

1. Plot each of the following complex numbers as vector in the complex plane:

a. $(-20-5j)$

b. $34e^{j45^\circ}$

c. $\frac{2+3j}{6} - \frac{3-2j}{4}$

d. $\frac{16}{4j^3}$

e. $\frac{-1+3j}{-5-4j}$

2. Give numerical answers to each of the following questions:

a. Rationalize $\frac{-10k \cdot (j2k)}{2k - j10k}$. Express your answer in rectangular form.

b. Find the polar form of $\left(\frac{5e^{j45^\circ}}{4+4j}\right)^* (30k - j10k)^*$ (Note: The asterisk means conjugate.)

c. Find the following phasor: $P[3\sin(3kt + 75^\circ)]$.

d. Find the magnitude of $\frac{(3-2j)2e^{-j60^\circ}}{2+j+4e^{j45^\circ}}$.

e. Find the imaginary part of $\frac{1-5j}{e^{-j60^\circ}(j1k+1k)}$.

3. a. Write phasors (as both $Ae^{j\phi}$ and $A\angle\phi$) for each of the following signals:

i. $v(t)=4\cos(5kt+60^\circ)V$

ii. $i(t)=16\sin(\omega t-123^\circ)mA$

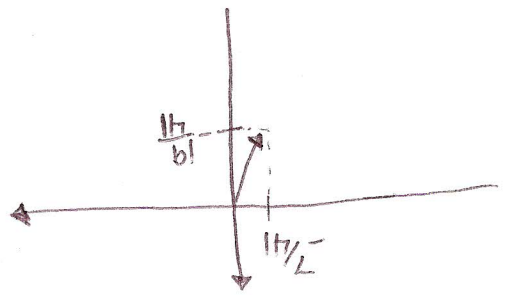
iii. $v(t)=\cos(10t+60^\circ)V+5\sin(10t-30^\circ)V$

b. Given $\omega=3\text{krad/sec}$, write inverse phasors for each of the following signals:

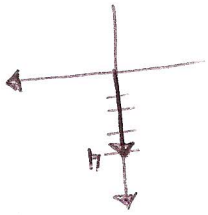
i. $\mathbf{I}=76.8e^{j15^\circ} \text{ A}$

ii. $\mathbf{V}=-8j^2 \text{ V}$

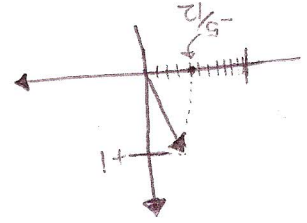
iii. $\mathbf{I}=5e^{+\frac{\pi}{2}-j30^\circ} \text{ A}$



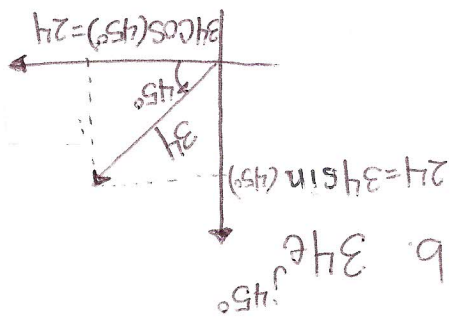
$$e. \frac{(-1+3j)(-5+4j)}{(-5+4j)(-5+4j)} = \frac{+5-4j-15j+12}{+25-20j+20j-16} = \frac{-7-19j}{25+16}$$



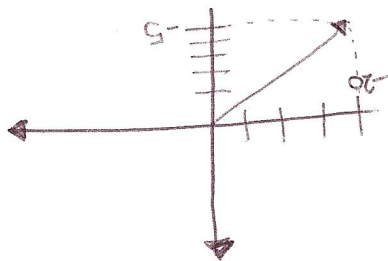
$$d. \frac{16}{4} \cdot \frac{4j}{4} = -4(-j) = +4j$$



$$c. \frac{2}{6} (2+3j) - \frac{4}{3} (3-2j) = \frac{4+6j-9+6j}{12} = \frac{-5+12j}{12}$$



