

1. Solve the following simultaneous equations for  $i_1$ ,  $i_2$ , and  $i_3$ :

$$\textcircled{1} \quad (-4i_1 + 3i_2) + (3i_2 - i_1 + 6i_3) - 2 = 0$$

$$\textcircled{2} \quad i_2 + 4(3i_1 + i_3) - 2i_1 = 0$$

$$\textcircled{3} \quad 5i_1 - 1 - i_3 = 0$$

from  $\textcircled{3}$   $5i_1 = 1 + i_3$

$$i_1 = \frac{(1+i_3)}{5}$$

from  $\textcircled{2}$  using  $\textcircled{3} \Rightarrow i_2 + 12\left(\frac{1+i_3}{5}\right) + 4i_3 - \frac{2}{5}(1+i_3) = 0$

$$i_2 = -\frac{12}{5} + \frac{2}{5} - \frac{12i_3}{5} + \frac{20}{5}i_3 + \frac{2}{5}i_3$$

$$i_2 = -\frac{10}{5} - \frac{30}{5}i_3 = -2 - 6i_3$$

From  $\textcircled{1}$  using  $\textcircled{2}$  and  $\textcircled{3}$  reduced above:

$$-\frac{4}{5}(1+i_3) + 3(-2-6i_3) + 3(-2-6i_3) - \frac{(1+i_3)}{5} + 6i_3 - 2 = 0$$

$$-\frac{4}{5}i_3 - \frac{18(5)}{5}i_3 - \frac{18(5)}{5}i_3 - \frac{1}{5}i_3 + \frac{30}{5}i_3 = \frac{4}{5} + \frac{30}{5} + \frac{30}{5} + \frac{1}{5} + \frac{10}{5}$$

$$-155i_3 = 75$$

$$i_3 = \boxed{\frac{-75}{155}} \text{ or } \frac{-15}{31}$$

$$\text{plug back into } \textcircled{2}: i_2 = -2 - \frac{6(-15)}{31} = \frac{-62+90}{31} = \boxed{\frac{28}{31}}$$

$$\text{plug back into } \textcircled{3}: i_1 = \frac{\left(1 + \frac{-15}{31}\right)}{5} = \frac{1}{5} - \frac{15}{(5)(31)}$$

$$i_1 = \frac{31-15}{5(31)} = \boxed{\frac{+16}{155}}$$

2. Perform the following calculations. Write the answers with appropriate prefixes (such as  $\mu$ , m, k etc.) for engineering units:

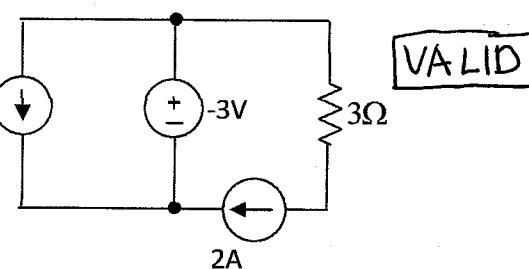
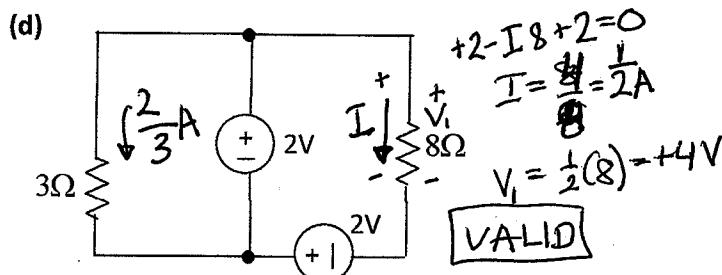
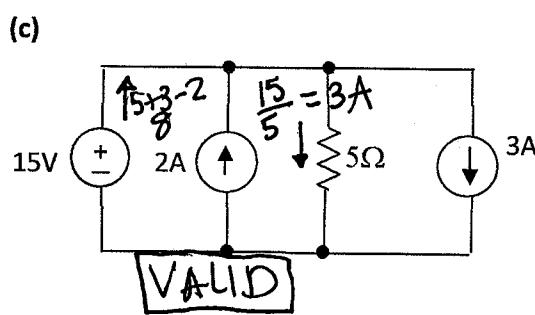
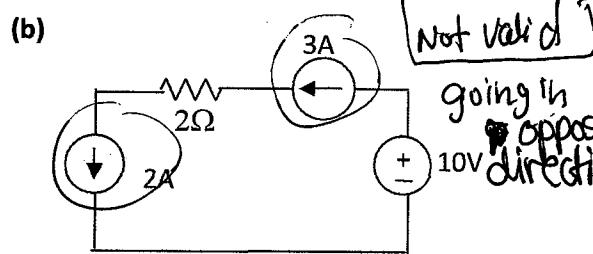
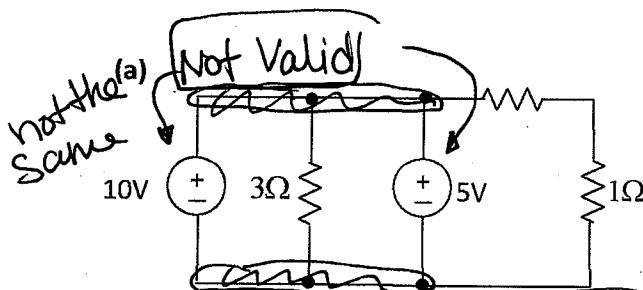
a)  $P = 7.2 \text{ mA} \times 6 \text{ mV}$  (Note:  $V \cdot A = W$ )

