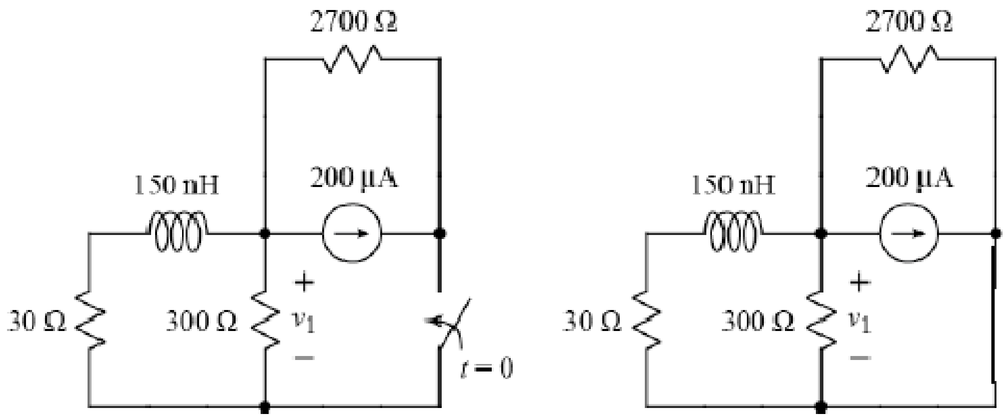
UNIVERSITY OF UTAH
ELECTRICAL & COMPUTER ENGINEERING DEPARTMENT

ECE 1270 **HOMEWORK #6** Summer 2011

1.

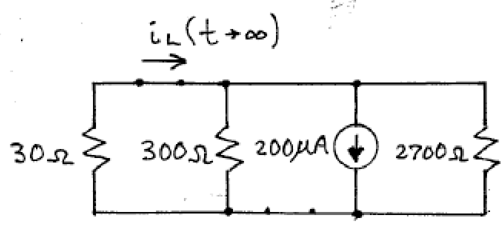


After being open(left circuit) for a long time, the switch closes at $t = 0$(right circuit).

a) Calculate the energy stored on the inductor as $t \to \infty$.

b) Write a numerical expression for $v_1(t)$ for $t > 0$.

sol'n: a) For $t \to \infty$, we model the L as a wire and the switch is closed.



We a current-divider, with the 200µA from the current source splitting between the 30Ω and the 300Ω‖2700Ω.

Q. Is there a minus sign in the current-divider formula?

A. No. If we follow the arrow for i_L around to the 200µA source, it points in the same direction as the arrow in the 200 µA source.

$$i_L(t \to \infty) = 200\mu A \cdot \frac{300 \| 2700\,\Omega}{300 \| 2700\,\Omega + 30\,\Omega}$$

Now, $300 \| 2700\,\Omega = 300\,\Omega \cdot 1 \| 9 = 300 \cdot \frac{9}{10}$

$$" \quad = 270\,\Omega.$$

$$i_L(t \to \infty) = 200\,\mu A \cdot \frac{270\ \Omega}{270 + 30\ \Omega} = 200\,\mu A \cdot \frac{270}{300}$$

$$" \quad = \frac{540}{3}\,\mu A = 180\,\mu A$$

Energy stored on L is $\frac{1}{2} L i_L^2$:

$$w_L = \frac{1}{2} \cdot 150\,n\,(180\,\mu)^2\ J$$

$$= \frac{1}{2} \cdot 150\,n \cdot 32.4\,k\,\mu^2\ J$$

$$= \frac{1}{2} \cdot 4.86\,\mu k \mu \mu\ J \qquad \text{since } 150n = 0.15\mu$$

$$\boxed{w_L = 2.43\ fJ} \qquad f \equiv \text{femto} = 10^{-15}$$

b) Using the circuit diagram from part (a) for $t \to \infty$, we see that $v_1(t \to \infty)$ is the voltage across all three resistors. The same voltage will be across the equivalent of the three resistors in parallel, and by Ohm's law the voltage will be the source current times the equivalent R.

