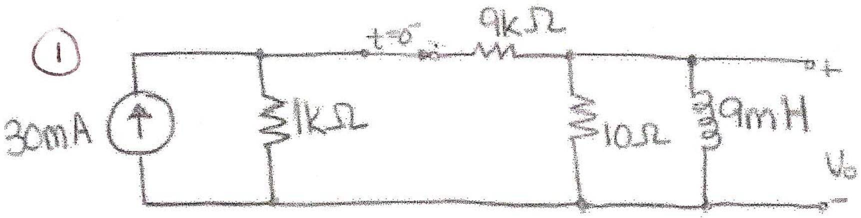
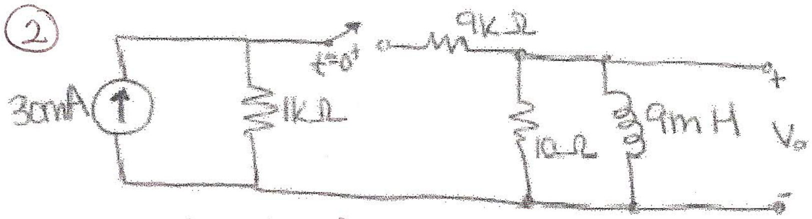
UNIVERSITY OF UTAH
ELECTRICAL & COMPUTER ENGINEERING DEPARTMENT

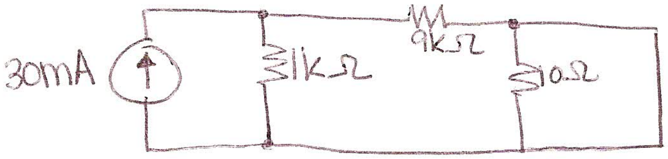
ECE 1270 **HOMEWORK #5**

1. After being closed(top circuit) for a long time, the switch is opened(bottom circuit) at t=0.
   (a) Find an expression for $V_o(t)$ for $(t \geq 0)$.
   (b) Find the energy stored in the inductor at time $t = 0^+$.





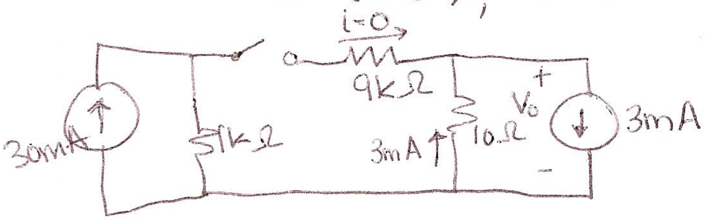
circuit ①: $(t=0^-)$, inductor becomes a wire, switch closed



$\downarrow i_L = \frac{30mA(1k)}{10k} = 3mA$

$i_L(t=0^-) = i_L(t=0^+) = 3mA$ (current source)

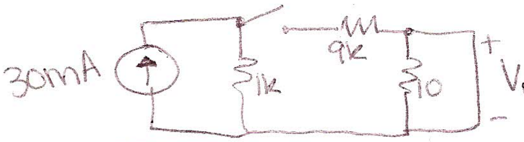
Circuit ②: $(t=0^+)$, inductor acts as a current source, switch opened



$V_o = -3m(10) = -30mV$

INITIAL VALUE

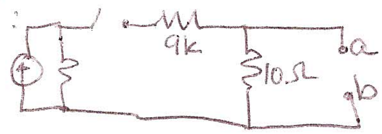
circuit ②: $(t \to \infty)$, inductor acts as a wire, switch opened



$V_o = 0$

FINAL VALUE

τ:



$R_{th} = 10\Omega$

$\tau = \frac{9m}{10}$

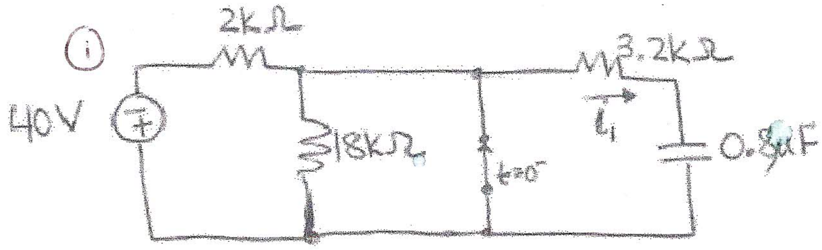
$\boxed{t \geq 0,\ V_o(t) = 0 + (-30mV - 0)e^{-t/(\frac{9m}{10})}\ V}$

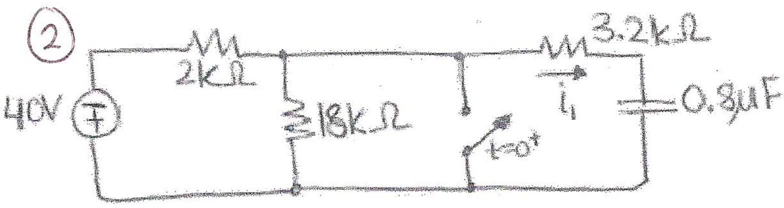
b) energy at $(t=0^+) = \frac{1}{2}(9m)(3m)^2$
$= \boxed{40.5nJ}$

2. After being closed(top circuit) for a long time, the switch is opened(bottom circuit) at t=0.

(a) Find an expression for $i_1(t)$ for $(t \geq 0)$.

