Solution

1. Solve the following simultaneous equations for i_1, i_2, and i_3:

$$① \quad 5(i_1+i_2)+(2i_2-i_3-4i_3)-20=0$$

$$② \quad -3(i_1+i_2)+2(3i_3)=0$$

$$③ \quad -5+i_1-2i_2=0$$

Rearranging ①:

$$5i_1+5i_2+2i_2-i_3-4i_3-20=0$$

$$5i_1+7i_2-5i_3-20=0$$

Solving ② for i_2:

$$-3i_1-3i_2+6i_3=0$$

$$\cancel{3}i_2=\left(\frac{-3i_1+6i_3}{3}\right) \Rightarrow i_2=-i_1+2i_3 \quad ②$$

Solving ③ for i_1 and plugging into ②:

$$-5+i_1-2i_2 \Rightarrow i_1=2i_2+5 \quad ③$$

into ②:

$$i_2=-2i_2-5+2i_3 \Rightarrow \cancel{3}i_2=\left(\frac{-5+2i_3}{3}\right) ④ \quad \{④\to③\}$$

$$\therefore i_1=\frac{-10}{3}+\frac{4}{3}i_3+5 \quad ⑤$$

[Now eq. ④ gives i_2 in terms of i_3 & ⑤ gives i_1 in terms of i_3] → usually try to put eq. in terms of 1 unknown

plug ④, ⑤ into ①

$$5\left(\frac{-10}{3}+\frac{4}{3}i_3+5\right)+7\left(-\frac{5}{3}+\frac{2}{3}i_3\right)-5i_3-20=0$$

$$i_3\left(\frac{20}{3}+\frac{14}{3}-\frac{15}{3}\right)=\frac{60}{3}+\frac{35}{3}-\frac{25(3)}{3}+\frac{50}{3}$$

problem #1 (cont.)

$$i_3\left(\frac{(34-15)}{3}\right) = \frac{(60+35-75+50)}{3}$$

$$i_3\left(\frac{19}{3}\right) = \left(\frac{70}{3}\right) \Rightarrow i_3 = \left(\frac{70}{\cancel{3}} \times \frac{\cancel{3}}{19}\right) = \boxed{\frac{70}{19}\,A} \approx (3.7)$$

use in 4:

$$i_2 = -\frac{5}{3} + \frac{2}{3}\left(\frac{70}{19}\right) = \frac{\overbrace{-5(19)}^{-95} + 2(70)}{3(19)} = \boxed{\frac{45}{57}\,A} \approx (0.8)$$

use in 5:

$$i_1 = -\frac{10}{3} + \frac{4}{3}\left(\frac{70}{19}\right) + \frac{15}{3} = \frac{\overbrace{5(19)}^{95} + \overbrace{4(70)}^{280}}{3(19)} = \boxed{\frac{375}{57}\,A} \approx (6.6)$$

check:

eq. 3 $\quad -5+6.6-1.6=0$ ✓

eq. 2 $\quad -3(6.6+0.8)+6(3.7)=0$ ✓

$\quad -22.2+22.2=0$

eq. 1 $\quad 5(6.6+0.8)+(2(.8)-3.7-4(3.7))-20=0$

$\quad 37+1.6-3.7-14.8-20 \cong 0$ ✓

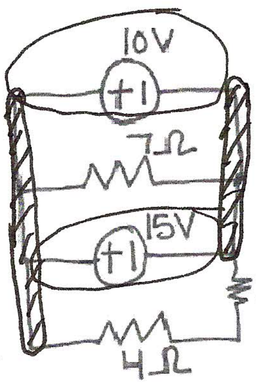
2. Perform the following calculations. Write the answers with appropriate prefixes (such as μ, m, k etc.) for engineering untis:

   a) $P = 5\,\mu A \times 6\,GV$ (Note: V*A=W) $= 5\times10^{-6}\cdot 6\times10^{9} = \boxed{30KW}$

   b) $R = 5.1\,k\Omega + 160\Omega = 5{,}100 + 160 = \boxed{5{,}260\,\Omega \text{ or } 5.26K\Omega}$

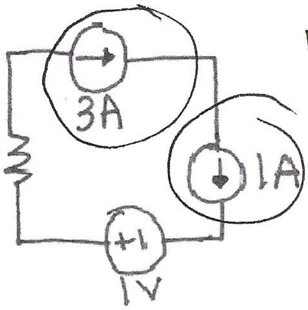
3. Determine whether each of the following circuits is valid or invalid.

