1.

Give numerical answers to each of the following questions:

a) Rationalize $\frac{3-j}{1-j2}$. Express your answer in rectangular form, $a + jb$.
Give the numerical values of a and b.

b) Find the rectangular form of $-j10e^{j90^\circ} - 7 - j3\sqrt{3}$.

c) Given $\omega = 120$k r/s, find the inverse phasor of $\frac{1}{1+j}$.

d) Find the magnitude of $\frac{e^{-j15^\circ}(e^{j15^\circ}+4)}{(e^{-j15^\circ}+4)}$.

e) Find the real part of $7 + j3e^{j\pi\cos 60^\circ}$.

sol'n: a) $\frac{3-j}{1-j2} \cdot \frac{1+j2}{1+j2} = \frac{3+2-j+j6}{1^2+2^2} = \frac{5+j5}{5}$

$= 1+j$

b) $-j10\underbrace{e^{j90^\circ}}_{=j} - 7 - j3\sqrt{3}$

$10 - 7 - j3\sqrt{3}$

$3 - j3\sqrt{3}$

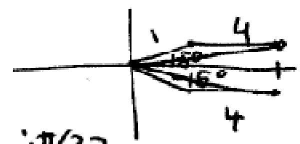
c) $\frac{1}{1+j} = \frac{1}{\sqrt{2}\angle 45^\circ} = \frac{1}{\sqrt{2}}\angle -45^\circ \rightarrow \frac{1}{\sqrt{2}}\cos(120kt - 45^\circ)$

or $\frac{1}{1+j} = \frac{1}{1+j}\frac{1-j}{1-j} = \frac{1}{2} - j\frac{1}{2} \rightarrow \frac{1}{2}\cos(120kt)$

$+ \frac{1}{2}\sin(120kt)$

d) $\left|\dfrac{e^{-j15°}(e^{j15°}+4)}{e^{-j15°}+4}\right| = \dfrac{|e^{-j15°}|\,\cancel{|e^{j15°}+4|}}{\cancel{|e^{-j15°}+4|}}$ these are conjugates with same magnitude

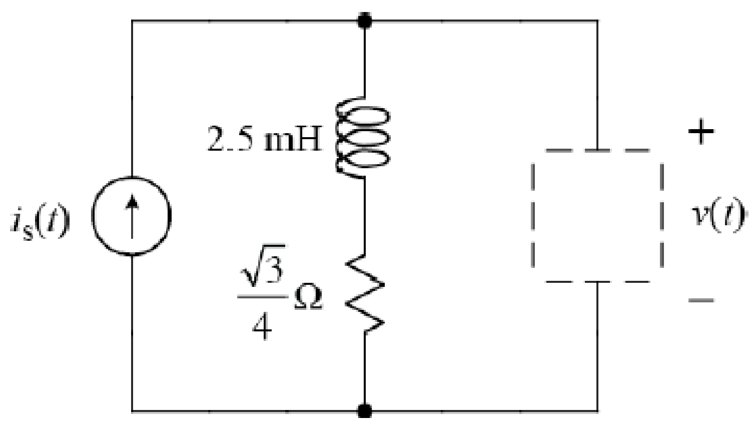
$= |e^{-j15°}| = 1$



e) $\mathrm{Re}\,[7 + j3e^{j\pi \cos 60°}] = \mathrm{Re}\,[7 + j3e^{j\pi/2}]$

$= \mathrm{Re}\,[7 + j3j] = \mathrm{Re}\,[7-3] = \mathrm{Re}\,[4] = 4$

2.



a) The current source in the above circuit has a value of

$$i_s(t) = 4\cos(100t)\,\mathrm{A}$$

Choose an R, an L, or a C to be placed in the dashed-line box to make

$$v(t) = V_o\cos(100t - 30°)$$

where V_o is a positive, (i.e., nonzero and non-negative), real constant with units of Volts. State the value of the component you choose.

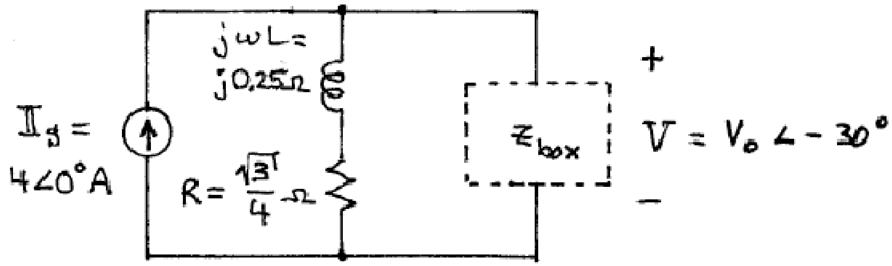
3. With your component from problem 2 in the circuit, calculate the resulting value of Vo.

sol'n: a) We convert to the frequency domain.