(b) Find the energy stored in the capacitor at time $t = 0^+$.

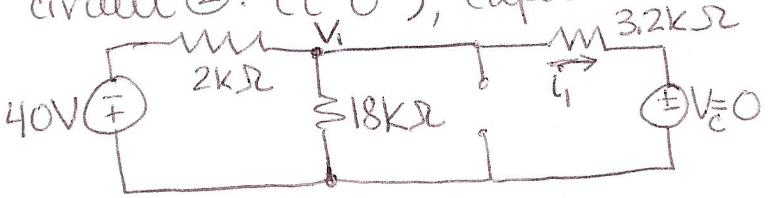




Circuit ①: $(t=0^-)$, capacitor open, switch closed ⇒

$V_c(t=0^-) = V_c(t=0^+) = 0V$

circuit ②: $(t=0^+)$, capacitor acts as Vsrc., switch open



$i_1 = \frac{V_1}{3.2k}$

node V_1:

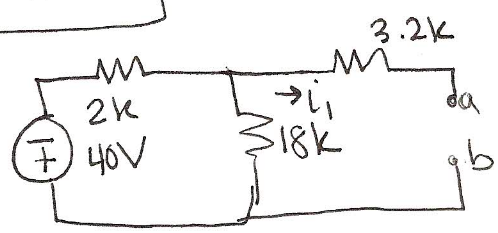
$\frac{V_1 + 40}{2k} + \frac{V_1}{18k} + \frac{V_1}{3.2k} = 0$

$V_1\left(\frac{1}{2k} + \frac{1}{18k} + \frac{1}{3.2k}\right) = \frac{-40}{2k}$

$V_1\,(868\mu) = 20m \rightarrow V_1 = -23V$

$\boxed{i_1 = -7.2mA}$ INITIAL VALUE

$(t \rightarrow \infty)$



$i_1 = 0$

$\tau = R_{th}C$

$R_{th} = 3.2k + 2k \| 18k$

$R_{th} = 5K$

2. (cont.)

I.V.

$i_1(t=0^+) = -7.2mA$

Final: $i_1(t\to\infty) = 0$

$R_{th} = 5k$

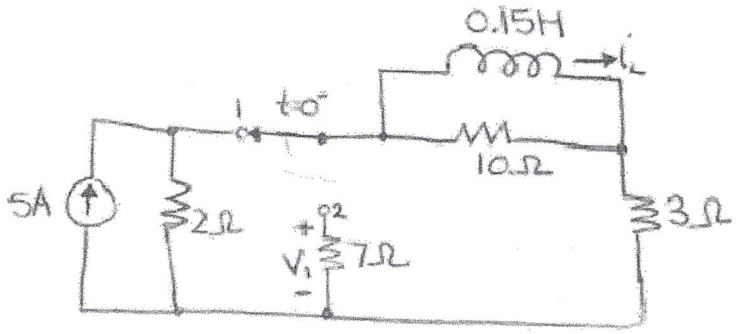
$(t \geq 0,)\ i_1(t) = 0 + (-7.2m - 0)e^{-t/5k(0.8\mu F)} = \boxed{-7.2mA\, e^{-t/4ms.}}$

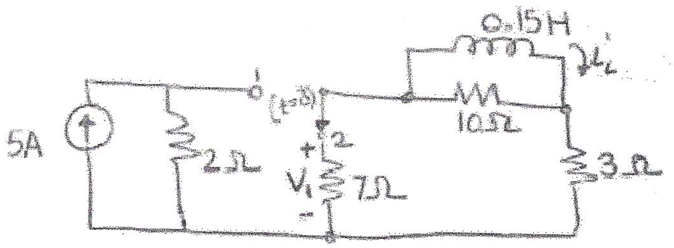
b) at $t=0^+$: $V_c(t=0^+) = 0$

$\therefore \boxed{w_c(t=0^+) = 0J}$

Use the circuits below for problems 3 and 4. After being in position 1 (top circuit) for a long time, the switch is moved to position 2(bottom circuit) at t=0.

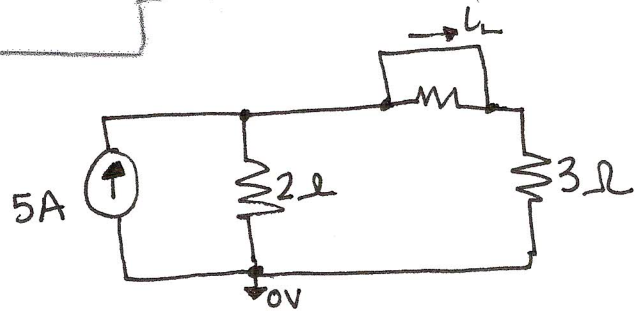
3. (a) Find an expression for $i_L(t)$ for $(t \geq 0)$.
   (b) Make a sketch of the expression for $i_L(t)$
4. (a) Find an expression for $V_1(t)$ for $(t \geq 0)$.
   (b) Make a sketch of the expression for $V_1(t)$