b)  $R = 4.5 \mu\Omega + 1600 \text{ n}\Omega$

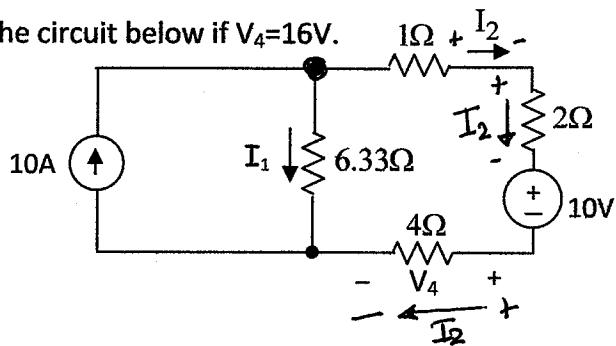
(a)  $P = 7.2 \times 10^{-3} \text{ A} \times 6 \times 10^{-3} \text{ V} = 43.2 \text{ mW}$

(b)  $R = 4.5 \times 10^{-6} \Omega + 1600 \times 10^{-9} \Omega = 6.1 \mu\Omega$

3. Determine whether each of the following circuits is valid or invalid.



5. Find  $I_1$  in the circuit below if  $V_4=16V$ .

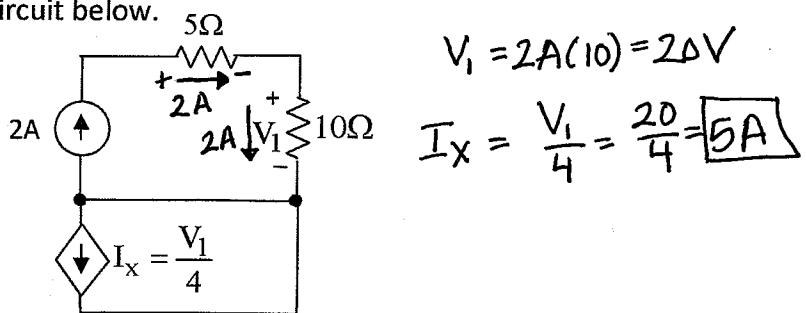


$$\text{Ohm's Law: } V_4 = I_2(4), \text{ if } V_4 = 16 \text{ then } 16 = I_2(4)$$

$$\therefore I_2 = 4A$$

Current summation at top node:  $-10 + I_1 + I_2 = 0$

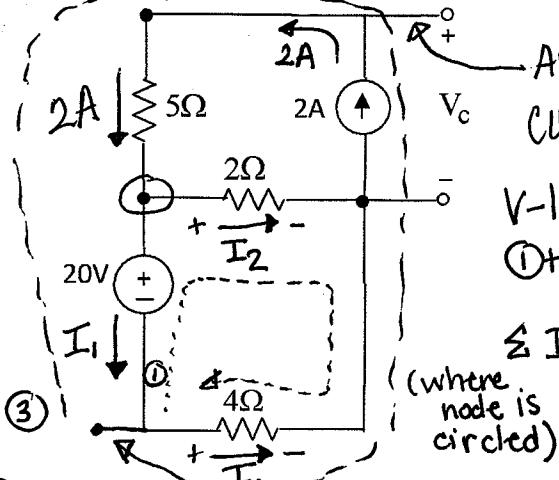
$$\therefore I_1 = 10 - I_2 = 10 - 4 = \boxed{6A}$$