$\omega = 100$ r/s from $i_s(t)$

$j\omega L = j \cdot 100 \text{ r/s} \cdot 2.5 \text{ mH} = j0.25\ \Omega$

$\mathbb{I}_s = 4\angle 0°$ A, $\mathbb{V} = V_0 \angle -30°$



By Ohm's law, $\mathbb{V} = \mathbb{I}_s \cdot Z_{tot} = \mathbb{I}_s \cdot (R + j\omega L) \parallel Z_{box}$

$$\mathbb{V} = \mathbb{I}_s \frac{1}{\dfrac{1}{R+j\omega L} + \dfrac{1}{Z_{box}}}$$

We consider only the angle's:

$$\angle \mathbb{V} = \angle \mathbb{I}_s + \angle \left(\frac{1}{\dfrac{1}{R+j\omega L} + \dfrac{1}{Z_{box}}} \right)$$

Using $\angle \dfrac{1}{A\angle\phi} = -\phi$, we have

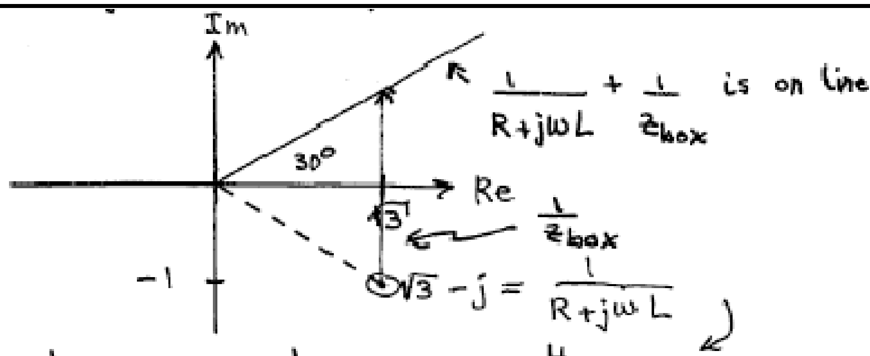
$$\angle \mathbb{V} = \angle \mathbb{I}_s - \angle \left(\frac{1}{R+j\omega L} + \frac{1}{Z_{box}} \right)$$

or

$$-30° = 0° - \angle \left(\frac{1}{R+j\omega L} + \frac{1}{Z_{box}} \right)$$

or

$$\angle \left(\frac{1}{R+j\omega L} + \frac{1}{Z_{box}} \right) = 30°$$



Now $$\frac{1}{R+j\omega L} = \frac{1}{\frac{\sqrt{3}}{4} + j\frac{1}{4}\,\Omega} = \frac{4}{\sqrt{3}+j\,\Omega}$$

$$= \frac{4}{\sqrt{3}+j}\,\frac{\sqrt{3}-j}{\sqrt{3}-j\,\Omega} = \frac{4(\sqrt{3}-j)}{\sqrt{3}^2+1^2\,\Omega} = \frac{\sqrt{3}-j}{\Omega}$$

From the above diagram, we see that we need $1/z_{box}$ to move us up to the 30° line. The real part of $\frac{1}{R+j\omega L} + \frac{1}{z_{box}}$ will be $\frac{\sqrt{3}}{\Omega}$. To be on 30° line,

we want $\frac{Im}{Re} = \tan 30° = \frac{1}{\sqrt{3}}$

or $Im = Re \cdot \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{\Omega} \cdot \frac{1}{\sqrt{3}} = \frac{1}{\Omega}$

Thus, $\frac{1}{z_{box}} = j\frac{2}{\Omega}$ or $z_{box} = -\frac{j}{2}\,\Omega$

$z_{box} = -\frac{j}{2}\,\Omega$ means we use a C.

$$\frac{1}{j\omega C} = -\frac{j}{2}\,\Omega$$

or $$C = \frac{2}{\omega \cdot \Omega} = \frac{2}{100}\,F = 20\ mF$$

$$\boxed{C = 20\ mF}$$

3.

We can work in terms of magnitudes.

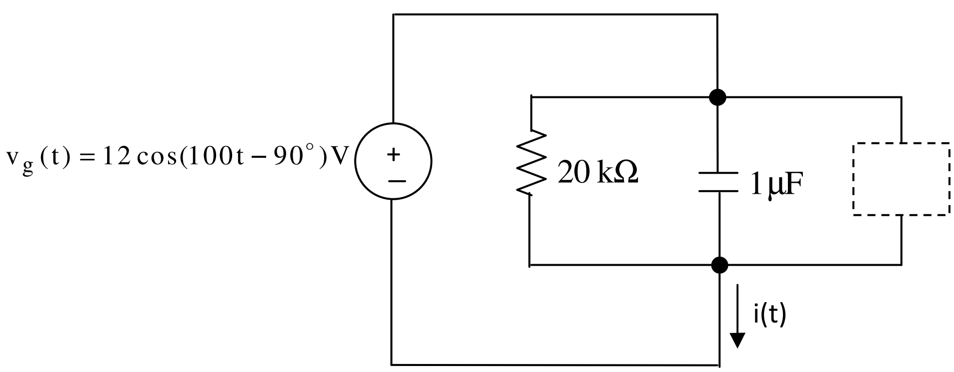
$$|V| = V_o = |I_s| \cdot |(R+j\omega L) \parallel Z_{box}|$$