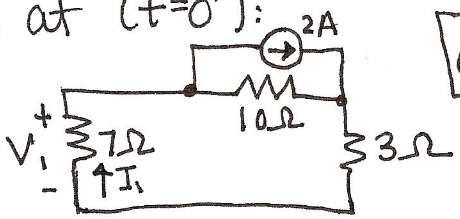


Circuit at $(t=0^-)$:

$$i_L = \frac{5(2)}{5} = 2A$$



circuit at $(t=0^+)$:

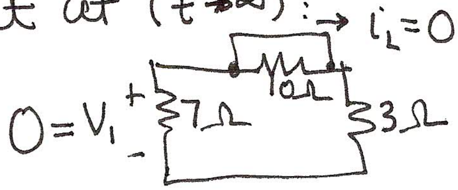


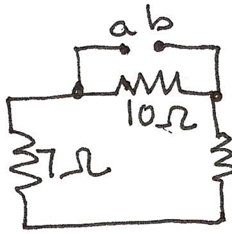
$\boxed{i_L(t=0^+)=2A}$

$$I_1 = \frac{2(10)}{20} = 1A$$

$$\therefore \boxed{V_1 = -1(7) = -7V}$$

circuit at $(t \to \infty)$: $\to i_L = 0$



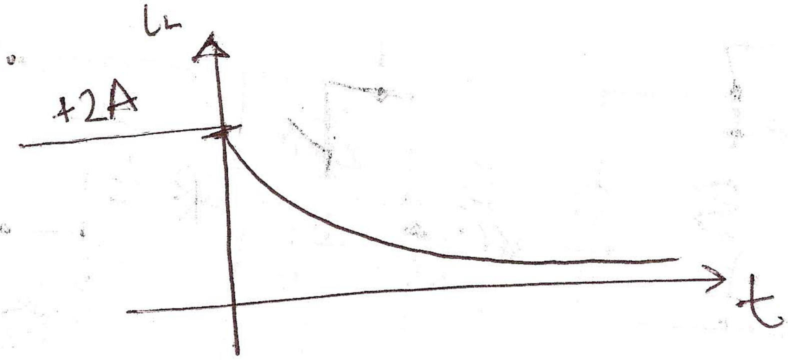


$R_{th} = 10 \| 10$

$R_{th} = 5\Omega$

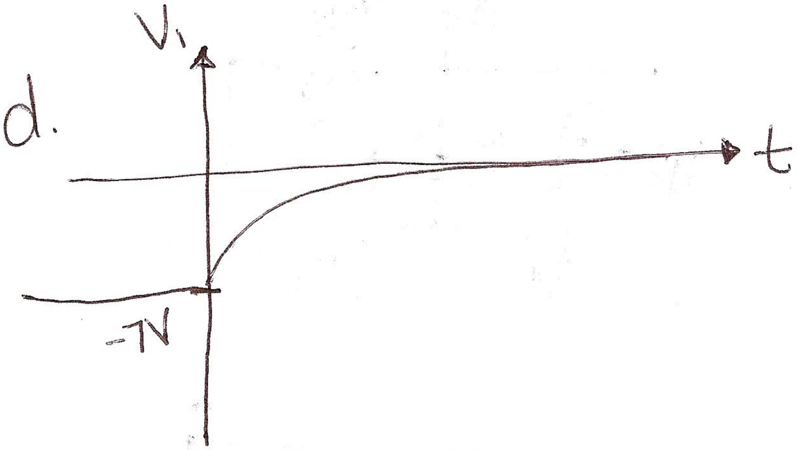
3 a. $i_L(t) = 0 + (2-0)e^{-t/(0.15/5)} = \boxed{2e^{-t/0.03s.}}$
(for $t \geq 0$)

b.



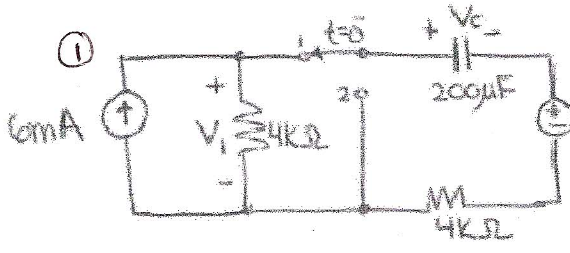
c. $V_1(t \geq 0) = 0 + (-7-0)e^{-t/.03} = \boxed{-7e^{-t/0.03s.}}$

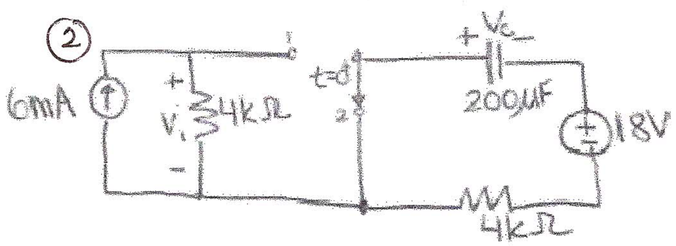
d.