4. Find  $I_x$  in the circuit below.

$$V_1 = 2A(10) = 20V$$

$$I_x = \frac{V_1}{4} = \frac{20}{4} = 5A$$

6. Use Kirchoff's laws and Ohm's Law to find the value of  $V_c$ .



An open wire does not conduct current. No current in this wire.

V-loop:

$$\textcircled{1} +20 -I_2(2) +I_1(4) = 0$$

$$\Sigma I: \textcircled{2} -2A +I_2 +I_1 = 0$$

(where node is circled)

Solving  $\textcircled{2}$  for  $I_1$ :  $I_1 = (2 - I_2)$

plug into  $\textcircled{1}$ :  $+20 -I_2(2) + (4)(2 - I_2) = 0$

$$20 + 8 - I_2(2) - 4I_2 = 0$$

$$I_2(6) = 28$$

$$I_2 = \frac{28}{6} \text{ A}$$

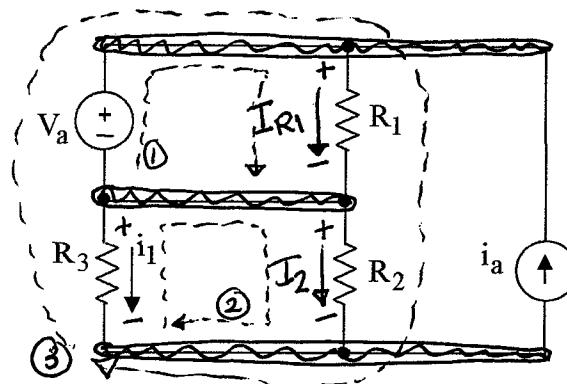
$$I_1 = (2 - I_2) = \frac{12}{6} - \frac{28}{6} = -\frac{16}{6}$$

Taking a loop with  $V_c$  (desired value) in it.  
 follow path  $\textcircled{3}$   $+20 + 2(5) - V_c + I_1(4) = 0$

$$V_c = 20 + 10 + \left(-\frac{16}{6}\right)(4)$$

$$V_c = 30 - 10.67 = \boxed{+19.3 \text{ V}}$$

7. Use Kirchoff's laws and Ohm's Law to find the expression for  $i_1$ . The expression can contain no other parameters than  $V_a$ ,  $i_a$ ,  $R_1$ ,  $R_2$ , and/or  $R_3$ .



1. Identify nodes

2. Label currents through all R's.
3. Take V-loops (use Ohm's Law)
4. Take current summations.
5. State Ohm's Law (if needed)
6. Solve equation set

V-loops:

$$\textcircled{1} +V_a - I_{R1} R_1 = 0 \Rightarrow I_{R1} = \frac{V_a}{R_1}$$

$$\textcircled{2} + i_1 R_3 - I_2 (R_2) = 0$$

$$\sum I: (\text{bottom node}) \textcircled{3} - i_1 - I_2 + i_a = 0$$

- \textcircled{1} in terms of only  $I_{R1}$
- solve \textcircled{2} for  $I_2$ :  $I_2 = \frac{i_1 R_3}{R_2}$

