1. Use the mesh-current method to find $i_1$ and $i_2$, and $i_3$.

![Circuit Diagram]

Procedure for solving for mesh-currents:

1. Label every mesh with a mesh current variable.
2. Label polarity on every R. If R is between 2 meshes, there will be polarities labeled for two mesh currents.
3. If an outside branch contains a current source (dependent or independent), the mesh current is equal to that current source value (+ if they are in the same direction or − if opposite directions).
4. If the circuit contains a dependent source, the dependent variable needs to be written in terms of the mesh currents.
5. If a current source is located between two meshes, a supermesh equation is written. The supermesh equation is the current source value equal to the two mesh currents going through it. (+ if the same direction, − if opposite directions).
6. A voltage loop is taken through pathways without current sources in them. The voltage loop equation will contain two currents going through a resistor located between 2 meshes.
7. Simultaneous equations are solved to obtain the mesh current values. These mesh current values can be used to determine any other unknown variable in the circuit.

Step 1 and 2 are shown in the figure above:

Step 3: $i_3 = 2A$

Step 4: skip

Step 5: skip

Step 6 (three voltage loops possible):

$$-i_1(5k) + 20 - i_4(10k) + i_4(10k) = 0$$

$$+i_4(10k) - i_2(10k) - 10 = 0$$

$$-i_1(5k) + 20 - 10 = 0$$

The last equation only has $i_4$ as an unknown, solving this gives:

$$-i_1(5k) + 20 - 10 = 0$$

$$i_1(5k) = 10 \quad \Rightarrow \quad i_1 = 2mA$$

Using the second equation and this known value yields:

$$+i_2(10k) = +i_4(10k) - 10$$

$$+i_2 = +i_1 - 1m$$

$$+i_2 = 2m - 1m = 1mA$$

$$i_1 = 2mA \quad i_2 = 1mA \quad i_3 = 2A$$
2. a. Use the mesh-current method to find $V_x$, $V_x$ must not be in equation.
   b. Find power dissipated by the dependent source.

Step 1 and 2 are shown in the figure at the right:

Step 3: $i_1 = -50mA$, $i_2 = +0.5A$

Step 4: $V_x$ is the voltage across the current source and the 500 ohm:

$$V_x = (+i_1 - i_2)500 = (+0.5 - i_2)500 = +250 - i_2 500$$

Step 5: No supermesh

Step 6 (only one voltage loop possible):

$$+2i_2 - i_2 1k + i_1 1k + 7 + V_x = 0$$

$$3(+250 - i_2 500) - i_2 1k + 50m(1k) + 7 = 0$$

$$i_2 2.5k = 807$$

$$i_2 = 282.8mA$$

Plugging this value into the equation for $V_x$ above gives:

$$V_x = +250 - (282.8m)500 = 108.6V$$

Power in the dependent source:

$$P = I \times V = (i_1 - i_2)V_x = (282.8m + (-50m))2(108.6) = 72.3W$$

3. Find the Thevenin equivalent circuit at terminals a-b.

Find $V_{th}$. This is the open circuit voltage between points a-b (Using mesh currents)

$i_1 = -5A$

$$+20 - i_1 5 - i_1 3 - i_2 2 + i_2 2 = 0$$

$$20 - i_1 15 + i_2 2 = 0$$

$$20 - i_1 15 + (-5)2 = 0$$

$$i_1 = \frac{2}{3}$$

An equation for $V_{th}$ is needed:

$$V_{th} - 0 - i_1(3) = 0$$

$$V_{th} = \frac{2}{3}(3) = 2V$$

$R_{th}$ is found by removing all the independent sources (20V becomes a wire, 5A becomes an open):

$$R_{th} = 0.6 + 3 \parallel (10 + 2) = 0.6 + \frac{1}{\frac{1}{3} + \frac{1}{12}} = 3\Omega$$
Using Node-Voltage Method – Procedure:

1. a. Place a reference point (0V).
   b. Label every essential node with a node-voltage.