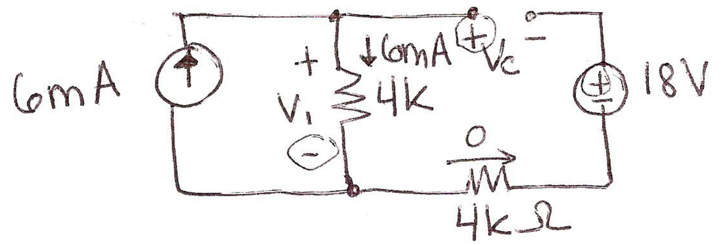
Use the circuits below for problems 5 and 6. After being in position 1 (top circuit) for a long time, the switch is moved to position 2(bottom circuit) at t=0.

5. (a) Find an expression for $V_C(t)$ for $(t \geq 0)$.
   (b) Make a sketch of the expression for $V_C(t)$
6. (a) Find an expression for $V_1(t)$ for $(t \geq 0)$.
   (b) Make a sketch of the expression for $V_1(t)$





circuit ① $(t=0^-)$:

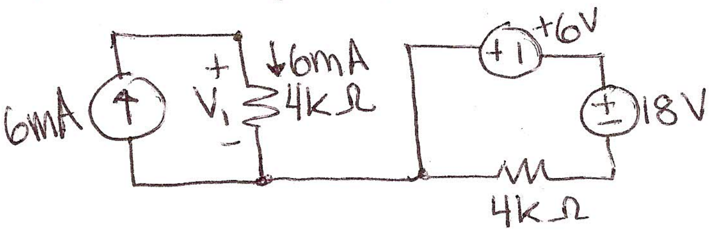


$V_1 = 6m(4k) = 24V$

$+V_c - 24 - 0 + 18 = 0$

$V_c = 24 - 18 = +6V$

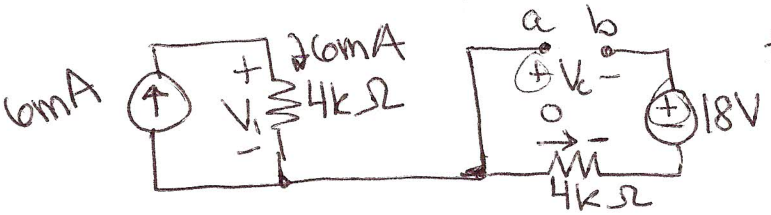
circuit ② $(t=0^+)$: INITIAL VALUE



$V_c(t=0^+) = 6V$

$V_1(t=0^+) = +24V$

circuit ② $(t \to \infty)$: FINAL VALUE



$+V_c - 0 + 18 = 0$

$V_c = -18V$

$V_1 = +24V$

τ:

$R_{th} = 4k$

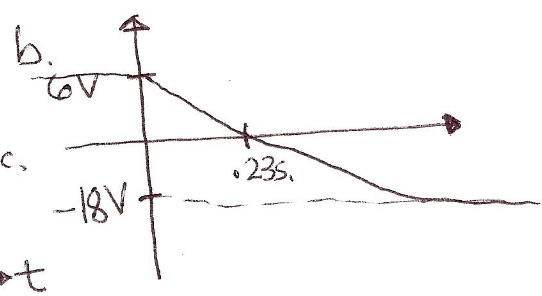
$\tau = 4k(200\mu)$

$\tau = 800ms.$

a. $V_c(t \geq 0) = -18 + (6 - (-18))e^{-t/(4k)(800\mu)}$

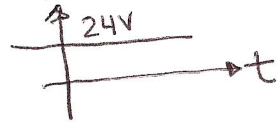
$\boxed{V_c(t \geq 0) = -18 + 24e^{-t/800ms}}$

b.

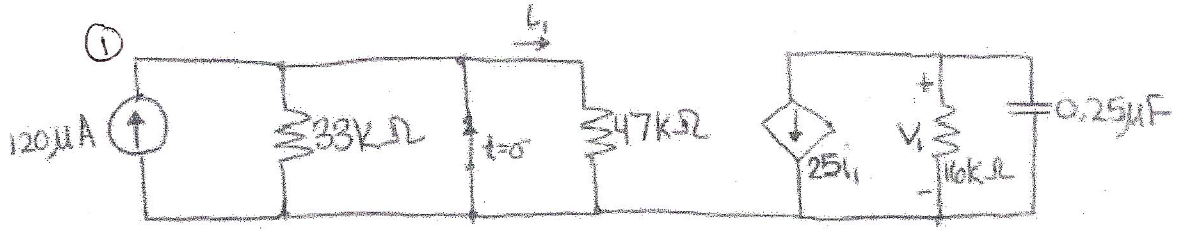


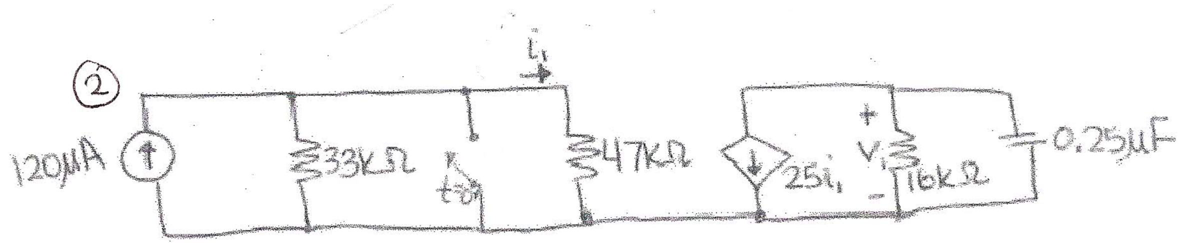
6. a. $V_1(t \geq 0) = +24 + (24 - 24)e^{-t/800msec.}$