$$= |I_s| \cdot \frac{1}{\left| \frac{1}{R+j\omega L} + \frac{1}{Z_{box}} \right|}$$

$$= 4A \cdot \frac{1}{\left| \frac{\sqrt{3}-j}{\Omega} + \frac{j}{2\Omega} \right|} = \frac{4}{|\sqrt{3}+1|} V = \frac{4V}{\sqrt{3+1}}$$

$$\boxed{V_o = 2\ V}$$

4.



Choose an R, an L, or a C to be placed in the dashed-line box to make

$$i(t) = \mathbf{I}_o \cos(100t - 45^\circ) A$$

where $\mathbf{I}_o$ is a real constant. State the value of the component you choose.

b. With your component from part (a) in the circuit, calculate the resulting value of I_o.

$\mathbf{I}_0$.

soln: a) Use conductance: $\mathbf{I} = \mathbf{I}_o \angle -45^\circ A = \mathbf{V}_g \cdot \overbrace{\left(\frac{1}{20k\Omega} + j100 \cdot \frac{1\mu}{\Omega} + \frac{1}{Z_{box}}\right)}^{G_{tot}}$
(and phasors)

Note: $\omega = 100$ from $v_g(t)$ where $\mathbf{V}_g = 12 \angle -90^\circ$ V

we have $\angle \mathbf{I} = \angle \mathbf{V}_g + \angle G_{tot}$ from phasor multiplication

$$-45^\circ = -90^\circ + \angle G_{tot}$$

$$\therefore \angle G_{tot} = 45^\circ \quad \text{or} \quad Re[G_{tot}] = Im[G_{tot}]$$

$$G_{tot} = \frac{50\mu}{\Omega} + j\frac{100\mu}{\Omega} + \frac{1}{Z_{box}}$$

we can choose $\frac{1}{Z_{box}} = \frac{50\mu}{\Omega} \Rightarrow \boxed{Z_{box} = 20k\Omega \text{ resistor}}$

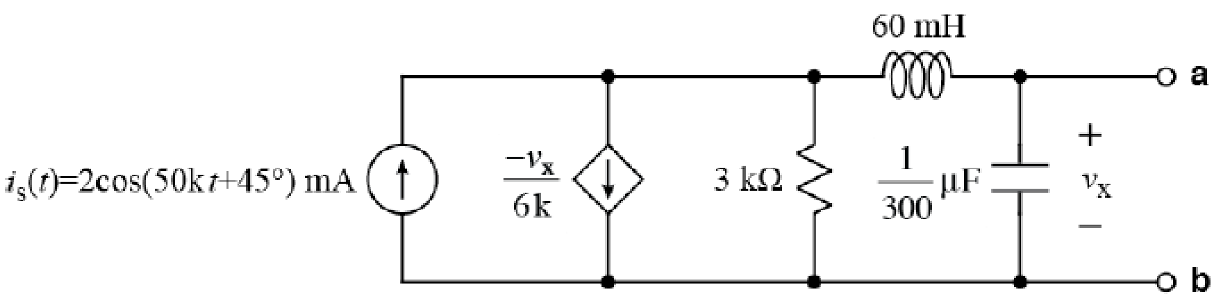
or $\frac{1}{Z_{box}} = \frac{-j50\mu}{\Omega} = \frac{-j}{\omega L} = \frac{-j}{100 \cdot L} \Rightarrow \boxed{Z_{box} = 200 \text{ H inductor}}$

Note: Either answer accepted but 20kΩ R is more sensible.

b) $\mathbf{I}_o = |\mathbf{I}| = |\mathbf{V}_g| \cdot |G_{tot}| = 12 \cdot \sqrt{2} \cdot 100\mu A = \boxed{\sqrt{2}\ 1.2\, mA \text{ for } 20k\Omega\ R}$

or $12 \cdot \sqrt{2} \cdot 50\,\mu A = \boxed{\sqrt{2}\ 600\,\mu A \text{ for } 200H\ L}$

5.



a. Draw a frequency-domain equivalent of the above circuit. Show a numerical phasor value for $i_s(t)$, and show numerical impedance values for R, L, and C. Label the dependent source appropriately.

b. Find the Thevenin equivalent (in the frequency domain) for the circuit from Problem 6. Give the numerical phasor value for $\mathbf{V}_{Th}$ and the numerical impedance value of z_{Th}.