2. Place all currents through every R.

3. If the circuit contains a dependent source, the dependent variable needs to be written in terms of the node voltages.

4. If only a single voltage source is located in a branch, a supernode equation is written. The supernode equation is the voltage source value equal to the two nodes on both sides of it. The voltage difference is taken as + at the positive side and – at the negative sign.

5. A current summation is taken at all node voltage variables. Each current term is a positive on the + side of the labeled current and – on the negative side of the labeled current.

6. Simultaneous equations are solved to obtain the node voltage values. These node voltage values can be used to determine any other unknown variable in the circuit.

In this case, the reference point is placed at b. The dependent variable is written in terms of $V_1$.

$$I_x = \frac{V_1}{(6+4)} = \frac{V_1}{10}$$

There are no supernodes for this circuit, so a current summation is written:

$$\frac{(V_1 - 68)}{6} + \frac{(V_1 - 4\left(\frac{V_1}{10}\right))}{2} + \frac{(V_1)}{10} = 0$$

$$V_1\left(\frac{1}{6} - \frac{1}{5} + \frac{1}{2} - \frac{1}{10}\right) = \frac{68}{6}$$

$$V_1 = \frac{(68)}{6} \cdot \frac{30}{17} = 20V$$

Using this solution for $V_1$:

$$I_x = \frac{V_1}{10} = \frac{20}{10} = 2A$$

Using Ohm's law:

$$V_{th} = I_x (4) = 2(4) = 8V$$
4. Find the Thevenin equivalent circuit at terminals a-b.

Solving using node voltage:

\[ I_x = \frac{V_1}{6} \]

\[ \frac{(V_1 - 68)}{6} + \frac{V_1 - 4(V_1)}{2} + \frac{V_1}{6} = 0 \]

\[ V_1(\frac{1}{6} + \frac{1}{2} - \frac{2}{6} + \frac{1}{6}) = \frac{68}{6} \]

\[ V_1 = \frac{68}{6} \cdot \frac{6}{3} = \frac{68}{2} = 33 \]

\[ I_{sc} = \frac{V_1}{6} = \frac{68}{3} = \frac{68}{18} = 3 \]

\[ R_{th} = \frac{V_{th}}{I_{sc}} = \frac{18}{68} \cdot \frac{8}{8} = \frac{144}{68} = \frac{36}{17} \]

Using a test source: (I used a vsrc to set a known current through 4Ω resistor)

\[ I_{test} = -I_x + \frac{1}{4} \cdot -I_{test} = 0 \]

\[ I_{test} = -I_x + \frac{1}{4} \]

where \[ I_x = \frac{V_1}{6} \]

\[ I_x = \frac{-1}{3} \cdot \frac{18}{6} = -\frac{1}{18} - \frac{3}{18} = -\frac{4}{18} = -\frac{1}{4} \]

\[ I_{test} = -\frac{4}{18} + \frac{1}{4} = \frac{16}{72} + \frac{18}{72} = \frac{34}{72} = \frac{17}{34} \]

\[ R_{th} = \frac{V_{test} = 1V}{I_{test}} = \frac{72}{34} = \frac{36}{17} \]

(same)
5. Determine the power in the dependent source if $R_L = 2k\Omega$

\[
\text{Power} = (2000I_x) \cdot I_1
\]

\[
\frac{(V_1 - 15)}{3k} + \frac{V_1}{6k} + \frac{V_1 - 2000}{4k + 2k} = 0
\]

\[
V_1 \left( \frac{2}{6k} + \frac{1}{6k} + \frac{1}{6k} - \frac{2000}{6k(6k)} \right) = \frac{15}{3k}
\]

\[
V_1 \left( \frac{12k + 6k + 6k - 2000}{6k(6k)} \right) = \frac{15}{3k} \cdot \frac{6k(6k)}{22k} = \frac{30(6k)}{22k} = 8.2V
\]

\[
I_1 = \frac{V_1 - 2000}{6k} = .91mA
\]

\[
\therefore \text{Power} = 2000 \left( \frac{8.2}{6k} \right) \cdot (.91mA) = 2.5mW
\]
For the circuit shown, write three independent equations for the node voltages \( v_1, v_2, \) and \( v_3 \). The quantity \( i_x \) must not appear in the equations.