$$v_1(t \to \infty) = -200\,\mu A \cdot 30\,\Omega \| 300\,\Omega \| 2700\,\Omega$$

$$" \quad = -200\,\mu A \cdot 30\,\Omega \| 270\,\Omega$$

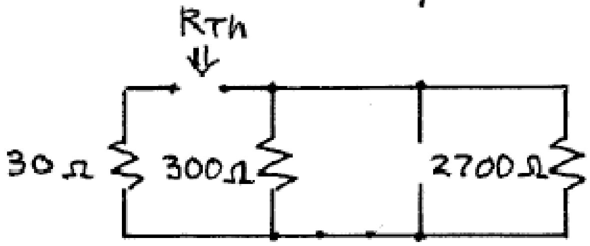
$$" \quad = -200\,\mu A \cdot 30\,\Omega \cdot 1 \| 9$$

$$v_1(t \to \infty) = -200\,\mu A \cdot 27\,\Omega$$

or

$$v_1(t \to \infty) = -5.4\ mV$$

For R_{Th}, we use the circuit for $t > 0$ with the L removed and the dependent source off.



$$R_{Th} = 30\,\Omega + 300\,\Omega \,\|\, 2700\,\Omega$$

$$= 30\,\Omega + 270\,\Omega$$

$$R_{Th} = 300\,\Omega$$

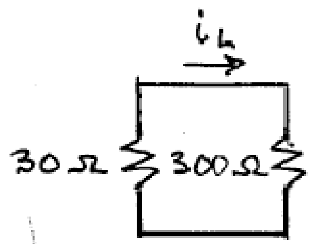
Our time constant is

$$\tau = \frac{L}{R_{Th}} = \frac{150\ nH}{300\ \Omega} = 0.5\ ns.$$

Last, we need $v_1(t = 0^+)$. To find this value, we need a model for the L. To find a model for the L, we consider $t = 0^-$, when the circuit is stable and the L acts like a wire.

The switch is open at $t = 0^-$, which means the circuit loop on the lower left is connected to the circuit loop on the upper right by a single point.

No current can flow between the two loops in the circuit without causing an accumulation of charge. Thus, the two loops have no influence on each other. Thus, we need only consider the loop on the lower left.



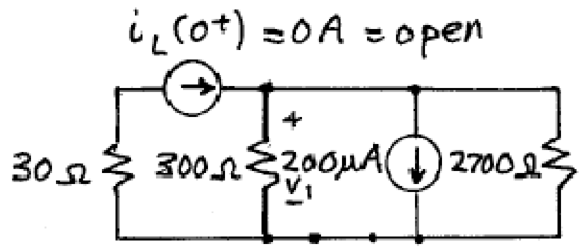
Since there is no power source,

$$i_L(0^-) = 0 \text{ A}.$$

Because i_L is an energy variable,

$$i_L(0^+) = i_L(0^-).$$

Now we can model the L as a current source at $t = 0^+$.



Because the L acts like an open, the 30Ω resistor dangles and has no impact on $v_1(t=0^+)$. $v_1(0^+)$ is given by the current source times the parallel resistance of 300 Ω and 2700 Ω, which is 270 Ω.

$$v_1(0^+) = -200\,\mu\text{A} \cdot 270\,\Omega = -54\,\text{mV}$$

We use the general form of solution to finish the problem.

$$v_1(t>0) = v_1(t\to\infty) + \left[v_1(0^+) - v_1(t\to\infty)\right] e^{-t/\tau}$$