$V_1(t \geq 0) = \boxed{+24V}$

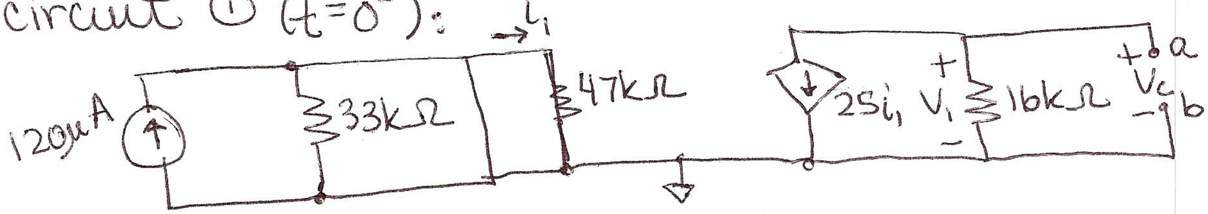


7. After being closed (top circuit) for a long time, the switch is opened (bottom circuit) at t=0. Find an expression for $V_1(t)$ for $(t \geq 0)$.



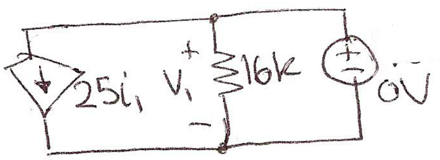


circuit ① $(t=0^-)$:



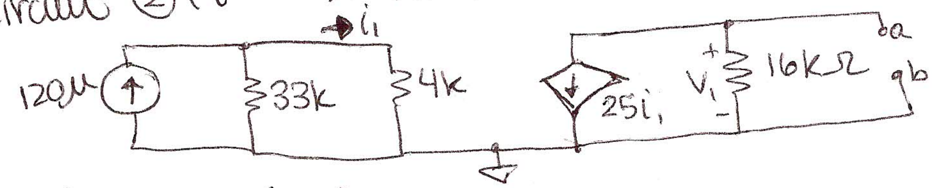
$i_1 = 0A, V_1 = 0V$ $\quad$ $V_1 = V_c = 0V$

circuit ② $(t=0^+)$: INITIAL VALUE



$V_1 = 0V$

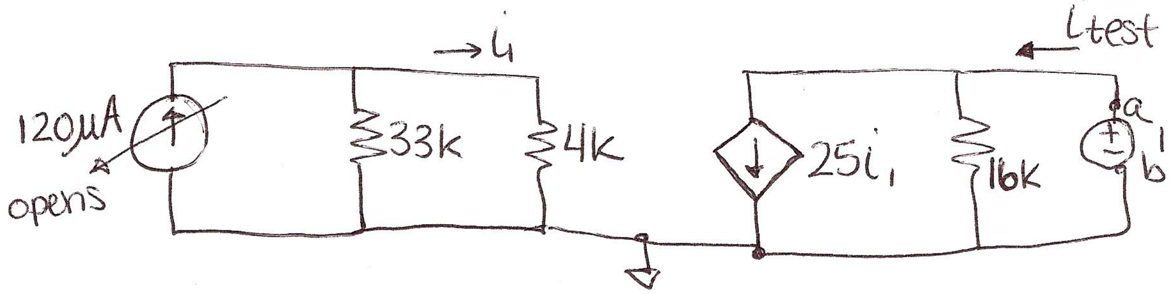
circuit ② $(t \to \infty)$: FINAL VALUE



$$i_1 = \frac{120\mu(33k)}{37k} = 107\mu A$$

$$V_1 = -25i_1(16k) = -25(107\mu)(16k) = -42.8V$$

circuit at ② $(t \to \infty)$: τ



Note: The dependent source has resistance!

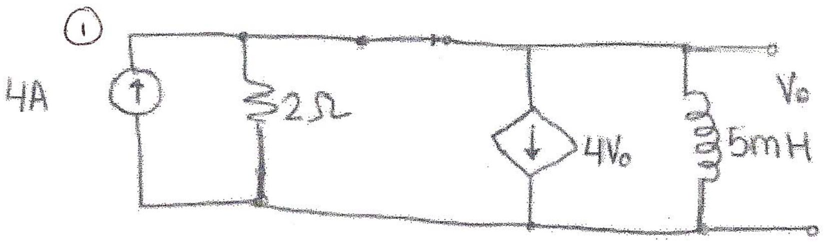
$i_1 = 0$, $25i_1 = 0$ (open

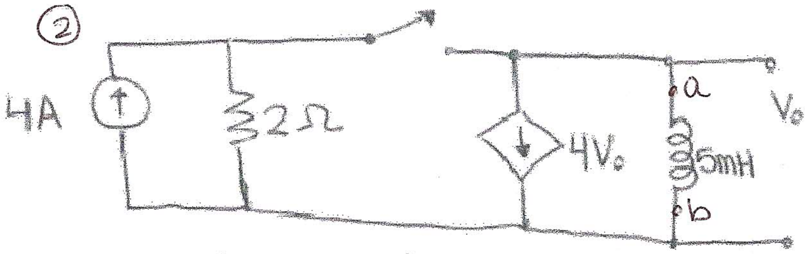
$\therefore \quad i_{test} = \dfrac{1}{16k}$