Make a consistency check on your equations for part 1(a) by setting resistors and sources to values for which the values of \( v_1, v_2, \) and \( v_3 \) are obvious. State the values of resistors, sources, and node voltages for your consistency check, and show that your equations for problem 1(a) are satisfied for these values. (In other words, plug the values into your equations for problem 1(a) and show that the left side and the right side of each equation are equal.)

6. Solution: Define \( i_x \) in terms of node \( v_1 \)’s:

\[
\begin{align*}
L_x &= \frac{C V - v_3}{R_2} = -\frac{v_3}{R_2}
\end{align*}
\]

Now we write three eqns for the node \( v_1 \)’s. We have a supernode for \( v_1 \) and \( v_3 \), so we sum the currents out of a bubble around \( v_1 \) and \( v_3 \) that contains the \( v_3 \) source:
The sum of currents out of the bubble has two terms that cancel out:

\[ \frac{v_1 - v_3}{R_1} + \alpha \left( \frac{-v_3}{R_2} \right) + \frac{v_3 - v_1}{R_1} + \frac{v_3}{R_2} = 0A \]

or

\[ v_3 \left( \frac{1}{R_2} - \frac{\alpha}{R_2} \right) = 0A \] (1)

We also get a voltage eq'n for the supernode:

\[ v_1 - v_3 = v_S \] (2)

Our third eq'n comes from node \( v_2 \):

\[-\alpha \left( \frac{-v_3}{R_2} \right) + \frac{v_2}{R_3} + i_3 = 0A \] (3)

7. Many different consistency checks are possible. One example is given here.

Let \( \alpha = 0 \), \( v_3 = 12V \), \( i_3 = 24A \), \( R_1 = 1\Omega \), \( R_2 = 2\Omega \), \( R_3 = 3\Omega \).

\[ V_1 = 12V \quad V_2 = -i_3 \cdot 3\Omega = -72V \]

We must have \( i_X = 0 \) or we will accumulate charge on the left side of the circuit. Thus, \( v_3 = 0V \).
Since the $v_3$ source connects $v_3$ to $V_1$, we have $V_1 = v_3 + v_3 = 0V + 12V = 12V$.

On the right side, $i_3$ flows around the loop, giving $v_2 = -i_3 \cdot 3\Omega = -24A \cdot 3\Omega = -72V$.

$V_1 = 12V, \quad v_2 = -72V, \quad v_3 = 0V$

Now we plug these values into our 3 eqns from part (a):

$$0V \left( \frac{1}{2\Omega} - \frac{0}{2\Omega} \right) = 0A \quad (1)$$

or

$$0A = 0A \checkmark$$

$$12V - 0V = 12V \checkmark \quad (2)$$

$$-0 \left( \frac{-12V}{2\Omega} \right) + \frac{-72V}{3\Omega} + 24A = 0A$$

or

$$-24A + 24A = 0A \checkmark \quad (3)$$

The eqns are all satisfied, completing the consistency check.
For the circuit shown, write three independent equations for the three mesh currents $i_1$, $i_2$, and $i_3$. The quantity $v_x$ must not appear in the equations.

**Soln: a)** The loops for $i_1$ and $i_2$ share a current source, $\alpha v_x$, meaning they form a super-mesh. For the voltage loop around the outside of the $i_1$ and $i_2$ loops, we have the following problem: the $i_1$ loop includes current source $i_3$. Thus, we must abandon the outer voltage loop.

Instead, we have $i_1 = i_3$, since $i_3$ is on the outside edge of the circuit. We need two eqns for the two loops, and the other eqn is the usual current source eqn for the $\alpha v_x$ source in terms of $i_1$ and $i_2$:

$$\alpha v_x = i_1 - i_2$$

We must write $v_x$ in terms of mesh currents, however:

$$v_x = (i_3 - i_1) R_2$$
Using this expression for $v_x$, we obtain a second equation for the super-mesh:

$$(i_3 - i_1)R_2 = i_1 - i_2$$

The voltage loop for $i_3$ gives the third and final equation:

$$-i_3R_4 - i_3R_2 + i_1R_2 + V_3 = 0V$$

Note: $i_1$ may be replaced by $i_3$ in the preceding two eqns.
Find the Thevenin equivalent circuit at terminals a and b. \( i_x \) must not appear in your solution. Note: \( \alpha > 0 \).