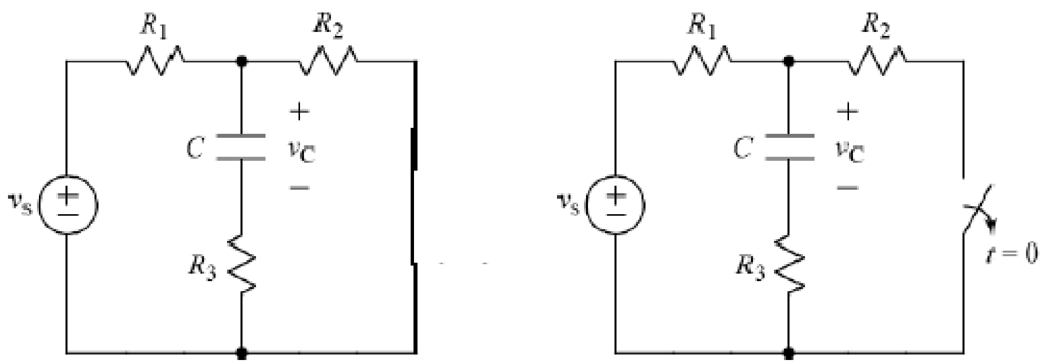
or

$$v_1(t>0) = -5.4\,mV + \left[-54\,mV - -5.4\,mV\right] e^{-t/0.5ns}$$

or

$$\boxed{v_1(t>0) = -5.4 - 48.6\, e^{-t/0.5ns} \quad mV}$$

2.



After being closed(left circuit) for a long time, the switch opens at $t = 0$(right circuit).

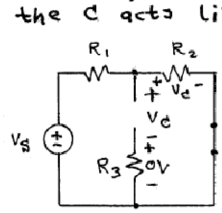
a) Write an expression for $v_C(t = 0^+)$.

b) Write an expression for $v_C(t > 0)$ in terms of no more than R_1, R_2, R_3, v_s, and C.

sol'n: a) Since v_C is an energy variable,

$$v_C(0^+) = v_C(0^-).$$

At $t = 0^-$, the switch is closed and the C acts like an open circuit.



No current flows thru R_3, so the voltage drop across R_3 is 0V. A voltage loop on the right gives v_C as the voltage drop across R_2. From the outside v-loop, we see that the voltage drop across R_2 is given by a voltage divider.

$$\boxed{v_C(0^+) = v_C(0^-) = v_s \frac{R_2}{R_1 + R_2}}$$

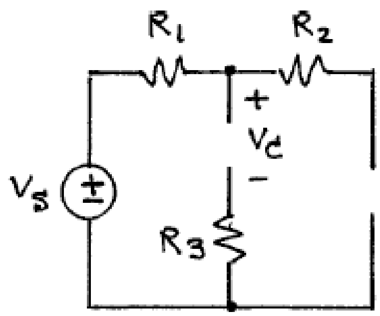
b) We use the general form of sol'n:

$$v_C(t>0) = v_C(t\to\infty) + \left[v_C(0^+) - v_C(t\to\infty)\right] e^{-t/\tau}$$

where $\tau = R_{Th}C$

We found $v_C(0^+)$ in part (a).

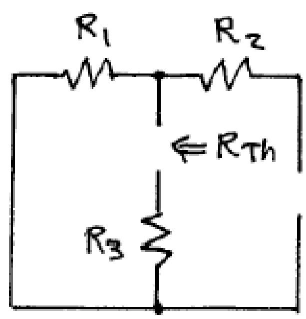
As $t\to\infty$, the C is open and the switch is open.



With the open circuits, no current flows. Thus, the v-drops for the R's are zero. For the v-loop on the left, we have

$$v_C(t\to\infty) = V_S.$$

To find R_{Th}, we turn off V_S and look into the circuit from the terminals where the C is connected. Because we are solving for v_C for $t>0$, the switch is open.