① a. $(-20-5j)$



$$(2) a. \frac{-10k(j2k)(2k+10kj)}{(2k-j10k)(2k+10kj)} = \frac{-20k^2(2kj) - 20k^2(10kj)^{-1}}{4k^2 + 20k^2j - 20k^2j - (10k)^2j^2}$$

$$\frac{+200 \times 10^9 - 40 \times 10^9 j}{104 \times 10^9} = \boxed{\frac{200 - 40j}{104}}$$

$$b. \left(\frac{5e^{j45^\circ}}{4+4j} \right)^* (30k-j10k)^* = \frac{5e^{j45^\circ}}{4-4j} \cdot (30k+j10k) =$$

$$\frac{5e^{-j45^\circ}}{\sqrt{4^2+4^2} e^{j \tan^{-1}(\frac{4}{4})}} \cdot \sqrt{(30k)^2 + (10k)^2} e^{j \tan^{-1}(\frac{10k}{30k})} \cong \frac{5e^{-j45^\circ} \sqrt{1 \times 10^9} e^{j18.4^\circ}}{\sqrt{32} e^{-j45^\circ}}$$

$$\boxed{\frac{5 \cdot \sqrt{1 \times 10^9}}{\sqrt{32}} \cdot e^{j18.4^\circ}}$$

$$c. \mathcal{P}[3 \sin(3kt + 75^\circ)] = \mathcal{P}[3 \cos(3kt + 75^\circ - 90^\circ)] = \boxed{3e^{j15^\circ}}$$

$$d. \left| \frac{(3-2j)2e^{j60^\circ}}{2+j+4e^{j45^\circ}} \right| = \frac{\sqrt{3^2+2^2} e^{j \tan^{-1}(\frac{-2}{3})} 2e^{-j60^\circ}}{2+j+4(\cos(45^\circ)+j\sin(45^\circ))} = \frac{2\sqrt{13} e^{j(-60^\circ-34^\circ)}}{4.8+3.8j}$$

$$\frac{2\sqrt{13} e^{-j94^\circ}}{\sqrt{4.8^2+3.8^2} e^{j \tan^{-1}(\frac{3.8}{4.8})}} = \frac{2\sqrt{13}}{\sqrt{37.5}} e^{-j94^\circ-38.4^\circ} \cong \left| 1.2 e^{-j132.4^\circ} \right| = \boxed{1.2}$$

$$e. \frac{(1-5j)}{e^{j60^\circ}(jk+1k)} = \frac{\sqrt{1^2+5^2} e^{j \tan^{-1}(\frac{-5}{1})}}{e^{j60^\circ} \sqrt{1k^2+1k^2} e^{j \tan^{-1}(1)}} = \frac{\sqrt{26}}{\sqrt{2} \times 10^6} e^{j78.7+60^\circ-45^\circ}$$

$$\text{Im} \left[\frac{\sqrt{26}}{\sqrt{2} \times 10^6} e^{j63.7^\circ} \right] = \frac{\sqrt{26}}{12 \times 10^6} \sin(-63.7^\circ) = \boxed{-3.2 \text{m}}$$

③ a. i. $4e^{j60^\circ}$ or $4\angle 60^\circ$

ii. $16\sin(\omega t - 123^\circ)\text{m} = 16\text{m}\cos(\omega t - 123^\circ - 90^\circ)$

$16\text{m}e^{-j213^\circ}$ or $16\text{m}\angle -213^\circ$

iii. $\cos(10t + 60^\circ) + 5\sin(10t - 30^\circ) = \cos(10t + 60^\circ) + 5\cos(10t - 30^\circ - 90^\circ)$
 $\cos(60^\circ) + \sin(60^\circ)j + 5\cos(-120^\circ) + 5\sin(-120^\circ)j =$

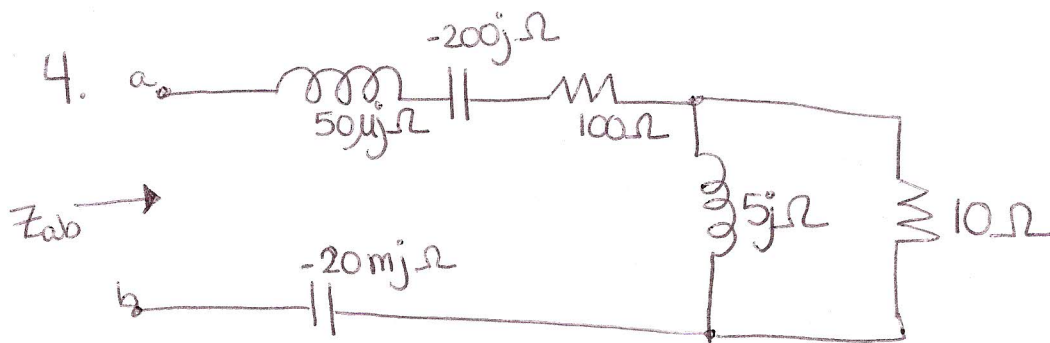
$0.5 + 0.866j - 2.5 - 4.33j = -2 - 3.5j = \sqrt{2^2 + (3.5)^2} e^{j\tan^{-1}(\frac{-3.5}{-2})}$
 $4e^{j(60^\circ + 180^\circ)} = 4e^{j240^\circ}$ or $4\angle 240^\circ$