(a)



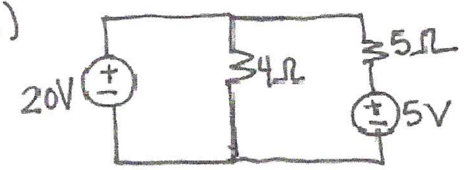
NOT VALID- 2 different voltages in parallel!

(b)



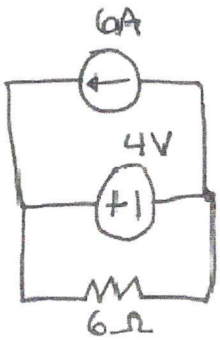
not valid - 2 different currents in 1 branch!

(c)



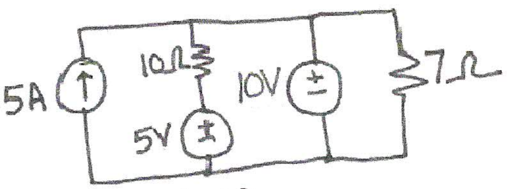
VALID

(d)



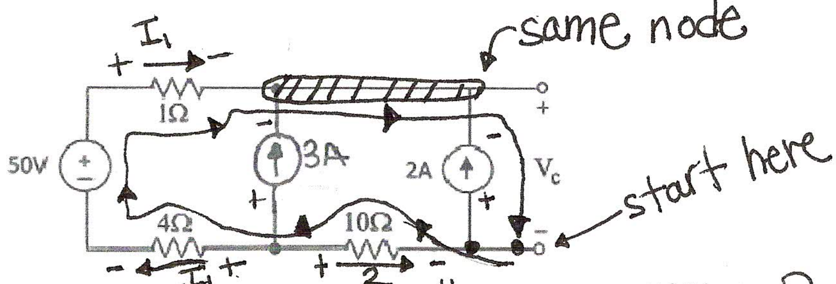
VALID

(e)



VALID

4. Use Kirchoff's laws and Ohm's Law to find the value of Vc. Note that it is also the voltage across the 3A current source.



1. Label a current through every R.

2. Take a current summation at top ~~no~~ node:

$$-I_1 - 3 - 2 = 0$$

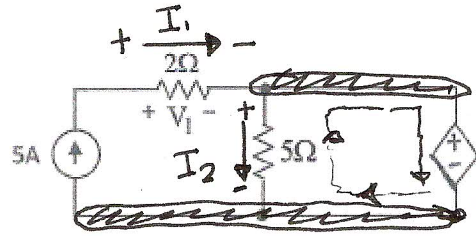
$$I_1 = -5A$$

3. Take a voltage loop ~~(Note:~~

$$\underbrace{+2(10)}_{V \text{ across } 10\Omega} - \underbrace{4(I_1)}_{\text{across } 4\Omega} + 50 - \underbrace{I_1(1)}_{\text{across } 1\Omega} - V_c = 0$$

$$\therefore V_c = 20 - 20 + 50 + 5 = \boxed{55\,V}$$

5. Use Kirchoff's laws and Ohm's Law to find the current through the 5Ω resistor. The current source is not ideal and so will have a voltage drop across it.



For dependent sources, always try to write an equation for unknown variable (V_1 in this case).