Sol'n: a) Frequency domain values:

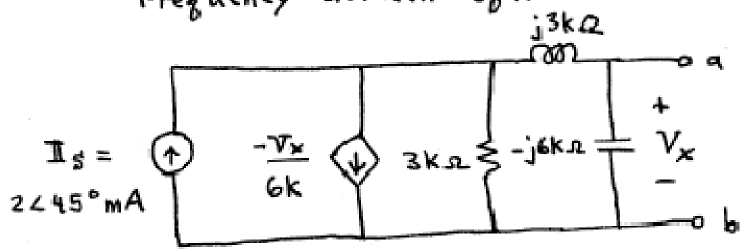
$$\mathbf{I}_s = 2\angle 45^\circ \text{ mA}$$

$$j\omega L = j\,50\text{k r/s} \cdot 60\text{ mH} = j\,3\text{k}\,\Omega$$

↑ from $i_s(t)$ frequency

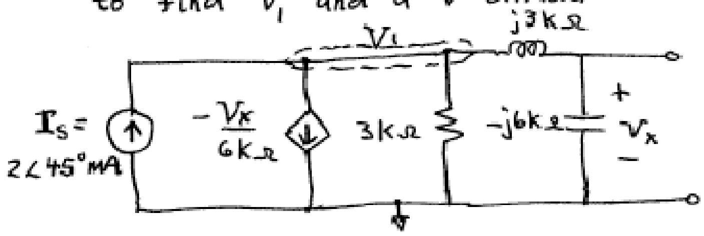
$$\frac{1}{j\omega C} = \frac{1}{j\,50\text{k} \cdot \frac{1}{300}\,\mu}\,\Omega = \frac{-j}{\frac{1}{6}\text{m}} = -j\,6\text{k}\,\Omega$$

Frequency domain equivalent circuit:



b) $V_{Th} = V_{a,b \text{ no load}} = V_{x \text{ no load}}$

One approach is to use node-V method to find V_1 and a V-divider to find $V_x = V_{Th}$:



$$V_x = V_1 \frac{-j\,6\text{k}\Omega}{j\,3\text{k}\Omega - j\,6\text{k}\Omega} = V_1 \frac{-2}{1-2} = 2V_1$$

It is interesting to note that V_x is larger than the voltage driving the V-divider.

$$\text{Node-V eq'n: } -2\angle 45^\circ\text{mA} - \frac{2V_1}{6\text{k}\Omega} + \frac{V_1}{3\text{k}\Omega} + \frac{V_1}{j3\text{k}\Omega - j6\text{k}\Omega} = 0\text{ A}$$

or

$$V_1\left(\cancel{-\frac{1}{3\text{k}\Omega}} + \cancel{\frac{1}{3\text{k}\Omega}} + \frac{1}{-j\,3\text{k}\Omega}\right) = 2\angle 45^\circ\text{ mA}$$

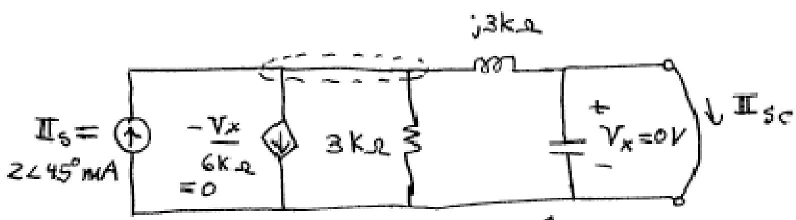
or

$$V_1 = 2\angle 45^\circ \text{ mA } (-j3k\Omega) = 2\angle 45^\circ \text{mA} \cdot 3\angle -90^\circ k\Omega$$

$$V_1 = 6\angle -45^\circ \text{ V}$$

$$V_{Th} = V_x = 2V_1 = 12\angle -45^\circ \text{ V}$$

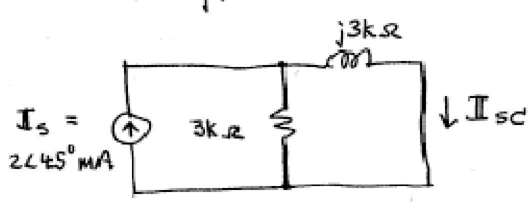
for Z_{Th}, using $Z_{Th} = \dfrac{V_{Th}}{I_{sc}}$ is convenient.



source = 0A since wire from a to b means $V_x = 0$ V

no current will flow in shorted C, so ignore C

New picture:



We have a current divider.

$$I_{sc} = I_s \frac{3k\Omega}{3k + j3k\Omega} = 2\angle 45^\circ \text{mA} \cdot \frac{1}{1+j}$$

