Find the absolute voltages at all the labeled nodes in the above circuit. Hint: Start at the bottom and work upwards.

Sol: Starting at the reference, which is 0 V, we work our way upwards. If we enter a source at the – sign, we add the voltage of the source. Otherwise, we subtract the voltage of the source.

\[ V_3 = -4V \]
\[ V_2 = +V_3 + 15V = -4 + 15 = 11V \]
\[ V_1 = -4 + 15 + (-3) = +8V \]

3. Using Ohm’s law and the node voltages found in Problem 2, find the currents for all the resistors in Problem 2.

Currents may be measured in one of two directions for each resistor. Here, they will all be measured with the arrow pointing down or to the right. The resistor currents are found by taking the difference of the node voltage on each end of the resistor and dividing by the resistance.

Across 8kΩ resistor:
\[ \frac{v_1 - 0}{8k} = \frac{-25}{8k} = -3.125mA \]

Across 10kΩ resistor:
\[ \frac{v_3 - 0}{10k} = \frac{-25}{10k} = -2.5mA \]

Across 5kΩ resistor:
\[ \frac{v_3 - v_1}{5k} = \frac{-25 - (-21)}{5k} = -0.8mA \]
Across 6kΩ resistor: \[ \frac{v_3 - v_4}{6k} = \frac{-25 - (-10)}{6k} = -2.5mA \]

Across 2kΩ resistor: \[ \frac{v_4 - v_5}{2k} = \frac{-10 - (-16)}{2k} = +3mA \]

Across 7kΩ resistor: \[ \frac{v_2 - v_4}{7k} = \frac{-13 - (-10)}{7k} = -0.43mA \]

4. Find the value of current, \( i_1 \), for each of the above circuits.

\[ i_1 = \frac{5 - (-4)}{9} = 1A \]

\[ v_2 = 5V \]
\[ 9Ω \]
\[ v_1 = -4V \]

To determine the signs, if we were to mark a + and – for a voltage measurement across the 9 Ω resistor that is in the passive sign direction for \( i_1 \), then we add \( v_2 \) (which is next to the + sign) and subtract \( v_1 \) (which is next to the – sign).

5. Find the value of current, \( i_1 \), for each of the above circuits.

The current must match the current source, but the polarity is inverted since the arrow in the current source is in the opposite direction of \( i_1 \). \[ i_1 = -2A \]

If there is a current source in a branch, (components in series), the current everywhere in that branch must be the same as the current source.

\[ i_1 = 4mA \]
6. Set the reference point at the bottom node and Vo as the top node. We set the sum of currents out of v1 equal to zero, group terms for Vo and then solve:

\[
\frac{V_o - 114}{12} + 2 + \frac{V_o - 0}{33} = 0
\]
\[
V_o \left( \frac{1}{12} + \frac{1}{33} \right) = \frac{114}{12} - 2
\]
\[
V_o \left( \frac{33}{12(33)} + \frac{12}{(12)33} \right) = \frac{114 - 24}{12}
\]
\[
V_o \left( \frac{45}{396} \right) = \frac{90}{12}
\]
\[
V_o = \frac{90}{12} \cdot \frac{396}{45} = \frac{35640}{540} = 66V
\]
\[
i = \frac{114 - V_o}{12} = \frac{114 - 66}{12} = 4A
\]

7. Use the node-voltage method to determine \( I_x \).

Using the bottom node as the reference node, two equations stated below are determined by setting the sum of currents(directions as shown) out each node equal to zero.

\[
\frac{V - 5.3}{4} + \frac{V_1}{3} + \frac{V_2 - V_2}{6} = 0 \quad \text{(V1 node)}
\]
\[
- \left( \frac{V_1 - V_2}{6} \right) + \frac{V_2}{12} - 2 \left( \frac{V_1 - V_2}{6} \right) = 0 \quad \text{(V2 node)}
\]
Solving the first equation for $V_1$:

$$V_1\left(\frac{1}{4} + \frac{1}{3} + \frac{1}{6}\right) = \frac{5.3}{4} + \frac{V_2}{6}$$

$$V_1\left(\frac{3}{12} + \frac{4}{12} + \frac{2}{12}\right) = \frac{5.3}{4} + \frac{V_2}{6}$$

$$V_1\left(\frac{9}{12}\right) = \left(\frac{5.3}{4} + \frac{V_2}{6}\right) \cdot \left(\frac{12}{9}\right) = \left(\frac{5.3}{3} + \frac{2V_2}{9}\right)$$