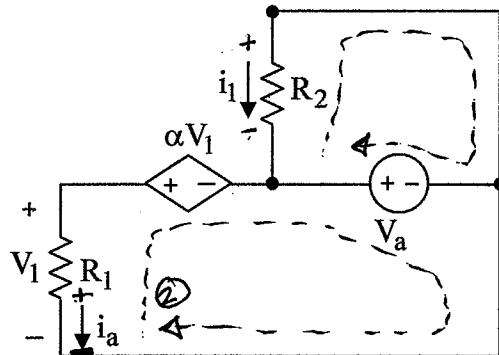
$$\text{solve } \textcircled{3} \text{ for } I_2 = i_a - i_1$$

plug into \textcircled{2}

$$i_1 R_3 - i_a R_2 + i_1 R_2 = 0$$

$$i_1 = \frac{i_a R_2}{(R_3 + R_2)}$$

8. Use Kirchoff's laws and Ohm's Law to find the expression for  $i_1$ . The expression can contain no other parameters than  $V_a$ ,  $\alpha$ ,  $R_1$ , and/or  $R_2$ . (Hint: Eliminate  $V_1$  from the expression)



$$\text{V-loop: } +i_1 R_2 + V_a = 0$$

$$i_1 = -\frac{V_a}{R_2}$$

If solving for  $i_a$ :

$$+i_a R_1 - \alpha V_1 - V_a = 0$$

Use I Law:  $V_1 = i_a R_1$

$$\therefore i_a R_1 - \alpha i_a R_1 - V_a = 0$$

$$i_a (1 - \alpha) R_1 - V_a = 0$$

$$i_a = \frac{V_a}{(1 - \alpha) R_1}$$

9. (a) Find  $i_1$ ,  $i_2$ ,  $i_3$ , and  $v_o$ .

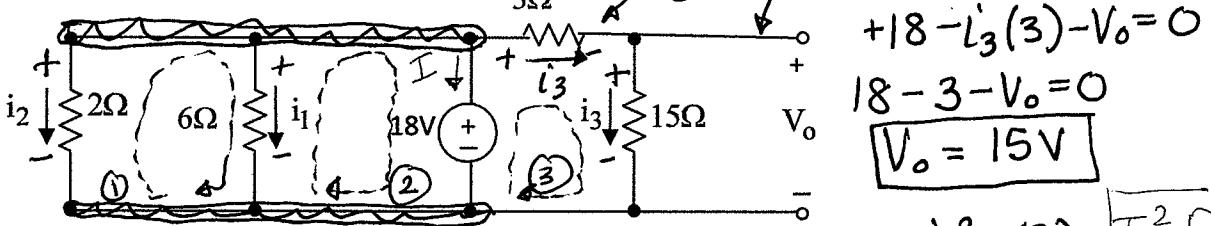
(b) Find the power dissipated in the  $3\Omega$  resistor and the power supply.

Note that  $18V$  is across each  $R$ . Using  $\Sigma I$  law:

$$\frac{18}{2} = 9A = i_2$$

$$\frac{18}{6} = 3A = i_1$$

$$\frac{18}{18} = 1A = i_3$$

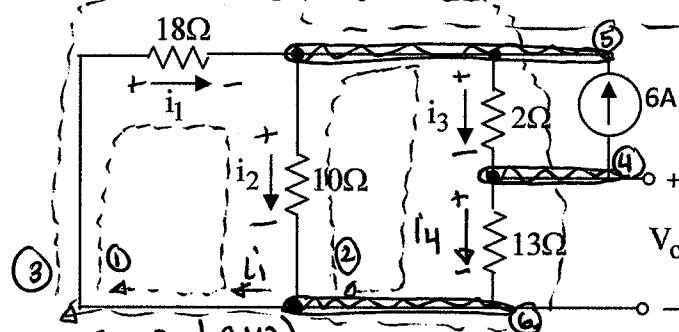


$$+18 - i_3(3) - v_o = 0$$

$$18 - 3 - v_o = 0$$

$$v_o = 15V$$

10. Find  $i_1$ ,  $i_2$ ,  $i_3$ , and  $v_o$ .



V-loops: (use  $\Sigma I$  Law)

$$\textcircled{1} \quad -i_1(18) - i_2(10) = 0$$

$$\textcircled{2} \quad +i_2(10) - i_3(2) - i_4(13) = 0$$

$$\textcircled{3} \quad -i_1(18) - i_3(2) - i_4(13) = 0$$

$\Sigma I$ :