$R_{th} = \dfrac{1}{\frac{1}{16k}} = \underline{\underline{16k\ \Omega}}$ $\qquad \therefore \ \tau = 16k(0.25\mu) = 4msec.$

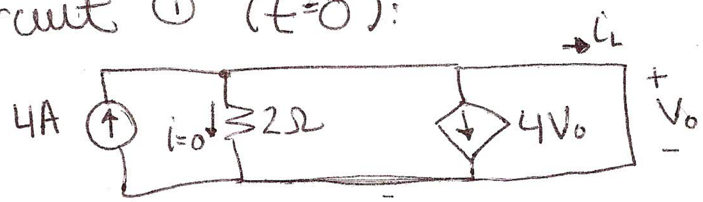
$V_1(t \geq 0) = -42.8 + (0 + 42.8)e^{-t/4ms.} = \boxed{[-42.8 + 42.8e^{-t/4msec.}]\,V}$

8. After being closed (top circuit) for a long time, the switch is opened (bottom circuit) at t=0. Find an expression for $V_o(t)$ for $(t \geq 0)$.



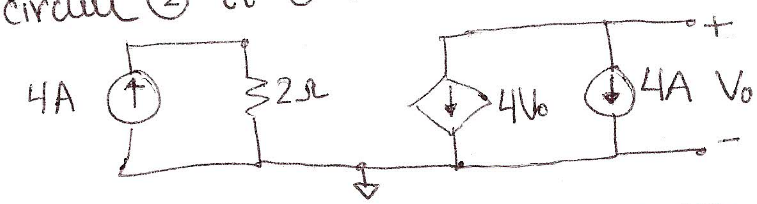


circuit ① $(t=0^-)$:



$i_L = 4A$

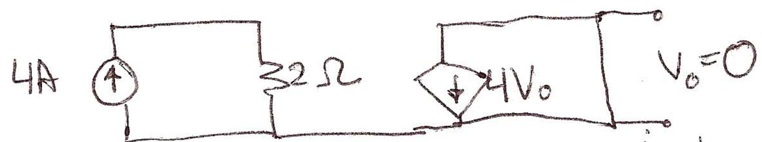
circuit ② $(t=0^+)$: INITAL VALUE



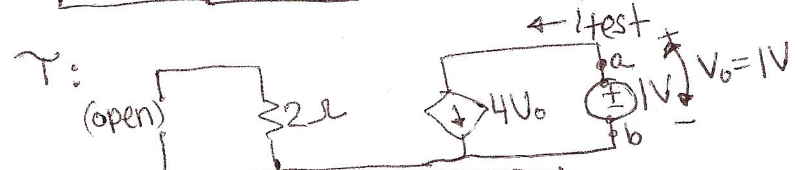
$-4V_o = 4A$

$\therefore V_o = -1V$

circuit ② $(t \to \infty)$: FINAL VALUE



τ:



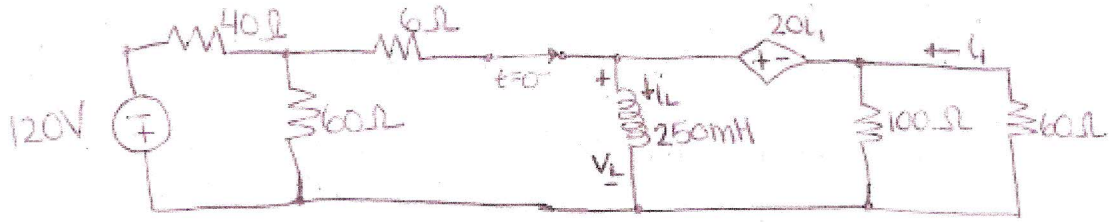
$i_{test} = 4(1) = 4A$

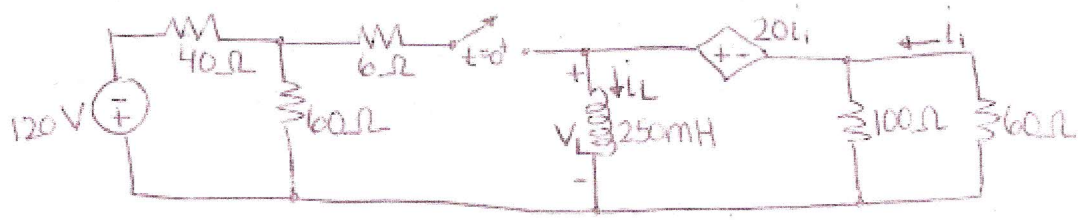
$R_{th} = \frac{1}{4} = \frac{1}{4}\Omega$