**Solutn:** \( V_{Th} = V_{ab} \) with nothing connected across terminals a and b. We can use the node-voltage method to find \( V_{Th} \).

We have one node: \( v_1 \). After defining \( i_x \) in terms of \( v_1 \), we sum the currents out of the \( v_1 \) node. For \( i_x \), however, the only eq'n we can write is the following:

\[
\begin{align*}
i_x &= -\alpha i_x \quad \text{where} \quad \alpha > 0 \\
or \quad i_x (1 + \alpha) &= OA \\
or \quad i_x &= OA
\end{align*}
\]
So no current flows in either $R_1$ or the dependent source. Thus, our node-voltage eq'n for $V_1$ is as follows:

$$-i_s + \frac{V_1}{R_2} = 0 \ A$$

or

$$V_1 = i_s R_2$$

Since $i_x = 0$, the voltage drop across $R_1$ is zero. It follows that

$$V_{Th} = V_1 = i_s R_2.$$  

One way to find $R_{Th}$ is to short a to b, find the current, $i_{sc}$, flowing in the short, and compute

$$R_{Th} = \frac{V_{Th}}{i_{sc}}.$$  

We use the node-voltage method again.
\( i_x \) defined in terms of \( v_1' \) is

\[
   i_x = -\frac{v_1'}{R_1}
\]

The summation of currents out of the \( v_1' \) node is as follows:

\[
   -i_s' + \frac{v_1'}{R_2} - \alpha\left(-\frac{v_1'}{R_1}\right) - \frac{v_1'}{R_1} = OA
\]

or

\[
   v_1'\left(\frac{1+\alpha}{R_1} + \frac{1}{R_2}\right) = i_s'
\]

or

\[
   v_1' = \frac{i_s'}{\frac{1+\alpha}{R_1} + \frac{1}{R_2}}\cdot\frac{R_1R_2}{R_1R_2}
\]

or

\[
   v_1' = \frac{i_s'R_1R_2}{(1+\alpha)R_2+R_1}
\]

and

\[
   i_x = -\frac{v_1'}{R_1} = -i_s'\frac{R_2}{(1+\alpha)R_2+R_1}
\]

We find \( i_{sc} \) by summing currents out of the top node.

\[
   \alpha i_x + i_x + i_{sc} = OA
\]

or

\[
   i_{sc} = -(1+\alpha)i_x = -\frac{(1+\alpha)i_s'R_2}{(1+\alpha)R_2+R_1}
\]

Thus,

\[
   R_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{i_s'R_2}{\frac{(1+\alpha)i_s'R_2}{(1+\alpha)R_2+R_1}} = \frac{(1+\alpha)R_2+R_1}{1+\alpha}
\]
Calculate the power consumed (i.e., dissipated) by the \( i_x/2 \) dependent source. Note:
If a source supplies power, the power it consumes is negative.

**sol'n:** Any method of sol'n is acceptable. Here, the sol'n employs the mesh-current method.

\[
i_x = i_1 \quad \text{so we have dependent src}
\]

**i_1 loop:**
\[
12V - i_1 \cdot 1k\Omega - 24V - i_1 \cdot 2k\Omega = 0V
\]

Solving for \( i_1 \), we have
\[
i_1 = \frac{24V - 12V}{1k\Omega + 2k\Omega} = \frac{-12V}{3k\Omega} = -4\,mA
\]

We have \( i_x = i_1 \), so the loop current \( i_2 \) is \( i_x/2 = i_1/2 \). (Note that \( i_2 \) is the same as the current in the dependent src, which is on the outside edge of the circuit.)

\[
i_2 = \frac{i_x}{2} = \frac{i_1}{2} = -2\,mA
\]

To find the voltage drop across the dependent source, we use a voltage loop on the right side.