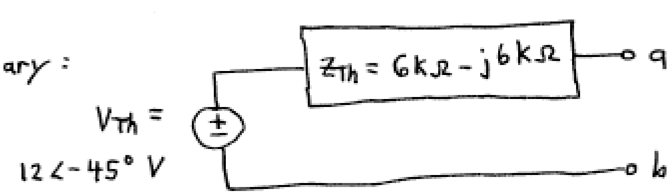
$$'' = 2\angle 45^\circ \text{ mA} \cdot \frac{1}{\sqrt{2}\angle 45^\circ} = \frac{2}{\sqrt{2}}\angle 0^\circ \text{mA}$$

$$'' = \sqrt{2}\angle 0^\circ \text{ mA}$$

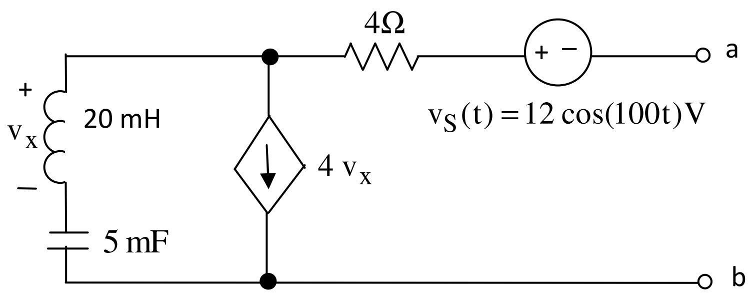
$$Z_{Th} = \frac{V_{Th}}{I_{sc}} = \frac{12\angle -45^\circ \text{ V}}{\sqrt{2}\angle 0^\circ \text{mA}} = 6\sqrt{2}\angle -45^\circ k\Omega$$

or $6k - j6k\ \Omega$

Summary:



6.



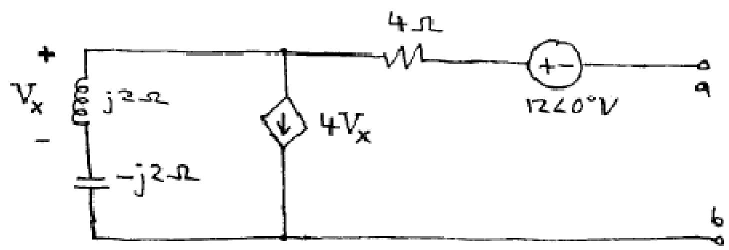
a. Draw a frequency-domain equivalent of the above circuit. Show a numerical phasor value for $v_S(t)$, and show numerical impedance values for R, L, and C. Label the dependent source appropriately.

b. Find the Thevenin equivalent (in the frequency domain) for the above circuit. Give the numerical phasor value for V_{Th} and the numerical impedance value of z_{Th}.

sol'n: a) $\omega = 100$ from $v_s(t)$ $\quad j\omega L = j\,100 \cdot 20\,\text{m}\,\Omega = j2\,\Omega$

$$\frac{-j}{\omega C} = \frac{-j}{100 \cdot 5\text{m}}\,\Omega = \frac{-j}{500\text{m}} = -j2\,\Omega$$

phasor $V_s \equiv P[12\cos(100t)]\,\text{V} = 12\angle 0^\circ$ V



b) $V_{Th} = V_{a,b}$ with no load.

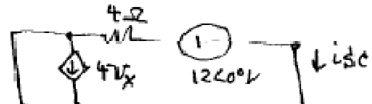
We have $z_L + z_C = 0\,\Omega$ so 0V across L & C together.

Also, no current in 4Ω ⇒ 0V across 4Ω.

Add the –12V for v-src to get $V_{Th} = -12$V

For z_{Th}, short a,b and measure i out of a terminal.

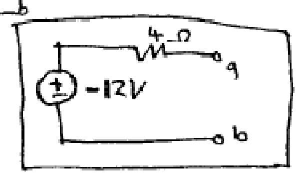
circuit model:



$4V_x$ irrelevant

$$i_{sc} = \frac{-12\text{V}}{4\,\Omega} = -3\text{A}$$

$$z_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{-12\text{V}}{-3\text{A}} = 4\,\Omega$$



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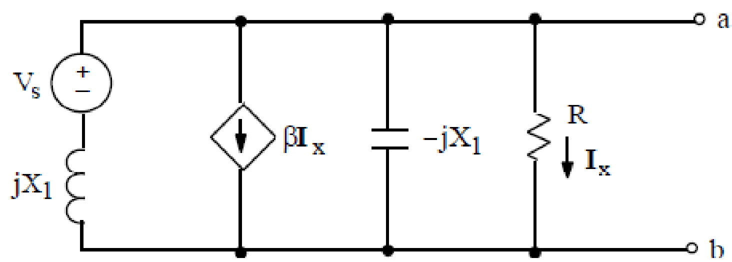
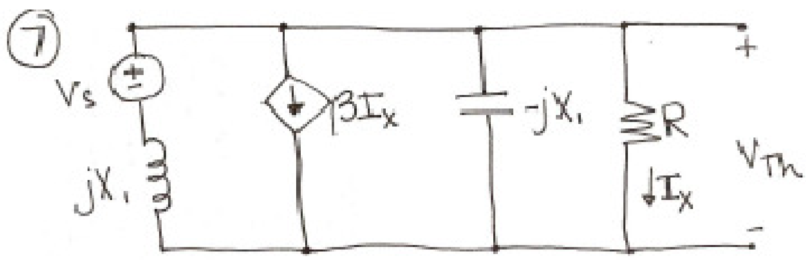


Construct a frequency-domain Thevenin equivalent circuit with respect to terminals a-b. Note that the L and C have impedances with equal magnitudes but opposite signs. Also, $\mathbf{I}_x$ must not appear in your answer.