b. $\omega = 3\text{krad/sec}$

i. $I = 76.8\cos(3kt + 15^\circ)\text{A}$

ii. $V = -8j^2 = +8$; $V = 8\cos(3kt)\text{V}$

iii. $I = 5(e^{90^\circ})e^{-j30^\circ}$; $I = 5e^{90^\circ}\cos(3kt - 30^\circ)\text{A}$



$$j5k(10n) = 50\mu\Omega j$$

$$j(5k)(1m) = 5j\Omega$$

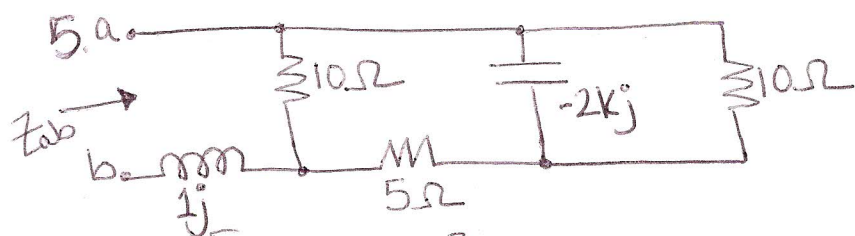
$$\frac{-j}{5k(1\mu)} = -200j\Omega$$

$$\frac{-j}{5k(10m)} = -0.02j\Omega = -20mj$$

$$Z_{ab} = 50\mu j - 200j + 100 + [5j \parallel 10] + -20mj$$

$$\frac{5j(10) \cdot (10-5j)}{(5j+10)(10-5j)} = \frac{500j - 250j^2}{50j^2 - 25j^2 + 100 - 50j} = \frac{(500j + 250)}{100 + 25}$$

$$Z_{ab} = 50\mu j - 200j - 20mj + 100 + \frac{500j}{\underbrace{125}_{4j}} + \frac{250}{\underbrace{125}_{2}} \approx \boxed{-196j + 102}$$



$$j(100)(10m) = 1j$$

$$\frac{-j}{5\mu(100)} = -2kj$$

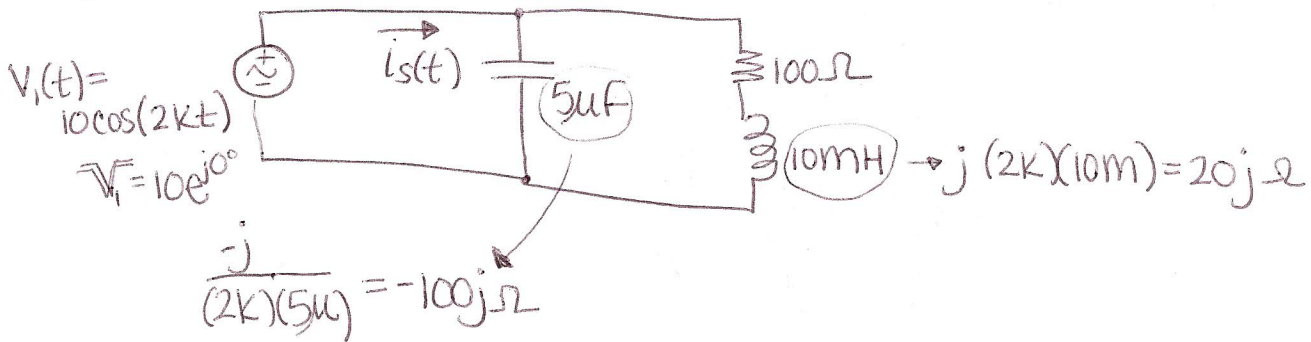
$$Z_{ab} = [(-2kj \parallel 10) + 5] \parallel 10 + 1j$$

$$\frac{-2kj(10)}{-2kj+10} = \frac{-20kj(10+2kj)}{(-2kj+10)(10+2kj)} = \frac{-200kj - 40k^2j^2}{100 + 4k^2j^2} = -0.05j + 10$$

$$Z_{ab} = [(-0.05j + 10 + 5) \parallel 10] + j = \frac{(-0.05j + 15)(10) \cdot (25 + 0.05j)}{(-0.05j + 25)(25 + 0.05j)} + j$$

$$Z_{ab} = \frac{-12.5j + 0.025 + 3750 + 7.5j}{(-1.25j + 2.5m + 1.25j + 625)} + j = \boxed{6 + 0.99j}$$

6.