$V_1 = I_1(2)$ where $I_1 = 5A$

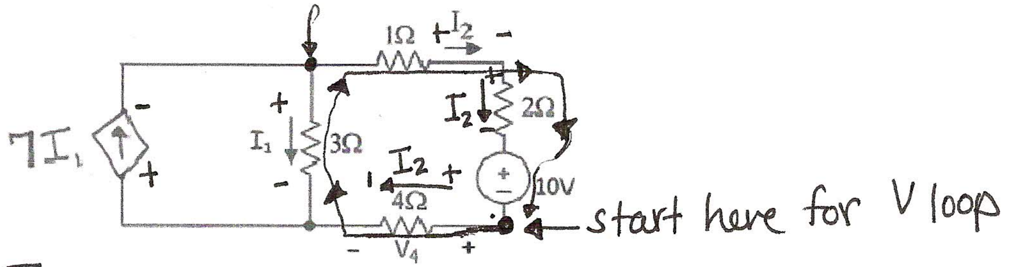
$\therefore \quad V_1 = 10V$

$+I_2(5) - 3V_1 = 0$ (V loop)

$I_2 = \frac{+30}{5} = \boxed{6\text{ A}}$

6. Use Kirchoff's laws and Ohm's Law to find I_2 and V_4 in the circuit below.

take current sum here.



ΣI:

$-7I_1 + I_1 + I_2 = 0 \longrightarrow I_2 = +6I_1$

V-loop:

$-I_2(4) + I_1(3) - I_2(1) - I_2(2) - 10 = 0$

$\cancel{-4I_2 + 18}$

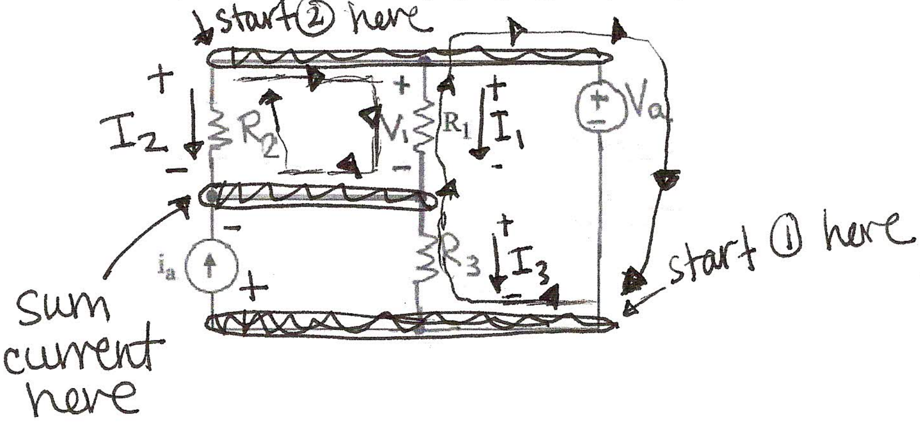
$-24I_1 + 3I_1 - 18I_1 - 10 = 0$

$I_1 = \frac{+10}{-39}$

$\boxed{I_2 = -\frac{60}{39} A}$

$V_4 = I_2(4) = \boxed{\frac{-240}{39} V}$

7. Use Kirchoff's laws and Ohm's Law to find the expression for V_1. The expression can contain no other parameters than V_a, i_a, R_1, R_2, and/or R_3.



$$-i_a - I_2 - I_1 + I_3 = 0 \longrightarrow I_3 = i_a + I_1 + I_2$$

V-loop:

① $+I_3R_3 + I_1R_1 - V_a = 0$

② $-I_1R_1 + I_2R_2 = 0$

$$\cancel{I_1 = \frac{I_2R_2}{R_1}} \quad I_2 = \frac{I_1R_1}{R_2}$$

plug into I_3 eq: $\cancel{I_3 = i_a + \frac{I_2R_2}{R_1} + I_2\frac{R_1}{R_1} = i_a + \frac{(R_1+R_2)}{R_1}I_2}$