Plugging this variable into the second equation and solving for $V_2$:

$$-\left(\frac{V_1-V_2}{6}\right) + \frac{V_2}{12} - 2\left(\frac{V_1-V_2}{6}\right) = 0$$

$$V_1\left(\frac{-1}{6} + \frac{-2}{6}\right) + V_2\left(\frac{1}{6} + \frac{1}{12} + \frac{2}{6}\right) = 0$$

$$\left(\frac{5.3}{3} + \frac{2V_2}{9}\right) \cdot \left(\frac{-3}{6}\right) + V_2\left(\frac{7}{12}\right) = 0$$

$$\left(-\frac{5.3}{6} + \frac{-V_2}{9}\right) + V_2\left(\frac{7}{12}\right) = 0$$

$$V_2\left(\frac{7}{12} + \frac{-1}{9}\right) = \left(\frac{5.3}{6}\right) \cdot \left(\frac{-9(12)}{7(9)-12}\right) = 1.9V$$

$$V_1 = \left(\frac{5.3(3)}{9} + \frac{2(1.87)}{9}\right) = 2.2V$$

Once $V_1$ and $V_2$ are solved, $I_x = 2I$ can be found:

$$I = \left(\frac{V_1-V_2}{6}\right) = \left(\frac{2.2-1.9}{6}\right) = 50mA$$

$$I_x = 2I = 100mA$$
8. Use the node-voltage method to find \(V_1\) and \(V_2\).

9. Determine the amount of power supplied by the voltage source.

(a) We derive two equations by setting the sum of currents out each node equal to zero.

\[
\begin{align*}
\left(\frac{v_1 - 6}{2}\right) + \frac{v_1}{8} + \frac{v_1}{8} + 3 = 0 & \Rightarrow \quad v_1 \left(\frac{4}{8} + \frac{1}{8} + \frac{1}{8}\right) = -3 + 3 \\
-3 + \left(\frac{v_2}{8}\right) + \frac{v_2}{8} - 4 = 0 & \Rightarrow \quad v_2 \left(\frac{1}{8} + \frac{1}{8}\right) = +7 \\
\end{align*}
\]

Solving the first equation for \(v_1\):

\[
\frac{6}{8} = 0 \Rightarrow \quad v_1 = 0
\]

\[
\frac{2}{8} = +7 \Rightarrow \quad v_2 = 7 \left(\frac{8}{2}\right) = 28V
\]

(b) Power \(= I \cdot V = I \cdot 6 = \left(\frac{v_1 - 6}{2}\right) \cdot 6 = \left(\frac{0 - 6}{2}\right) \cdot 6 = -18W\) (generating)

Use the node-voltage method to find \(v_1\) and \(v_2\).
Sol’n: Write current summations for $v_B$ and $v_E$ nodes. The second equation is the supernode equation. The first equation is due to a supernode between $v_B$ and $v_E$

$$\frac{V_B - 0}{3k} + \frac{V_B - 12.6}{1k} + \frac{V_E - 2}{330} - 99i_b = 0$$

$$V_B - V_E = 0.7$$

The first equation still has $i_b$ in it. This needs to be removed by writing $i_b$ in terms of $v_B$ and $v_E$. This can be done by a current summation for the $v_B$ node:

$$\frac{V_B - 0}{3k} + \frac{V_B - 12.6}{1k} + i_b = 0$$

$$i_b = \frac{-V_B}{3k} + \frac{-V_B}{1k} + \frac{12.6}{1k}$$

Plugging this into the first equation above to remove $i_b$ from it:

$$\frac{V_B - 0}{3k} + \frac{V_B - 12.6}{1k} + \frac{V_E - 2}{330} - 99\left(\frac{-V_B}{3k} + \frac{-V_B}{1k} + \frac{12.6}{1k}\right) = 0$$

$$V_B - V_E = 0.7$$

$$V_E = V_B - 0.7$$

$$\frac{V_B - 0}{3k} + \frac{V_B - 12.6}{1k} + \frac{(V_B - 0.7) - 2 - 99\left(\frac{-V_B}{3k} + \frac{-V_B}{1k} + \frac{12.6}{1k}\right)}{330} = 0$$

$$V_B = 9.3$$

$$V_E = V_B - 0.7 = 9.3 - 0.7 = 8.6V$$

$$i_L = \frac{V_E - 2}{330} = \frac{8.6 - 2}{330} = 20mA$$