7.

$$\tilde{I}_s = \frac{\tilde{V}_1}{-100j \parallel [100 + 20j]}$$

$$\frac{-100j \cdot (100 + 20j)}{[100j + 20j + 100]} \cdot \frac{(100 + 80j)}{(100 + 80j)} = \frac{(-1 \times 10^6 j) + 2 \times 10^5 + 8 \times 10^5 + (1.6 \times 10^5 j)}{10,000 + 6,400} =$$

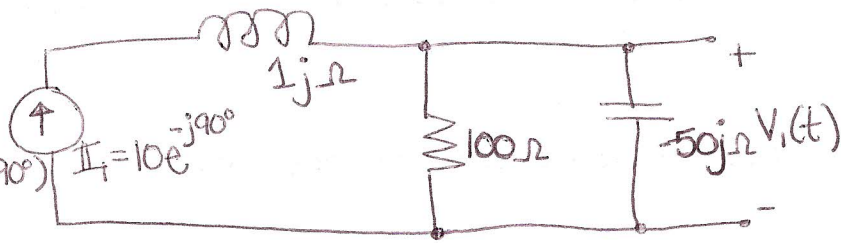
$$\tilde{I}_s \approx \frac{10 \cdot (61 + 51j)}{(61 - 51j)(61 + 51j)} = \frac{10(61 + 51j)}{3721 + 2601} \approx 96.5m + 80.7mj$$

$$\tilde{I}_s = \sqrt{(96.5m)^2 + (80.7m)^2} e^{j \tan^{-1} \left(\frac{80.7m}{96.5m} \right)} \approx 126mA e^{j40^\circ}$$

$$i_s(t) = 126m \cos(2kt + 40^\circ) A$$

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$$I_1(t) = 10 \cos(1kt - 90^\circ)$$



$$\frac{-j}{1k(20\mu)} = -50j$$

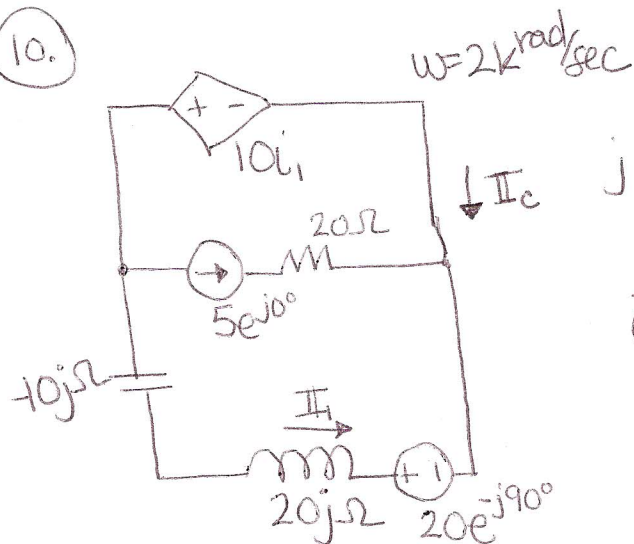
$$j(1k)(1m) = 1j$$

$$9. \quad V_1 = I_1 \cdot (100 \parallel -50j) = 10e^{-j90^\circ} \cdot \frac{(100)(-50j)}{(100-50j)} \cdot \frac{(100+50j)}{(100+50j)} = \frac{2.5 \times 10^6 - 5 \times 10^6 j}{12,500}$$

$$V_1 = -400 - 200j = 447 e^{-j153^\circ}$$

$$V_1(t) = 447 \cos(1kt - 153^\circ) \text{ or } 447 \cos(1kt + 207^\circ) \text{ V}$$

10.



$$j(2k)(10m) = 20j \Omega$$

$$\frac{-j}{(2k)(50\mu)} = -10j \Omega$$

$$-I_c - 5 - I_1 = 0$$

$$I_1 = \frac{0 - (-10I_1) - 20(-j)}{-10j + 20j} \Rightarrow +10j I_1 = +10I_1 + 20j$$

$$(10j - 10) I_1 = +20j$$

$$I_1 = \frac{+20j}{(10j - 10)} \cdot \frac{(-10 - 10j)}{(-10 - 10j)} = \frac{-200j - 200j^2}{-100j - 100j^2 + 100 + 100j}$$

$$I_1 = \frac{+200 - 200j}{100 + 100} = 1 - j$$

$$I_c = -5 - 1 + j = -6 + j = \sqrt{36+1} e^{j \tan^{-1}(\frac{1}{-6})}$$

$$I_c = \sqrt{37} e^{-j9.5^\circ}$$

$$I_c = \sqrt{37} \cos(2kt - 9.5^\circ)$$