$$I_3 = i_a + I_1 + I_1\frac{R_1}{R_2} = i_a + \frac{(R_1+R_2)}{R_2}\cdot I_1$$

plug into ①

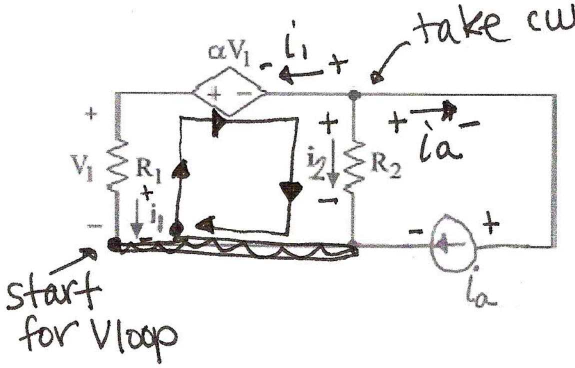
$$i_aR_3 + \frac{(R_1+R_2)R_3}{R_2}\cdot I_1 + I_1R_1 - V_a = 0$$

$$I_1\left[\frac{(R_1+R_2)R_3 + R_1R_2}{R_2}\right] = V_a - i_aR_3$$

$$\boxed{V_1 = I_1R_1 = \frac{(V_a - i_aR_3)R_1\cdot R_2}{(R_1+R_2)R_3 + R_1R_2}}$$

8. Use Kirchoff's laws and Ohm's Law to find the expression for i_1. The expression can contain no other parameters than i_a, α, R_1, and/or R_2. (Hint: Eliminate V_1 from the expression)

take current sum here



current sum:

$$+i_1 + i_2 + i_a = 0 \rightarrow i_2 = -i_a - i_1$$

V-loop:

$$+i_1 R_1 - \alpha V_1 - i_2 R_2 = 0$$

where $V_1 = i_1 R_1$

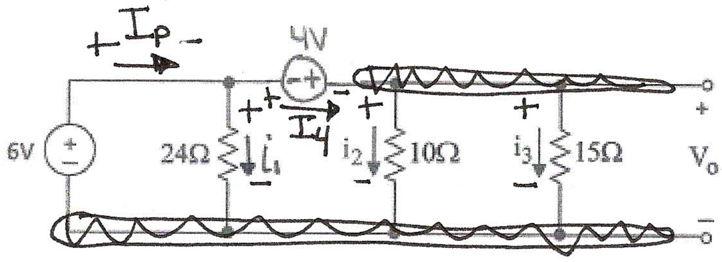
$$\therefore \; i_1 R_1 - \alpha i_1 R_1 - i_2 R_2 = 0$$

$$i_1 (R_1(1-\alpha)) + i_a R_2 + i_1 R_2 = 0$$

$$\boxed{i_1 = \frac{i_a R_2}{R_1(1-\alpha) + R_2}}$$

9. (a) Find i_1, i_2, i_3, and v_o.

(b) Find the power dissipated in the 24Ω resistor and the 6V power supply.



$$+6 - i_1(24) = 0$$

$$i_1 = \frac{6}{24} = \boxed{\frac{1}{4}A}$$

$$+i_1(24) + 4 - i_2(10) = 0$$

$$\frac{1}{4}(24) + 4 - i_2(10) = 0$$

$$\therefore i_2 = \frac{10}{10} = \boxed{1A}$$

$$+i_2(10) - i_3(15) = 0$$

$$i_3 = \frac{i_2(10)}{15} = 1 \cdot \frac{10}{15} = \boxed{\frac{2}{3}A}$$

$$V_o = i_3(15) \text{ or } i_2(10) = \frac{2}{3}(15) = \boxed{10V}$$

power in 24-Ω R:

$$p = i_1^2 R = \left(\frac{1}{4}\right)^2 \cdot 24 = \frac{\cancel{24}}{\cancel{16}} \frac{\cancel{6}}{\cancel{4}} \boxed{\frac{3}{2}W}$$

ΣI: $-I_p + i_1 + I_4 = 0$

$$-I_4 + i_2 + i_3 = 0 \rightarrow I_4 = i_2 + i_3$$

$$-I_p + i_1 + i_2 + i_3 = 0$$

$$I_p = i_1 + i_2 + i_3 = \frac{1(3)}{4(3)} + \frac{4(3)}{4(3)} + \frac{2(4)}{(4)3} = \frac{\overbrace{(3+12+8)}^{23}}{12}$$

$$P_{6V supply} = -\frac{23}{12} \cdot 6 = \boxed{-\frac{23}{2}}$$