_s R_1}{(R_1 + R_3)} + \frac{-i_s R_1 R_3}{R_1 + R_3}}$$