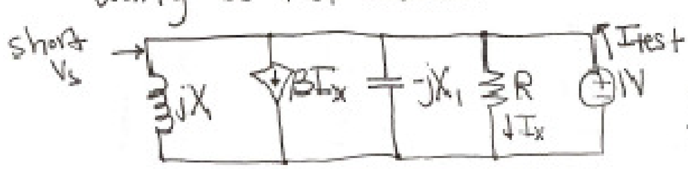


$$\frac{V_{Th}-V_s}{jX_1} + \beta \underset{\frac{V_{Th}}{R}}{\underset{\downarrow}{I_x}} + \frac{V_{Th}}{-jX_1} + \frac{V_{Th}}{R} = 0$$

$$V_{Th}\left(\cancel{\frac{1}{X_1}(-j)} + \frac{\beta}{R} + \cancel{\frac{1}{X_1}(+j)} + \frac{1}{R}\right) = \frac{V_s}{jX_1}$$

$$V_{Th} = \frac{V_s \cdot R}{X_1 j(\beta+1)} = \boxed{\frac{-V_s R j}{X_1(\beta+1)}}$$

Using a test source: $I_x = \frac{1}{R}$

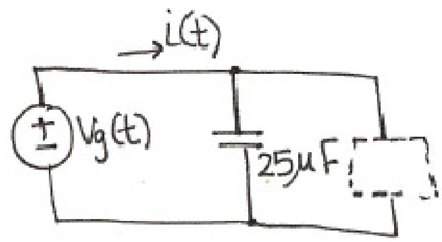


$$\frac{1}{jX_1} + \beta\left(\frac{1}{R}\right) + \frac{1}{-jX_1} + \frac{1}{R} = I_{test}$$

$$I_{test} = \frac{(\beta+1)}{R} + \cancel{\frac{(-j)}{X_1}} + \cancel{\frac{(+j)}{X_1}}$$

$$Z_{th} = \frac{1}{I_{test}} = \boxed{\frac{R}{(\beta+1)}}$$

(8)



$i(t) = 2\cos(2\text{k}t + 45^\circ)\text{A}$

$25\mu\text{F} \Rightarrow \frac{-j}{2\text{k}(25\mu)} = -20j$

$V_g(t) = 120\sin(2\text{k}t + 45^\circ) = 120\sin(2\text{k}t + 45^\circ - 90^\circ)$

$\mathbb{V}_g = 120e^{-j45^\circ}$

$$\mathbb{I} = \frac{\mathbb{V}_g}{-20j \| Z} = \frac{\mathbb{V}_g(-20j + Z)}{-20j(Z)} = \frac{120e^{-j45^\circ}(-20j + Z)}{-20j(Z)}$$

Need $\frac{(-20j + Z)}{-20j(Z)}$ to give an angle of $+90^\circ$

which is $+j$.

If R: $\angle\frac{(-20j + R)}{-20j \cdot R}$ will give $\frac{\angle -45^\circ}{\underbrace{\angle -400j}_{-90^\circ}}$ (if R=20)

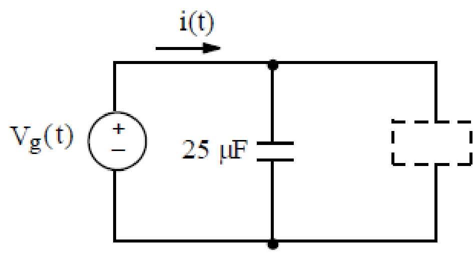
$\angle -45^\circ \angle +90^\circ = \angle +45^\circ$ (Not possible for $+90^\circ$

If C: $\frac{\angle(-20j - C_1 j)}{\angle -20j(-C_1 j)} = \frac{\angle -90^\circ}{\angle +20C_1 \underbrace{j^2}_{-1}} = \frac{\underbrace{\angle -90^\circ \angle -180^\circ}}{\angle -270^\circ}$

If L: $\frac{\angle(-20j + j(2\text{k})L_1)}{\angle \underbrace{-20j(j(2\text{k})L)}_{0^\circ}}$ will yield $+90^\circ$ if

$j(2\text{k})L_1 > +20j \Rightarrow \boxed{L > \frac{+20}{2\text{k}}}$ ~~(can't be~~

8.



$$V_g(t) = 120 \sin(2000\,t + 45°)\ \text{V}$$

Choose one R, one L, or one C to be placed in the dashed-line box to make

$$i(t) = 2\cos(2000\,t + 45°)\ \text{A}.$$

State the type and value of the component you choose.