$\tau = \frac{5m}{(\frac{1}{4})} = 20ms.$

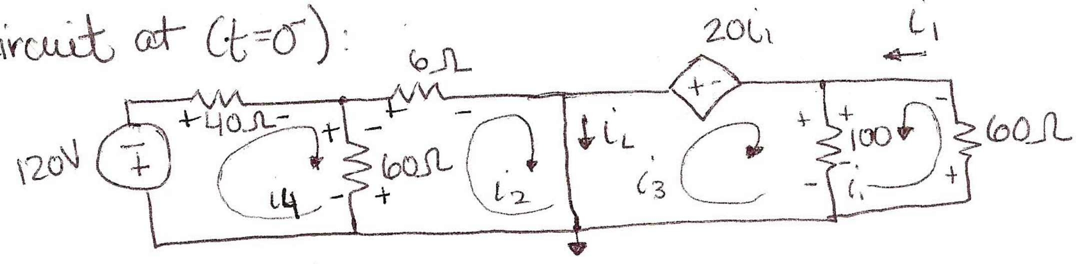
$V_o(t \geq 0) = \boxed{0 + (-1+0)e^{-t/20ms.}\ V}$

9. After being closed (top circuit) for a long time, the switch is opened (bottom circuit) at t=0. Find an expression for $V_L(t)$ and $i_L(t)$ for $(t \geq 0)$.





#9. circuit at $(t=0^-)$:



$$-i_2+i_L+i_3=0 \rightarrow i_L=(i_2-i_3)$$

① $-20i_4-100(i_3+i_1)=0$

② $+100(i_3+i_1)+i_1(60)=0$

③ $-120-i_4(40)+60(i_2-i_4)=0$

④ $+60(i_4-i_2)-i_2(6)=0$

From ② $\quad 100i_3=-\frac{160i_1}{100}=-\frac{8}{5}i_1$

plug into ①:

$$-20i_1-100i_1-(100)(-\tfrac{8}{5}i_1)=0$$

$$\therefore i_1=0 \rightarrow i_3=0$$

From ④:

$$60i_4=\frac{66i_2}{60}=\frac{11}{10}i_2$$

plug into ③:

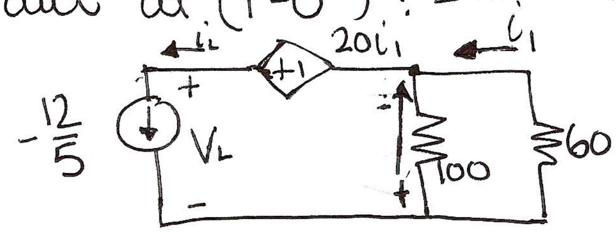
$$-120-\tfrac{11}{10}i_2(40)+60i_2-60(\tfrac{11}{10})i_2=0$$

$$120=-44i_2+60i_2-66i_2=+60i_2-110i_2$$

$$-\frac{120}{50}=-\cancel{50}i_2$$

$$\therefore i_L=i_2=-\frac{12}{5}\text{A}$$

#9. circuit at $(t=0^+)$: INITIAL VALUE



$$i_1 = \frac{+\left(-\frac{12}{5}\right)100}{160} = \frac{-12(\cancel{20})}{8\,\cancel{160}} = \frac{-3}{2}A = 1.5A$$

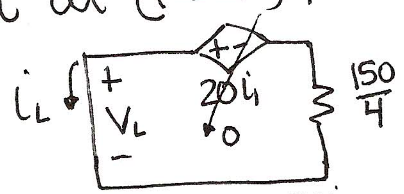
$$100 \parallel 60 = \frac{\overset{50}{\cancel{100}}(\overset{3}{\cancel{60}})}{4\,\cancel{8}\,\cancel{160}} = \frac{150}{4} = 37.5$$

$$+V_L - 20i_1 + \left(-\frac{12}{5}\right)\left(\frac{150}{4}\right) = 0$$

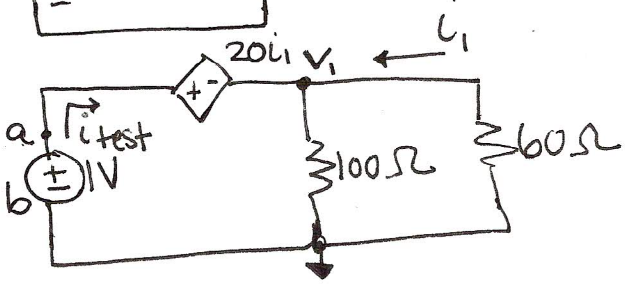
$$V_L = +20\left(\frac{-3}{2}\right) + 3(3) = -30+9 = \underline{\underline{-21V}}$$

$$i_L = -\frac{12}{5}$$

circuit at $(t\to\infty)$:



$V_L = 0,\ i_L = 0$



$$+1 - 20i_1 - V_1 = 0 \rightarrow V_1 = (+1 - 20i_1)$$

$$-i_{test} + \frac{(1-20i_1)}{100} + \frac{(1-20i_1)}{60} = 0$$

$$i_1 = \frac{-1+20i_1}{60} \rightarrow 60i_1 - 20i_1 = -1 \rightarrow i_1 = \frac{-1}{40}$$

$$i_{test} = \frac{1}{100} + \frac{1}{60} - \frac{20}{100}\left(-\frac{1}{40}\right) - \frac{20}{60}\left(-\frac{1}{40}\right)$$

#9.

$$i_{test} = \frac{6}{600} + \frac{10}{600} + \frac{1}{200} + \frac{1}{120} = \frac{16}{600} + \frac{3}{600} + \frac{5}{600} = \frac{24}{600}$$

$$i_{test} = \frac{1}{25}$$

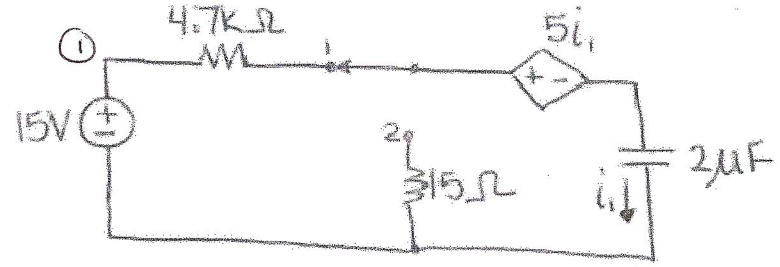
$$R_{th} = \frac{1}{\frac{1}{25}} = 25\Omega$$

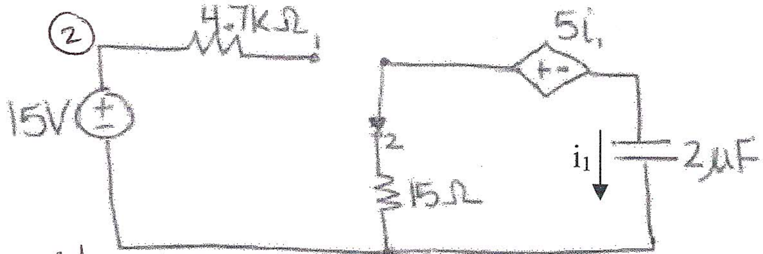
$$\tau = L/R_{th} = \frac{250mH}{25\Omega} = 10ms.$$

$$V_L(t \geq 0) = 0 + (-21 - 0)e^{-t/10ms.} = \boxed{-21V e^{-t/10ms.}}$$

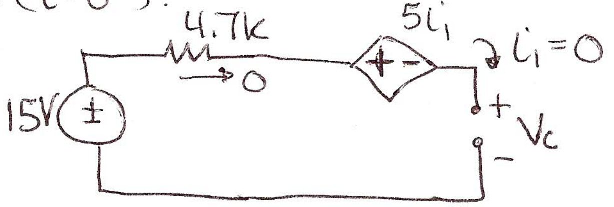
$$i_L(t \geq 0) = 0 + (-\tfrac{12}{5} - 0)e^{-t/10ms.} = \boxed{-\frac{12}{5} e^{-t/10ms.}}$$

10. After being in position 1 (top circuit) for a long time, the switch is moved in position 2 (bottom circuit) at t=0. Find an expression for $i_1(t)$ for $(t \geq 0)$.



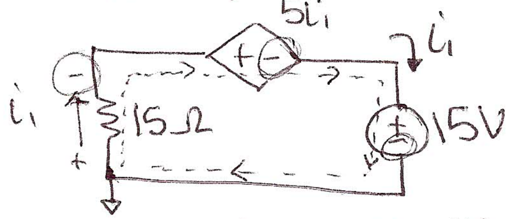


circuit at $(t=0^-)$:



$V_c = 15V$

Circuit at $(t=0^+)$: INITIAL VALUE

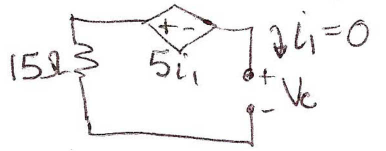


$-15i_1 - 5i_1 - 15 = 0$

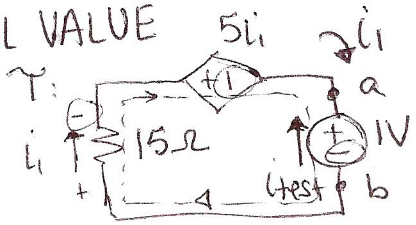
$+20i_1 = -15$

$\therefore i_1 = \frac{-15}{20} = \frac{-3}{4} A$

circuit at $(t \to \infty)$: FINAL VALUE



τ:



$i_1 = -i_{test}$

$-15i_1 - 5i_1 - 1 = 0$

$i_1 = -\frac{1}{20}$

$\therefore R_{th} = \frac{1}{(\frac{1}{20})} = 20\Omega$

$\tau = 20(2\mu) = 40\mu s.$

$$\boxed{i_1(t \geq 0) = -\frac{3}{4} e^{-t/40\mu s.} A}$$