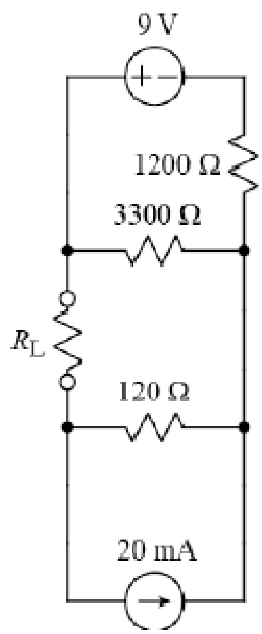
Because the switch is open, R_2 has no effect on R_{Th}. Since R_1 and R_3 are in series, we have

$$R_{Th} = R_1 + R_3$$

Substituting into the general form of solution, we have the following:

$$v_C(t>0) = v_S + \left[v_S \frac{R_2}{R_1+R_2} - v_S \right] e^{-t/(R_1+R_3)C}$$

3.

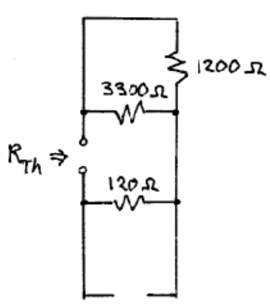


a) Calculate the value of R_L that would absorb maximum power.

b) Calculate that value of maximum power R_L could absorb.

soln: a) $R_L = R_{Th}$ for max pwr transfer.

We remove R_L, turn off the independent sources, and look in from the terminals for R_L to find R_{Th}.



$$R_{Th} = 120\,\Omega + 3300\,\Omega \| 1200\,\Omega$$

We have $3300 \,\|\, 1200\,\Omega = 100 \cdot 33 \,\|\, 12\,\Omega$

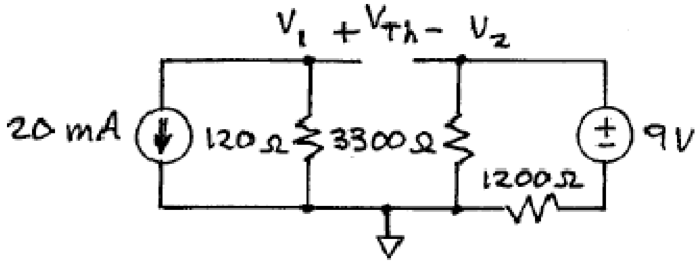
$$= \overset{20}{\cancel{100}} \cdot \frac{\cancel{396}^{\,44}}{\cancel{45}\ \cancel{9}}\,\Omega$$

$$= 880\,\Omega$$

So $\boxed{R_L = R_{Th} = 120\,\Omega + 880\,\Omega = 1\,k\Omega}$.

b) $$P_{max} = \frac{V_{Th}^2}{4R_{Th}}$$

We find V_{Th} at the terminals where R_L is connected but without R_L. Since V_{Th} is squared, we may measure V_{Th} in either direction.



No current flows in the bottom wire, since otherwise charge would accumulate on one side. So the two sides act as separate circuits.

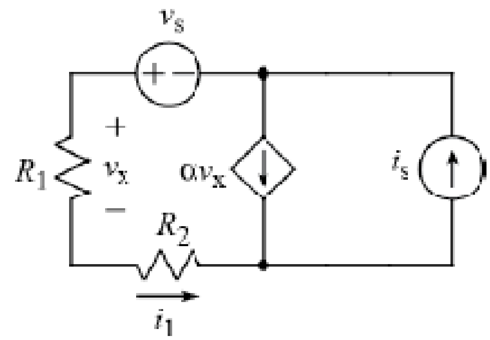
$V_1 = -20\,mA \cdot 120\,\Omega = -2.4V$ (ohm's law)

$V_2 = 9V \cdot \dfrac{3300\,\Omega}{3300+1200\,\Omega} = 6.6V$ (v-divider)