8 cont. If C:

To get $I = 2e^{j45°} = \dfrac{120e^{-j45°}\left(-20j + \frac{-j}{2kC}\right)}{-20j\left(\frac{-j}{2kC}\right)}$

$$I = 2e^{j45°}\left(\frac{+20\cancel{j}^{\,2}}{2kC}\right) = 120e^{-j45°}\left[j\left(+20 + \frac{+1}{2kC}\right)\right]$$

$$2e^{j45°}\left(\frac{+10}{\cancel{kC}}\right)e^{j180°} = (1kC)\,120e^{-j45°}e^{-j90°}\left[\frac{20(2kC)+1}{2kC}\right]$$

$$20e^{j225°} = 120e^{-j135°}\left[\frac{20(2kC)+1}{2}\right]$$

$$\frac{20(2)}{120} = e^{-j360°}\,(20(2kC)+1) \qquad (e^{-j360°} \to 0°)$$

$$-\frac{1m}{60} = \frac{-2}{6k(20)} = \frac{\frac{1}{3}-\frac{3}{3}}{20(2k)} = C$$

(can't be negative!) NOT POSSIBLE

If L:

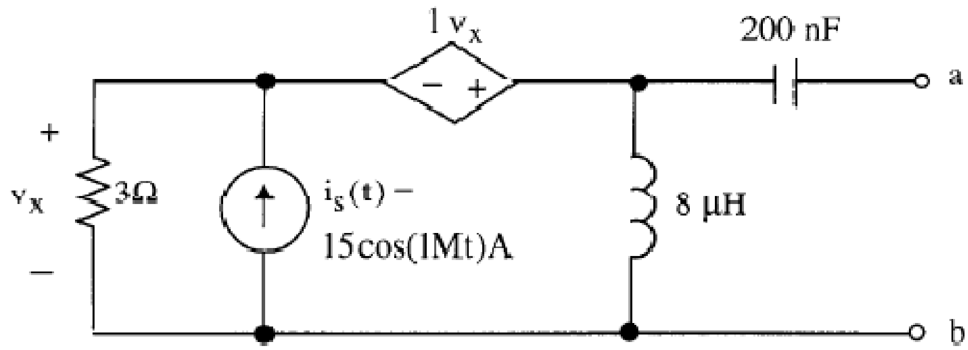
$$I = 2e^{j45°} = \frac{120e^{-j45°}(-20j + j(2k)L)}{-20j(+j(2k)L)} = \frac{120e^{-j45°}(-20j + j2kL)}{+20(2k)L}$$

$$\frac{2(20)(2k)\,L\,e^{(j45°+45°)}}{120\,e^{-j45°}} + 20j = j2kL - \frac{80k}{120}jL$$

$$20\cancel{j} = \cancel{j}\left(2kL - \frac{80k\,L}{120}\right) = L\left(2k - \frac{80k}{120}\right)$$

$$\therefore\ L = \frac{20(120)}{2k(120) - 80k} = \boxed{15\text{mH}}$$

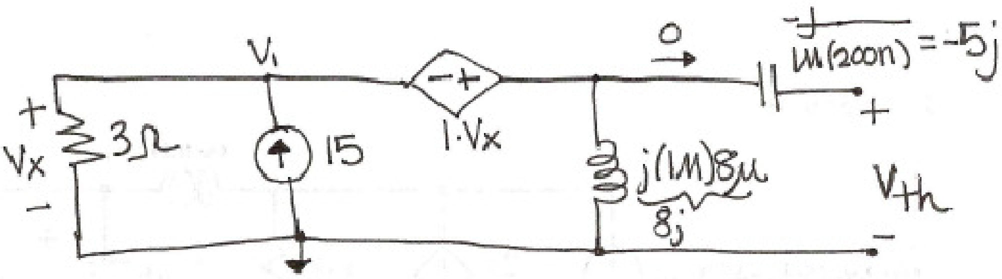
9.



a) Draw a frequency-domain equivalent of the above circuit. Show a numerical phasor value for $i_S(t)$, and show numerical impedance values for R, L, and C. Label the dependent source appropriately.

b) Find the Thevenin equivalent (in the frequency domain) for the above circuit. Give the numerical phasor value for V_{Th} and the numerical impedance value of Z_{Th}.