$$\textcircled{4} \quad -i_3 + 6 + i_4 = 0$$

$$\textcircled{5} \quad -i_1 + i_2 + i_3 - 6 = 0$$

$$\textcircled{6} \quad +i_1 - i_2 - i_4 = 0$$

Use  $\textcircled{1}$  to solve for  $i_2 \Rightarrow i_2 = \frac{-i_1}{10} 18$

Use  $\textcircled{4}$  to solve for  $i_3 \Rightarrow i_3 = (6 + i_4)$

plug  $\textcircled{1}, \textcircled{4}$  into  $\textcircled{2}$ :

$$-\frac{i_1(18)}{10} 10 - 12 - 2i_4 - i_4(13) = 0$$

$$i_4 = \frac{[-18i_1 - 12]}{15}$$

$i_3$  because no current

$$+18 - i_3(3) - v_o = 0$$

$$18 - 3 - v_o = 0$$

$$v_o = 15V$$

$$\text{power}_{3\Omega} = i_3^2 \cdot (3) = I^2 R$$

$$\text{power}_{3\Omega} = (1)^2 (3) = 3W$$

+ value means  
"absorbing"

$$\text{power power supply} = I \times V$$

$$\Sigma I: +i_2 + i_1 + i + i_3 = 0$$

$$I = -(9 + 3 + 1) = -13$$

$$\text{power} = (-13)(18) = -234W$$

$$\text{plug into } \textcircled{6}: i_1 + \frac{18}{10} i_1 + \frac{12}{15} i_1 + \frac{12}{15} = 0$$

$$i_1 \left( \frac{150}{150} + \frac{18(15)}{150} + \frac{12(10)}{150} \right) = -\frac{12}{15}$$

$$i_1 \left( \frac{150 + 270 + 180}{150} \right) = -\frac{12}{15}$$

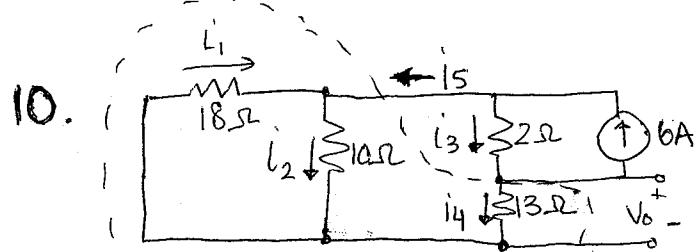
$$i_1 = -\frac{12}{15} \cdot \frac{150}{600} = -0.2A$$

$$\therefore i_4 = -\frac{18}{10}(-0.2) - \frac{12}{15} = -0.56A$$

$$\therefore i_2 = \frac{-18(-0.2)}{10} = 0.36A$$

use ohm's law:

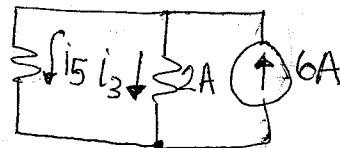
$$v_o = i_4(13) = -0.56(13) = -7.3V$$



Using a current divider

Redraw: Combine

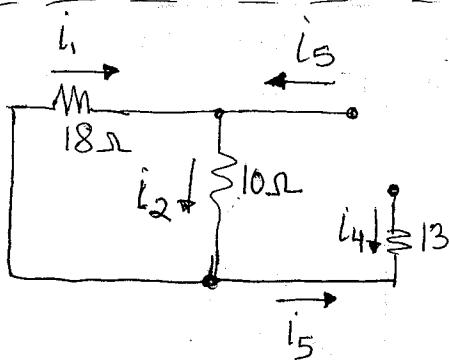
$$\frac{(18||10) + 13}{R}$$



$$i_3 = \frac{6 \left( \frac{544}{28} \right)}{\frac{544}{28} + 2} = 5.4 \text{ A}$$

$$\frac{18(10)}{28} + \frac{13(28)}{28} = \frac{544}{28}$$

$$i_5 = 6 - 5.4 = 0.6 \text{ A}$$



$$i_4 = -i_5 = -0.6 \text{ A}$$

$$i_1 = -\frac{i_5(10)}{28} = -\frac{0.6(10)}{28} = -0.2 \text{ A}$$

$$i_2 = \frac{i_5(18)}{28} = 0.4 \text{ A}$$

$$V_o = i_4(13) = -0.6(13) = -7.8 \text{ V}$$