$V_{Th} = V_1 - V_2 = -2.4V - 6.6V = -9V$

$$\boxed{P_{max} = \frac{(-9V)^2}{4 \cdot 1k\Omega} = 20.25\,mW}$$

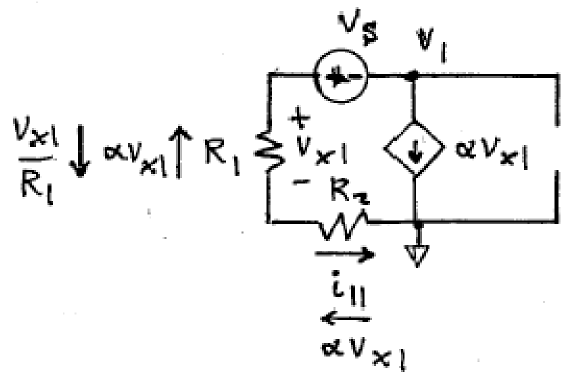
4.



Using superposition, derive an expression for i_1 that contains no circuit quantities other than i_s, v_s, R_1, R_2, and α. Note: $\alpha > 0$.

sol'n: We turn on one independent source at a time. Dependent sources are always on.

case I: v_s on, i_s off = open



Because of the open circuit, the current αv_{x1} flows thru R_2 and up thru R_1.

The current flowing down thru R_1 is v_{x1}/R_1.

$$\frac{v_{x1}}{R_1} = -\alpha v_{x1}$$

The only possible sol'n is $v_{x1} = 0$, $i_{11} = 0\text{A}$.

Or we can use the node-voltage method.

v_1 node: $\dfrac{v_1 + v_s}{R_1 + R_2} + \alpha v_{x1} = 0A$

where $v_{x1} = (v_1 + v_s)\dfrac{R_1}{R_1 + R_2}$

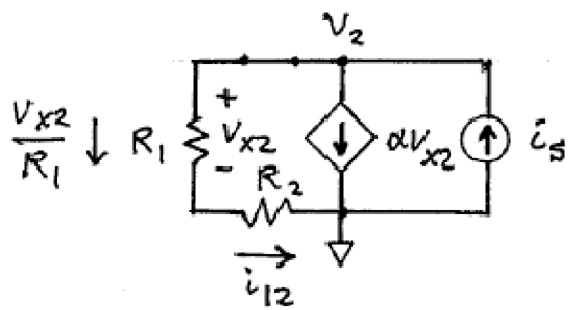
So $(v_1 + v_s)\left(\dfrac{1 + \alpha R_1}{R_1 + R_2}\right) = 0A$

Since $\alpha > 0$, $\dfrac{1 + \alpha R_1}{R_1 + R_2} \neq 0$.

Thus, $v_1 + v_s = 0V$ or $v_1 = -v_s$.

This means the voltage drop across $R_1 + R_2$ is 0V, giving $i_{l1} = 0A$.

case II: v_s off = wire, i_s on



Using the node-voltage method, we have a current sum at v_2:

$$\frac{v_2}{R_1 + R_2} + \alpha v_2 \frac{R_1}{R_1 + R_2} - i_s = 0A$$

or $v_2 \dfrac{1 + \alpha R_1}{R_1 + R_2} = i_s$

or $v_2 = i_s \dfrac{R_1 + R_2}{1 + \alpha R_1}$

$$i_{12} = \frac{v_2}{R_1 + R_2} = \frac{i_s}{1 + \alpha R_1}$$

Or we could use a current sum directly in terms of v_x:

$$i_s = \frac{v_{x2}}{R_1} + \alpha v_{x2} = v_{x2}\left(\frac{1}{R_1} + \alpha\right)$$

or

$$v_{x2} = i_s \frac{R_1}{1 + \alpha R_1}$$

$$i_{12} = \frac{v_{x2}}{R_1} = \frac{i_s}{1 + \alpha R_1}$$

The total i_1 is the sum of i_{11} and i_{12}.

$$i_1 = i_{11} + i_{12} = 0 + i_{12} = \frac{i_s}{1 + \alpha R_1}$$

$$\boxed{i_1 = \frac{i_s}{1 + \alpha R_1}}$$