9



$$V_1 = V_X$$

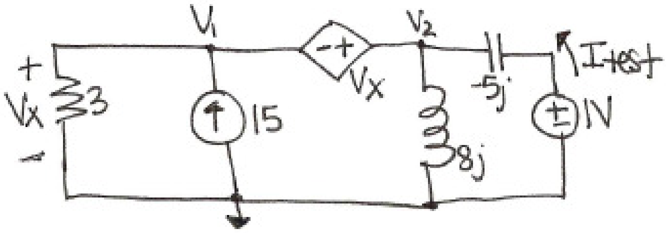
$$\frac{V_1}{3} - 15 + \frac{(V_1 + V_1)}{8j} = 0$$

$$V_1\left(\frac{8}{24} + \frac{-3j(2)}{24}\right) = +15$$

$$\therefore \quad V_1 = \frac{15(24)}{8-6j}$$

$$(+V_1 + V_1) = +V_{th}$$

$$\therefore \quad V_{th} = \frac{2(15)(24)}{\sqrt{8^2+6^2}\, e^{j\tan^{-1}\left(-\frac{6}{8}\right)}} \cong \frac{720}{10} e^{+j37°} \cong \boxed{72\, e^{j37°}}$$



$$V_2 - V_1 = V_x$$
$$V_1 = V_x$$
$$\therefore V_2 = V_x + V_x = 2V_x$$

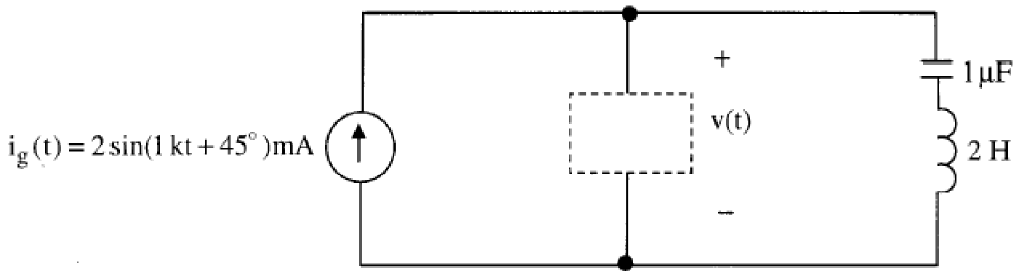
$$\frac{V_2}{8j} + \frac{V_2 - 1}{-5j} - 15 + \frac{V_1}{3} = 0 \Rightarrow \frac{2V_x}{8j} + \frac{2V_x}{-5j} + \frac{1}{5j} - 15 + \frac{V_x}{3} = 0$$

$$V_x\left[\frac{1}{4j} + \frac{2}{-5j} + \frac{1}{3}\right] = +15 - \frac{1}{5j} \Rightarrow V_x\left[\frac{-j5}{20} + \frac{8j}{20} + \frac{1}{3}\right] = \left(15 + \frac{j}{5}\right)$$

$$V_x = \frac{(15 + j/5)}{(1/3 + 3j/20)} \qquad \therefore I_{test} = \frac{1 - 2V_x}{-5j}$$

$$Z_{th} = \frac{1}{I_{test}} \cong \boxed{0.06e^{j114^\circ}}$$

10.

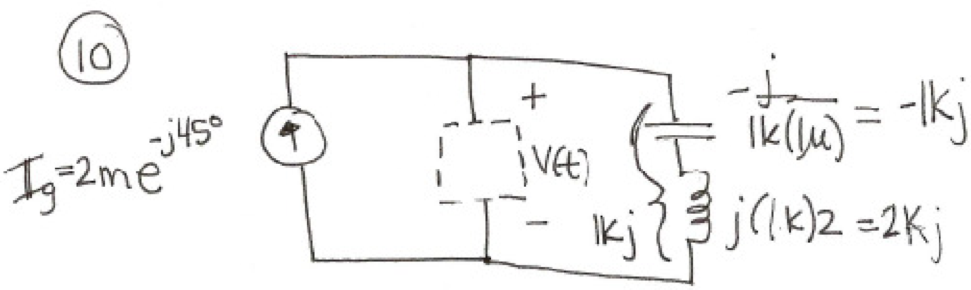


a. Choose an *R*, an *L*, or a *C* to be placed in the dashed-line box to make

$$V(t) = V_o \cos(1k\,t)$$

where V_o is a positive, (i.e., nonzero and non-negative), real constant with units of Volts. State the value of the component you choose.

b. Calculate the resulting value of V_o.



$V(t) = V_o\cos(1kt)$

$$V = \frac{2me^{-j45^\circ}(1kj)}{1kj + Z} = \frac{2me^{-j45^\circ}(1k)e^{j90^\circ}}{1kj + Z} = \frac{2e^{+j45^\circ}}{(1kj + Z)}$$

need e^{j45° to achieve $\angle 0^\circ$ of end result form.

to get e^{j45°: $1kj + Z$

$Z = \boxed{R = 1k}$

$$V = \frac{2e^{j45^\circ}}{1k\sqrt{2}\,e^{j45^\circ}} = \boxed{\frac{2}{1k\sqrt{2}} = V_o}$$